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# On combined inverse problems of geophysics for multidisciplinary earthquake prediction studies*

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A method for the determination of the integral critical parameter (precursor) of the rocks' strength and destruction has been developed using the dilatancy physical–mathematical model for the destruction of rocks in the earthquake source. This parameter is density of cracks in the unit of the volume. It is determined on the basis of the data of monitoring of anomalous geophysical fields of different nature and the numerical solution of the combined multidisciplinary inverse problems for the periodic determination of this integral precursor.

Some examples of specific combined inverse problems of geophysics for the multidisciplinary earthquake prediction studies are considered.

## 1. Basic components of the model

In the 20th century earthquakes were the cause of deaths of more than 2 million people. This is more than a half of the whole number of victims of natural catastrophes (see Appendix A). For South Asia and the countries of the western part of the Pacific Ocean there are more than 85% of deaths. In spite of intensive investigations of seismologists and the existence of expensive national and international scientific programs on the prediction of earthquakes, people continue to die from them, the prediction problem being still not solved.

Most intensive research of the prediction problems is being conducted at present by the Chinese seismologists. After the earthquake in Xingtai in 1966 they created a broad multidisciplinary observation network and studied in detail more than 60 large earthquakes and their precursors. China became the first country where the state seismologic service gives the official prediction of large earthquakes. A number of earthquakes was predicted successfully. Control of effectiveness of the prediction service, correction of

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the prediction methods and scientific investigations are being also carried out. The Chinese concept of prediction is based on the maximally reliable prediction of large earthquakes. This leads to the use of the largest possible number of precursors which provides the comprehensive control of the process of preparation of large earthquakes.

However, the use of multifactor criteria complicates the conditions of the reliable application of the statistical methods and the techniques of pattern recognition, because in this case it is necessary to have a larger volume of *a priori* information. Moreover, "logical deadlocks" appear in cases when the multifactor and multidisciplinary criteria in different combinations give contrasting estimates of the fact itself of earthquake preparation and its tempo.

Many Chinese seismologists consider that "extremely important is the development of the physical prediction method" (Mei Shirong, 1992). Russian seismologists add, "It is necessary to orient the earthquake prediction strategy to getting reliable results on the basis of unreliable individual data" (G. Sobolev, 1992).

In order to advance in these directions, we must use:

1. Special models and methods and a sufficiently representative mathematical model of mechanical and geophysical processes occurring in the zone of earthquake preparation and reflecting in the geophysical fields at the Earth's surface;
2. Physical–mathematical model for the destruction of rocks in the earthquake source;
3. Model of the earthquake prediction on the basis of methods for the determination of critical parameters of the rocks' strength using the data of monitoring of anomalous geophysical fields and the solution of the combined (multidisciplinary) inverse geophysical problems;
4. "Computational monitoring" method of the numerical solution of inverse problems for the periodic determination of integral precursors (quantitative indicators of the beginning of destruction) on the basis of data of geophysical monitoring (recording of geophysical fields at the Earth's surface).

As to the problems (1) and (2), rich literature and the well-known models exist at present. Thus, the most developed "dilatancy model" of earthquake preparation is based on the assumption that an earthquake is the result of accumulation of stresses, deformations, microcracks, with the formation of large cracks at the final stage of destruction. There are two similar models of earthquake preparation: the dilatancy model of C. Sholz

[8] (1968) and the model of V. Myachkin, G. Sobolev and others [9] (1975) from the Institute of the Earth's Physics, Russian Academy of Sciences. These models use the equation of the static elasticity theory in order to describe the process of concentration of stresses and deformations; equations of the seismic waves' propagation for the determination of velocity distribution in the earth's crust; the gravics equation and qualitative representations, dependencies of the electric conductivity and the ground water level on the extent of the volume extension of a medium with the increase of the cracks' density.

The dilatancy models mentioned above can quantitatively describe the variations of the characteristics of geophysical fields in the zone of earthquake preparation on the basis of the accumulated experience in the solution of the direct problems for these equations and the generalization of numerous laboratory experiments. So, on the basis of the general statement that an earthquake is a consequence of destruction of a medium and using the model for the accumulation of microcracks due to the increase of deformation in the stressed zone, the processes in the earthquake preparation zone can be connected with the variations of geophysical precursors. Appendix B represents the types of destruction processes. Variations of characteristics of geophysical fields at the Earth's surface are shown in Appendix C, and an example of real data of monitoring of the geophysical fields' variations is given in Appendix D (reproduced from the Journal of Earthquake Prediction). A qualitatively good agreement between the model of formations of anomalies and the real data can be seen.

However, the direct problems of geophysics give only the description of transformation of the variations of the medium properties into the changes of the fields-precursors. The reverse procedure is necessary for the earthquake prediction – passing from the fields-precursors to the mechanical parameters of the medium. Let us consider the associated problems.

It is necessary to use the quantitative and single-valued transformation of the observed anomalies into the reliable complex precursor of rheological and geometrical nature. These complex parameters cannot be measured directly and must be obtained by means of the numerical solution of the inverse problems of geophysics.

There are different ways and possibilities. Transformation of separate (individual) geophysical anomalies into some physical characteristics of a medium can be performed by solving the inverse problems of the corresponding geophysical methods. For example, the location of the zones of stresses' concentration in the crust can be determined using the recording of deformations of the Earth's surface; or the location and sizes of the zones of anomalies of seismic velocities can be determined periodically using the hodographs of seismic waves on the basis of their anomalous behaviour. There exist inverse problems on the determination of behaviour of the zones where the density of a medium (the problem of gravics) or the electric conductivity (the problem of geoelectrics) change. Such problems are very useful for the study of the process of earthquake preparation. At present a good theory of such problems and many numerical methods for their solution are available. The contribution of Novosibirsk mathematicians and geophysicists into the study of these problems is considerable [3–6].

However, there are two important questions in the problem of earthquake prediction that call for additional investigations.

Firstly, in the geophysical monitoring of a seismic prone zone, each method of observation can give only insufficient reliability and few details in the result of the solution to the corresponding inverse problem when the time and coordinates of the expected earthquakes are not definite.

Secondly, the recorded anomalies of geophysical fields which are used as precursors, form a very large and contradictory set of data. Thus, the detailed analysis of about 60 large earthquakes of China in 1966 – 1988 recorded by the multidisciplinary network of stations gives 75 different precursors (see Appendix E). Each time not all the precursors indicate the approach of an earthquake. Some of them give the opposite information. The question is, what set of precursors is true and how to transform and compress the data into a small number of the medium parameters which are "responsible" for the control of the seismic prone zone.

Answers to these questions can be found using the possibility to construct the integral precursor that can quantitatively estimate the place and time of the beginning of medium destruction. The monitoring of this integral precursor can be realized by means of the solution to the multidisciplinary inverse problem using *all the observed geophysical anomalies* as initial data.

It is reasonable to take the velocity of increase of the cracks' density in a medium as the periodically determined integral precursor. Other integral precursors also can be chosen, with no change of the scheme of further reasoning. The criterion of rock destruction consisting in the fact of the slowing of the number (and density) of cracks has deep theoretical and empirical grounding.

The general scheme of search for the rheological and geometrical integral parameters using the measured anomalies of geophysical fields is shown in Appendix F.

The difference of this problem from the usual inverse problems for individual geophysical methods lies in the fact that it uses not one physical-mathematical model and one corresponding field, but the whole totality of methods and measurements. The number $m$ of individual methods can be different in different models. If we are not afraid of computational difficulties, – the more $m$, the better.

Appendix G gives the determination of the combined inverse problem as the generalized problem for all fields. It should be noted that the set of individual inverse problems is not equivalent to one combined inverse problem. In some cases individual methods cannot uniquely solve the geophysical problem. If we combine them even without any additional information, we can sometimes get the unique, stable and reliable solution. This property is called "the property of complementability".

In Appendix H the most typical causes of "complementability" are listed. These are: similar geometry of objects when their physical properties are different; presence of the same coefficients in the equations for different methods; statistical or deterministic relations between the coefficients.

An elegant example of "complementability" was given by the Bulgarian geophysicist I. Nedialkov (1957). Appendix I shows a gravitational anomaly of a cylinder. Its depth, radius and density cannot be determined by individual gravitational methods taken separately. It is only possible to find a two-parameter family of bodies.

By analogy, the geometrical parameters and conductivity of a cylinder cannot be determined by the anomaly of electrical potential (Appendix J). But if we consider the multidisciplinary statement of the problem and remember that this is the same body, the superposition of two families gives the unique determination of all 4 unknown parameters.

The requirement formulated by G. Sobolev ("obtaining of reliable integral results from unreliable and insufficient individual data") is exactly fulfilled in this situation.

In accordance with our choice of integral precursor, the key value here is the dilatancy itself. The increase in the number of cracks will be considered to be approximately proportional to the linear extension of the elastic medium.

Volume dilatation should be computed with maximal accuracy and reliability using the property of "complementability" of other precursors of different physical nature (seismic, gravic, geoelectric).

Reduction of dilatancy and the beginning of the stage when the number of the cracks decreases is closely connected with the formation of main cracks and destruction.

The succession of time intervals $T_i$ $(i = 1, 2, \dots, n)$ is determined for the monitoring of anomalies and precursors. We must solve the multidisciplinary inverse problem in each such interval.

The following set of equations (Appendix K) is used for the multidisciplinary inverse problem:

- equation of elastodynamics for the anisotropic medium of Eshelby taking into account compressibility and gravitation by A. Love;
- equation of static elasticity for the medium of Love–Eshelby for the investigation of stress-strain;
- equation of gravitation;
- equation of electric conductivity.

The solution of the combined inverse problem for these equations may give a reliable determination of the seismic velocities $V_p$ and $V_s$ and the real value of dilatancy $\Theta(x, y, z)$. The density of cracks $e(T_i)$ in the monitoring periods can be found from here for the Eshelby medium (Appendix L) as the integral precursor $\dot{e}(T_i)$ using the optimization technique.

There exist numerous problems of testing and improvement of the medium destruction models and the choice of the most informative, short-period integral precursor. Detailed work on the testing of quantitative models by means of the processing of real materials that have been accumulated in Russia, Japan, USA and China should be organized.

## 2. The combined 2D inverse problem for seismic and gravic fields

In order to illustrate the advantage of the multidisciplinary approach, let us consider the combined inverse problem for seismic and gravic fields. The physical background of this statement is as follows: When the process of earthquake preparation begins, usually there appears the localized area of concentration of strong elastic stresses. Appearance of a lot of cracks in this area is accompanied by the increase of intensity of these stresses. This process of cracking is inevitably followed by the qualitative and quantitative changes of coefficients in the Hooke law (for example, isotropic media may transform into anisotropic media with an arbitrary level of anisotropy) and, due to the cracking, also by the change of density (because these cracks are usually empty or filled with fluids, which are more light in comparison with the background) (see Appendix F).

Here we are not going to take into account the transformation of isotropic media into anisotropic media during the cracking process and will assume that the background is isotropic with the Lame parameters $\lambda_0$, $\mu_0$ and density $\varrho_0$. Then, in the process of stress concentration (and, hence, cracking), these parameters in the localized area are, for some current moment, changing into $\lambda_0 + \delta\lambda$, $\mu_0 + \delta\mu$ and the density is changing into $\varrho_0 + \delta\varrho$. This will cause the change of the gravitational potential on the free surface (the boundary Earth–Atmosphere) which may be represented by the Newton potential:

$$\delta U(x_1, x_2) = \int_V \frac{\delta\varrho(\xi_1, \xi_2, \xi_3) dV_\xi}{\sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + \xi_3^2}} \tag{1}$$

where $V$ is the cracking area. As to the process of elastic wave propagation, it is described now (after the Fourier transform with respect to time) by the following system of differential equations:

$$\operatorname{div}\left((\mu_0 + \delta\mu)\frac{\partial(\vec{u}_0 + \delta\vec{u})}{\partial x_j}\right) + \operatorname{div}\left((\mu_0 + \delta\mu)\operatorname{grad}(u_{0j} + \delta u_j)\right) +$$
$$\frac{\partial}{\partial x_j}\left((\lambda_0 + \delta\lambda)\operatorname{div}(\vec{u}_0 + \delta\vec{u})\right) + (\varrho_0 + \delta\varrho)\omega^2(\vec{u}_0 + \delta\vec{u}) = 0, \tag{2}$$
$$j = 1, 2, 3,$$

the boundary conditions

$$l\langle\vec{u}_0 + \delta\vec{u}\rangle\big|_{x_3=0} = \vec{f}(x, y; \omega) \tag{3}$$

and the principle of vanishing attenuation. $\vec{u}_0$ in (2) and (3) is the wave field for the background free from the process of stress concentration.

If the seismic monitoring is performed, one can assume that the background (i.e., $\lambda_0$, $\mu_0$ and $\varrho_0$) is known and the problem consists in the reconstruction of the perturbations $\delta\lambda$, $\delta\mu$ and $\delta\varrho$ only by the data

$$\delta\vec{u}\big|_{x_3=0} = \delta\vec{u}_0(x_1, x_2; \omega), \quad \omega_1 \le \omega \le \omega_2, \tag{4}$$

because the elastic wave field $\vec{u}_0$ on the free surface is also known from the previous observations (i.e., one can use the perturbation of the wave field only in comparison with the previous experiments).

Using some assumptions, one can linearize (2)–(3) with respect to the background (the well-known Born approximation, for example, may be reasonable here, as usually very low time frequencies are used for the investigation of seismic prone zones), and come to the following system for the perturbations of the seismic wave field:

$$\operatorname{div}\left(\mu_0\frac{\partial\delta\vec{u}}{\partial x_j}\right) + \operatorname{div}(\mu_0 \operatorname{grad}\delta u_j) + \frac{\partial}{\partial x_j}(\lambda_0 \operatorname{div}\delta\vec{u}) + \varrho_0\omega^2\delta\vec{u} =$$
$$\operatorname{div}\left(\delta\mu\frac{\partial\vec{u}_0}{\partial x_j}\right) + \operatorname{div}(\delta\mu \operatorname{grad} u_j) + \frac{\partial}{\partial x_j}(\delta\lambda \operatorname{div}\vec{u}_0) + \omega^2\delta\varrho\vec{u}_0 \tag{5}$$

and the boundary conditions

$$l(\delta\vec{u}_0)\big|_{x_3=0} = 0. \tag{6}$$

Taking into account (4) and (5), one comes to the following representation of the perturbed wave fields on the free surface

$$\delta\vec{u}_0 = \int_V G(x_1, x_2, 0; \xi_1, \xi_2, \xi_3; \omega)\vec{R}(\xi_1, \xi_2, \xi_3; \omega)d_\xi V$$

with the Green matrix $G$ for the known background $\lambda_0$, $\mu_0$, $\varrho_0$ and $\vec{R}$ – the right-hand side of (5). If we apply the Gauss–Ostrogradsky formula, we come to the system of integral equations of the first kind with respect to the perturbations $\delta\lambda$, $\delta\mu$ and $\delta\varrho$:

$$\delta\vec{u}_0 = \int_V \tilde{G} \begin{pmatrix} \delta\lambda \\ \delta\mu \\ \delta\varrho \end{pmatrix} dV \tag{7}$$

and have the combined gravic–seismic system of linear integral equations (1), (7) of the first kind with respect to the unknown perturbations of Lame parameters and density.

**Remark.** The same approach may be applied also to a more general situation when isotropic media transform into anisotropic media. Then a more complicated system of linear integral equations of the first kind with respect to perturbations of the generalized Lame parameters is obtained. This system may be reduced in the usual way to the normalized system, and it is possible to search for its general normal solution.

In order to show some of the advantages of the seismic–gravic combined inverse problem, let us consider the simplest model: let us assume that the half-space $x_3 > 0$ is filled with the homogeneous "elastic fluid", i.e., $\mu_0 = 0$, and $\lambda_0$ and $\varrho_0$ are constants, but there is the localized area $V$, where only the density is perturbed ($\delta\lambda = \delta\mu = 0$). Next, let us consider the wave process within this medium caused by the normally incident plane wave with the spectrum of the impulse $F(\omega) \neq 0$ within the interval $(\omega_1, \omega_2)$. Then, after the Fourier transform with respect to $x_1$ and $x_2$, the system (1), (7) reduces to the following:

$$F(\omega)\int_{z_1}^{z_2} \delta\varrho(k_1, k_2; x_3)\exp\left\{ix_3(\frac{\omega}{V_p} + \sqrt{(\omega/V_p)^2 - k_1^2 - k_2^2}\right\}dx_3 =$$
$$-\frac{\varrho_0}{(\omega/V_p)^2 - k_1^2 - k_2^2}\delta\hat{u}_{03}(k_1, k_2; \omega), \tag{8}$$

$$\int_{z_1}^{z_2} \delta\varrho(k_1, k_2; x_3)\exp\left\{-x_3\sqrt{k_1^2 + k_2^2}\right\}dx_3 = \delta\hat{U}(k_1, k_2). \tag{9}$$

where $\omega \in (\omega_1, \omega_2)$ and $0 \leq k_1^2 + k_2^2 \leq \omega^2$.

It should be noted that the equation (9) itself allows us to determine uniquely the variation of density $\delta\varrho$ at least theoretically. But in trying to solve it numerically, one comes to the situation when this solution becomes crucially unstable. The reason is that from (9) one can find only the part of the spatial spectrum of $\delta\varrho$, and this part depends on the range of the time frequencies $(\omega_1, \omega_2)$. For example, for $\omega_1 > 0$ there is no possibility to recognize very smooth, slow spatial variations of the density (it is the well-known problem of the reconstruction of the trend component). But, at the same time, one can see that the second, gravic equation (9) is responsible for such type of variations and not for the high oscillating variations. So, these two equations complete each other and allow us to improve the stability of the solution and its reliability, too.

In order to solve the system (8)–(9) numerically, the approach based on the concept of $r$-solution was used. The main pecularities of this approach are the following.

Let us consider the integral equation of the first kind

$$(Kx)(s) \equiv \int_a^b K(s,t)x(t)dt = y(s), \qquad s \in [c,d], \tag{10}$$

with the operator $K$ acting from $X = L_2(a,b)$ into $Y = L_2(c,d)$. The first important problem for the numerical solution of (10) is its approximation. We used here the Galerkin technique. To do this, we choose the orthonormal bases $\{\varphi_j\}_{j=1}^{\infty}$ and $\{\psi_j\}_{j=1}^{\infty}$ in the Hilbert spaces $X$ and $Y$ with the projectors $P_m$ and $Q_m$ on the linear spans formed by the first $m$ vectors of these bases and consider the equation

$$K_m x = Q_m K P_m x = Q_m y = y^{(m)}. \tag{11}$$

Under the condition $y \in R(K) \oplus N(K^*)$, the solution of this equation converges to the general normal solution of (10). But (11) is equivalent to the system of linear algebraic equations

$$\mathcal{K}_m x = y^{(m)}$$

with the matrix $\mathcal{K}_m = \left((K\varphi_j, \psi_i)\right)_{i,j=1}^{m}$ and the right hand side

$$y^{(m)} = \left((y, \psi_i)\right)_{i=1}^{m}. \tag{12}$$

The question now is how close the solutions of (10) and (12) are to each other. Of course, there is no convergence at all, but there exists convergence of their $r$-solution, i.e., the projections of their exact solutions to the linear spans formed by the first $r$ singular vectors of these operators. Moreover, one can estimate not only the residuals, but the accuracy of these two $r$-solutions in the dependence of the accuracy of approximation, the distribution of singular values, and so on.

In order to solve (1), (7), the Haar basis was used. We used two intervals of the time frequencies: $0 \div 5$ Hz and $2 \div 5$ Hz (see Appendix M). One can see that there is no reason to apply gravics to the first situation, because there is the zero time frequency for the seismic data, which makes it possible to recognize very smooth, slow oscillations of the density. But if there are no time frequencies in the vicinity of zero, no possibility of the reconstruction of the "trend" component only by the seismic data exists, while their combination with the gravic data allows us to do this reliably.

In the solution of the individual problem of seismics (5) we obtain the condition number $10^9$. In the combined problem, the condition number in the determination of density is equal to $9 \cdot 10^7$. (This example of the solution of the combined inverse problem was given by V.A. Tcheverda.)

As for the problem of multidisciplinary earthquake prediction, there is a whole number of applications of active seismology methods using vibrosources and adaptable portable observation systems. Vibroseismic monitoring of the rheological state of seismic prone zones can be the major application here. Techniques of monitoring have two stages: detection of an anomalously stressed area in a seismic prone zone; local seismic and multi-disciplinary monitoring of the fine structure of the earthquake preparation process.

At the first stage it is sufficient to use the standard regional network of digital seismic stations (distance between them being approximately $30 \div 70$ km) and $3 \div 4$ stationary points of radiation of vibrosignals with the distance between the points of about $150 \div 200$ km.

This network will make it possible to detect the zone of anomalous accumulation of stresses and deformations in the rectangle 5 km $\times$ 10 km (the future earthquake prone zone) of the area 150 km $\times$ 150 km. This is the task of the first stage of monitoring.

At the second stage of monitoring, a more dense observational network can be organized in the zone of development of rheological anomalies detected at the first stage. Detailed observation systems with transportable seismic stations and groups of large mobile vibrators on chassis for the seismic prospecting can be created. Methods for the investigation can be taken from the well-developed field of oil geophysical exploration, both for the study of the medium structure and for the monitoring of the anomalous zone evolution (in oil seismic prospecting – evolution of the deposit during its exploitation). The methods listed above are advanced and there are facilities for observation, digital processing and visual display of three-dimensional objects in seismic tomographic movies.

Such methods give new powerful means [6] for seismology, including one of the most important and difficult problems of natural sciences – prediction of earthquakes. Gradually, even more effective means associated with the multidisciplinary approaches to the solution of the inverse problems of the complex geophysical monitoring [7] are being created.

There exist numerous problems of testing and improvement of the medium destruction models and the choice of the most informative, short-period integral precursor. Detailed work on the testing of quantitative models by means of the processing of real materials that have been accumulated in Russia, Japan, USA and China should be organized. Now we are preparing such plan and wish to discuss it with everybody who will show interest in it.

## 3. Conclusion

1. A variant of model construction of the integral earthquakes' precursor using the solution to the multidisciplinary inverse geophysical problem was considered;
2. The solution of the multidisciplinary inverse problem has the following important aspects:
   (a) use of all observational information on anomalies for the quantitative data processing;
   (b) use of the mechanism of *complementability* of methods owing to which unreliable individual data serve to increase the degree of reliability;
   (c) possibility of systematization and compression of complex information on precursors of different physical nature. Processing is carried out in order to get the most comprehensive combined precursor.
3. The scheme proposed is invariant to the choice of model variants for the evaluation of the earthquake source. It allows their classification and the "computational monitoring" of the key rheological precursors that cannot be directly recorded.
4. Testing of the model on real data by means of computational experiments can show how to form more comprehensive physical–mathematical models of earthquake prediction.

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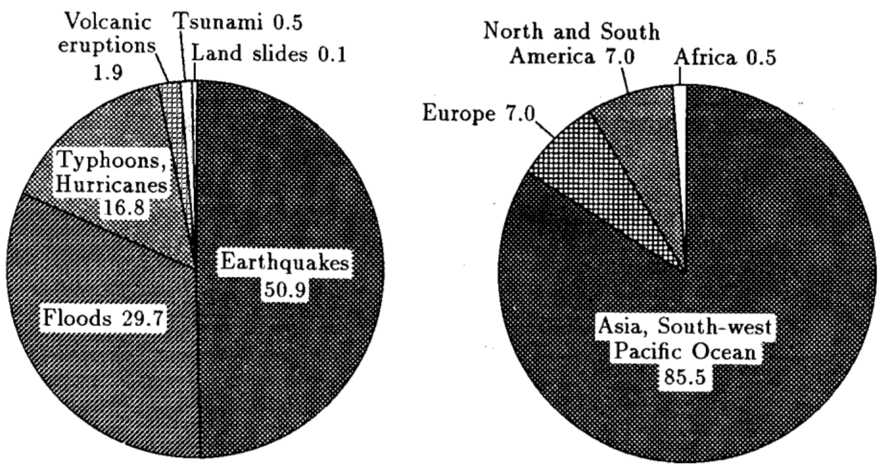
# Appendix A

## Shares of damage caused by natural disasters worldwide in the 20th century

Human Lives Lost




Share of 4.08 million total death (%)

Earthquakes, floods, typhoons and hurricanes are big disasters. Most disasters occur in Asia and South–west Pacific Ocean.

# Appendix B

## Types of destructive processes in rocks

Basic types of destructive processes:

- destruction during tension;
- formation of multiple cracks and a main fracture;
- homogeneous flow;

were established and described in the works of:

H.F. Reid (1911), A. Griffith (1920), P. Bridgman (1949),
H. Benioff (1951), K. Kasahara (1957), H. Honda (1957),
J. Eshelby (1957), K. Jida (1959) , A.V. Vvedenskaya (1961),
K. Aki (1966), K. Mogi (1967), C. Sholz (1968),
J.D. Byerlee (1970), V. Myachkin and G. Sobolev (1975),
T. Rikitake (1976), H. Kanamori (1977), etc.

Two types of models for the development of an earthquake source:

- dilatancy models (C.H. Sholz, 1968),
  (J.Ph.Z. model of V. Myachkin and G. Sobolev, 1975);
- non-regular sliding (J.D. Byerlee, 1970)

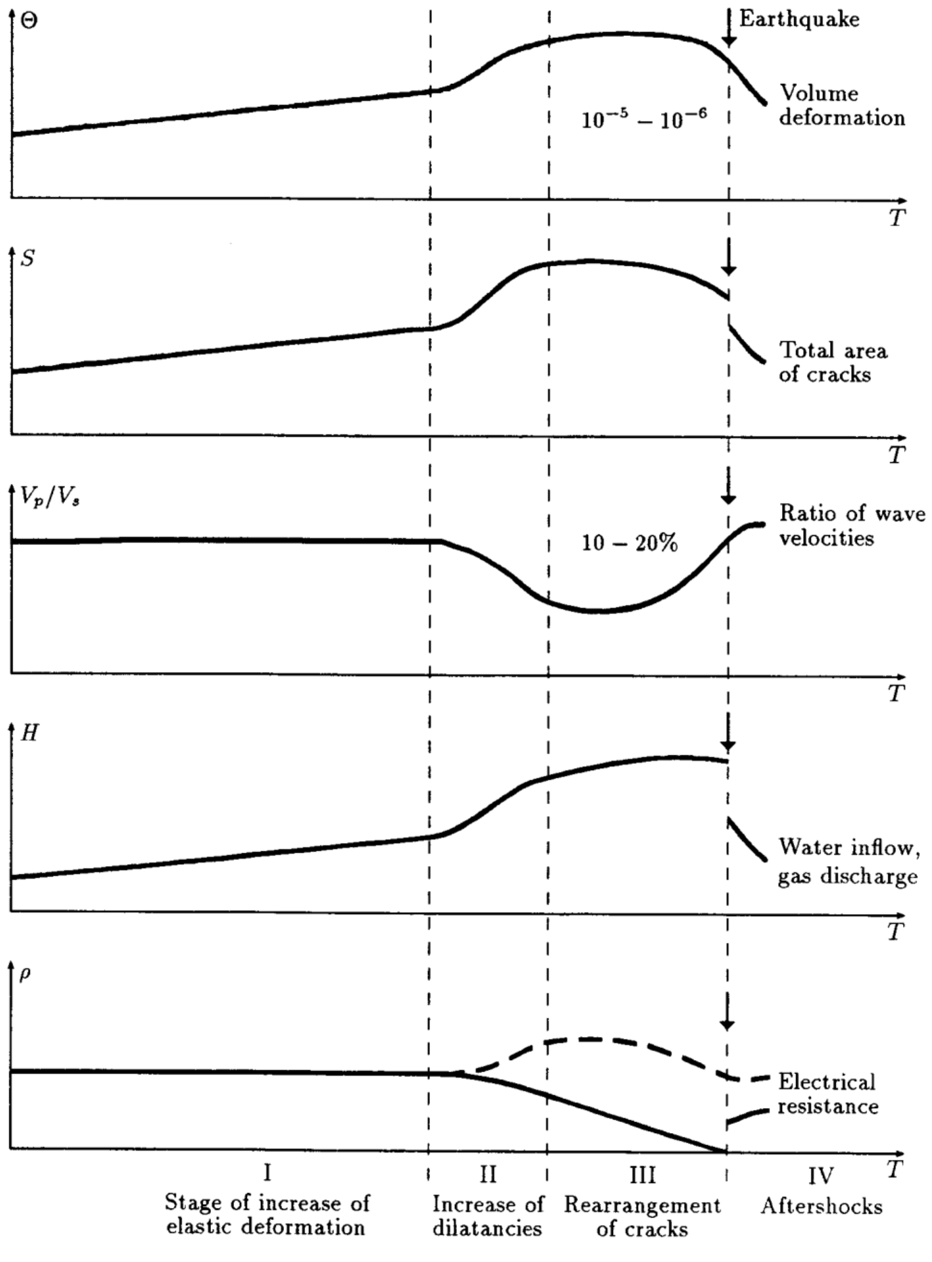
**dilatancy models** – intraplate processes
(Europe, Russia, China);

**non-regular sliding** – interplate processes
(Japan, America, subduction zones).

# Appendix C

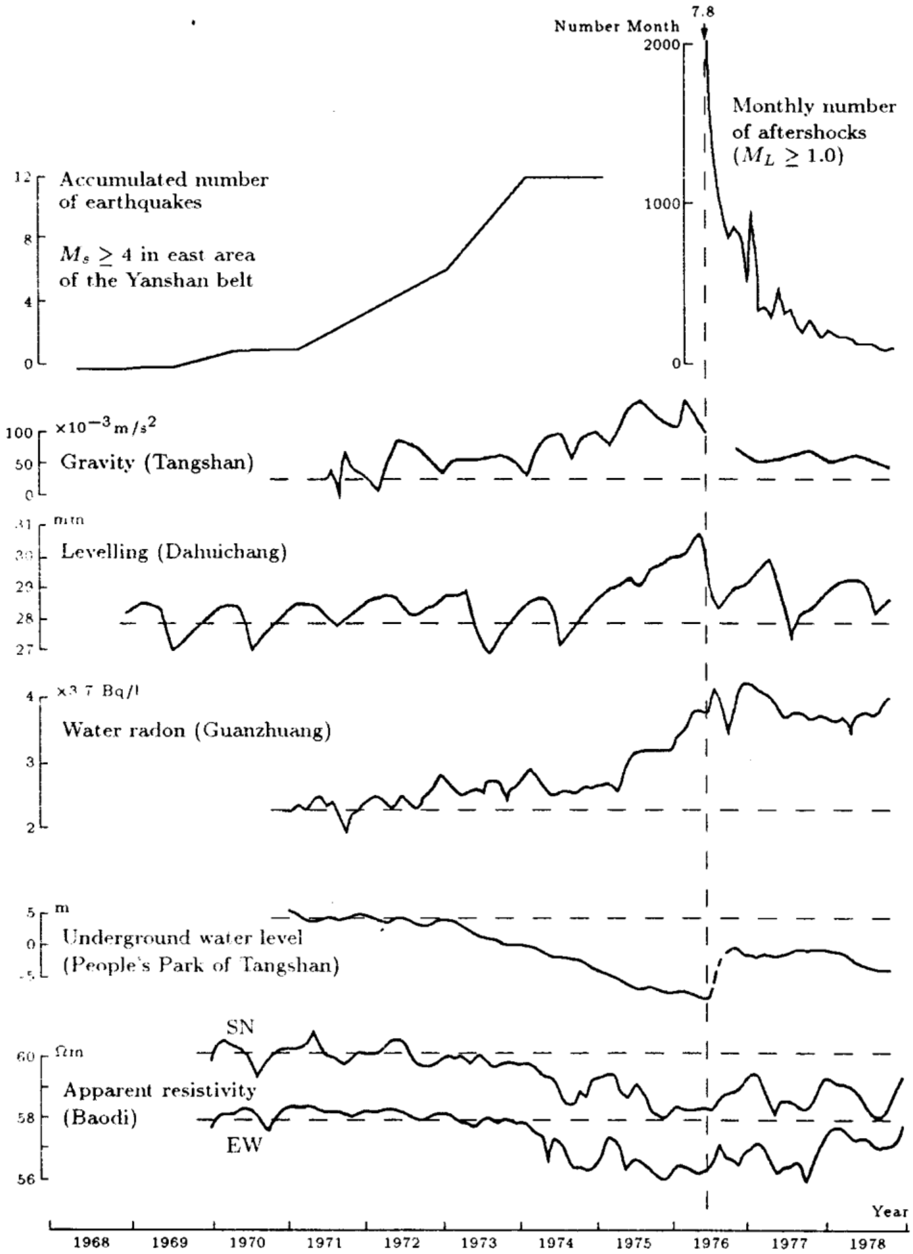
## Variations of physical characteristics in zone of earthquake preparation

(the model of the Institute of Physics of Earth of the Russian Academy of Sciences)



# Appendix D

# Data on the Tangshan earthquake [25]



# Appendix E

## The list of observation means and precursor items [25]

| Observation means | Numbers and names of precursor items |
|---|---|
| Seismometry | 1. Seismic band, 2. Seismic gap (segment), 3. Seismicity pattern, 4. Precursory earthquake (or swarm), 5. Swarm activity, 6. Area of earthquake coverage ($A$ value), 7. Strain release (energy release), 8. Earthquake frequency, 9. $b$ value, 10. $h$ value, 11. Seismic window, 12. Earthquake deficiency, 13. Foreshock, 14. Total area of fault plane ($\sum t$), 15. Index ($A(b)$ value) of earthquake regime, 16. $\gamma$ value of seismicity, 17. $\eta$ value of seismicity, 18. $D$ value of seismicity, 19. Composite fault plane solution of small earthquakes, 20. Sign contradiction ratio of $P$ wave onsets, 21. Stress drop, 22. Quality factor ($Q$ value), 23. Wave velocity, 24. Wave velocity ratio, 25. $S$ wave polarization, 26. $\tau_H/\tau_\nu$, 27. Amplitude ratio, 28. Microseisms, 29. Seismic waveform, 30. E, N and S criteria, 31. Modulation ratio of small earthquakes |
| Deformation | 32. Levelling measurements (long levelling), 33. Fixed levelling (short levelling), 34. Mobile levelling, 35. Sea level, 36. Fixed baseline (short baseline), 37. Mobile baseline, 38. Tilt |
| Gravity | 39. Fixed gravity, 40. Mobile gravity |
| Geoelectricity | 41. Apparent resistivity |
| Geomagnetism | 42. Z variation, 43. Amplitude difference, 44. Low-point shift of daily variation, 45. Distortion of daily variation, 46. Total intensity, 47. Mobile geomagnetic observation, 48. Tilt |
| Hydrochemistry | 49. Water radon, 50. Air radon, 51. Soil radon, 52. Total water hardness, 53. Water conductivity, 54. Total air flux, 55. $CO_2$, 56. $H_2$, 57. $H_2S$, 58. $SiO_2$, 59. $CL^-$, 60. $F^-$ |
| Hydrodynamics | 61. Groundwater level, 62. Groundwater level and lake water level, 63. Water (spring) flow, 64. Water temperature |
| Strain / Stress | 65. Stress (electric inductance), 66. Stress (steel-string), 67. Strain (vibrating-string), 68. Volumetric strain |
| Meteorology | 69. Air temperature, 70. Drought, 71. Waterlogging |
| Other microscopic variation | 72. Variation of oil well, 73. Geotherm, 74. Electromagnetic radiation |
| Macroscopic variation | 75. Macroscopic phenomena |

# Appendix F

## A general scheme for the determination of mechanical characteristics of a seismic prone area using geophysical anomalies with the help of the multidisciplinary combined inverse problem

(integral geophysics)

Equation system:

$$L_\nu(U_\nu, \alpha_\nu, \beta\nu) = f_\nu(x,t), \qquad \nu = 1, 2, \ldots, m.$$

Conditions:

$$l_\nu(U_\nu, \alpha_\nu, \beta_\nu)\big|_S = h_\nu(s,t); \quad U_\nu\big|_{t<0} \equiv 0, \qquad \nu = 1, 2, \ldots, m.$$

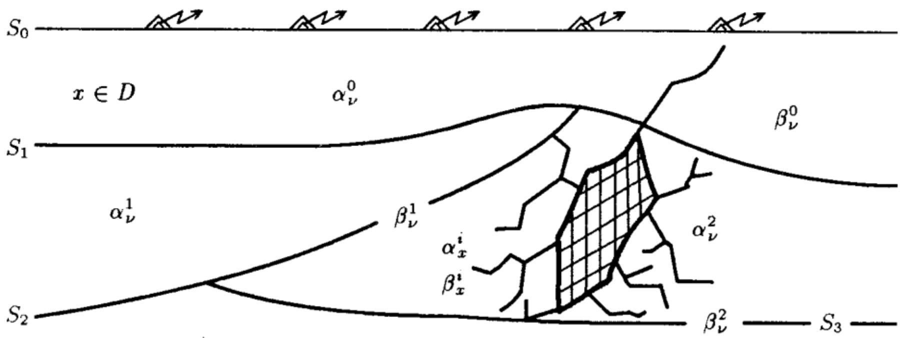
Initial information:

$$U_\nu\big|_S = U_\nu^0(s,t), \qquad \nu = 1, 2, \ldots, m.$$

It is necessary to find:

$$\alpha_\nu(x), \;\; \beta_\nu(x), \qquad \nu = 1, 2, \ldots, m.$$

One or several fields $U_\nu(x,t)$ are monitored:




$\alpha_\nu^i$ – physical parameters of the medium
$\beta_\nu^i$ – geometrical parameters of the same body

It is required to find the parameters:

$$(\alpha_x^i, \beta_x^i), \qquad i = 1, 2, \ldots, m_x,$$

and make their "computational monitoring".

# Appendix G

# Definition of combined inverse problems

The combined inverse problem is an inverse problem in which the total system of equations for $m$ fields is considered as a complex equation of the generalized geophysical process, and the totality of $m$ fields at the surface of a body is considered as the generalized initial data for it.

**Remark.** The totality of $m$ independent inverse problems is not equivalent to the combined inverse problem.

The conditions of uniqueness and stability of solutions for all individual problems do not coincide with the conditions for the combined problem. We may give many examples of individual problems which have no uniqueness. However, the corresponding combined problem has the unique and stable solution. An important feature of the combined inverse problem is the property of *geophysical complementability.*

# Appendix H

# Property of "Geophysical Complementability"

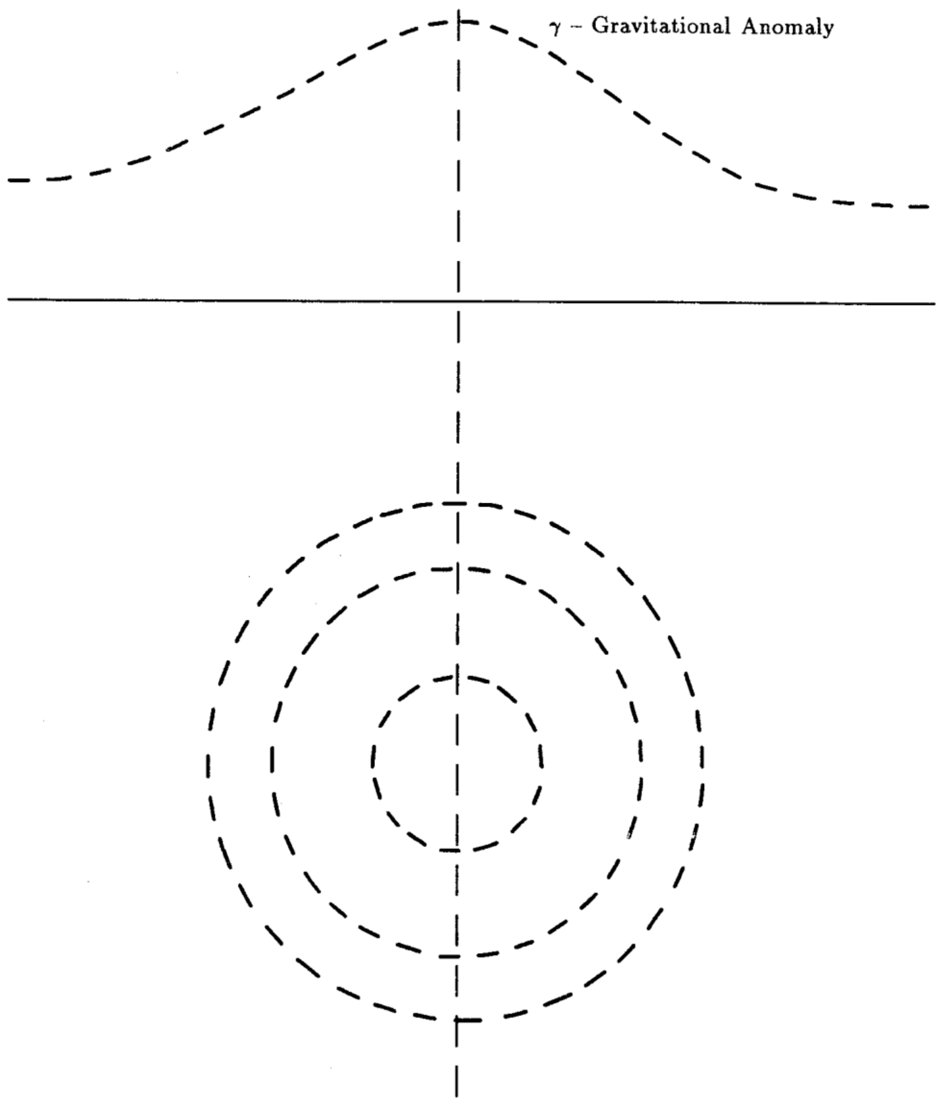
(examples)

1. A number of geometrical characteristics of geological objects for certain geophysical methods are the same (in particular, for an isolated geological object). *I.P. Nedialkov, Complex interpretation of potential fields, Dokl. BAN, Vol. 10, No 6, 1957* (see Appendices I, J).
2. A number of coefficients of mathematical equations for different geophysical methods are the same or have functionally determined relations (for example, the medium density is simultaneously present in seismic and gravitational equations). *A.S. Alekseev, B.A. Bubnov, On one combined statement of inverse problems of seismics and gravimetry, Dokl. Akad. Nauk SSSR, Vol. 251, No 5, 1981.*
3. There are statistical relations between several coefficients in the combined system of equations. *A.S. Alekseev, G.N. Erokhin, Cooperative approach to solving inverse problems in integral geophysics (theoretical aspects), Geophysical Data Inversion Methods and Application, Proceedings of the 7th International Seminar, Free University of Berlin, 1989.*

# Appendix I

## Gravitational family of cylinders equivalent to the given anomaly $\gamma$

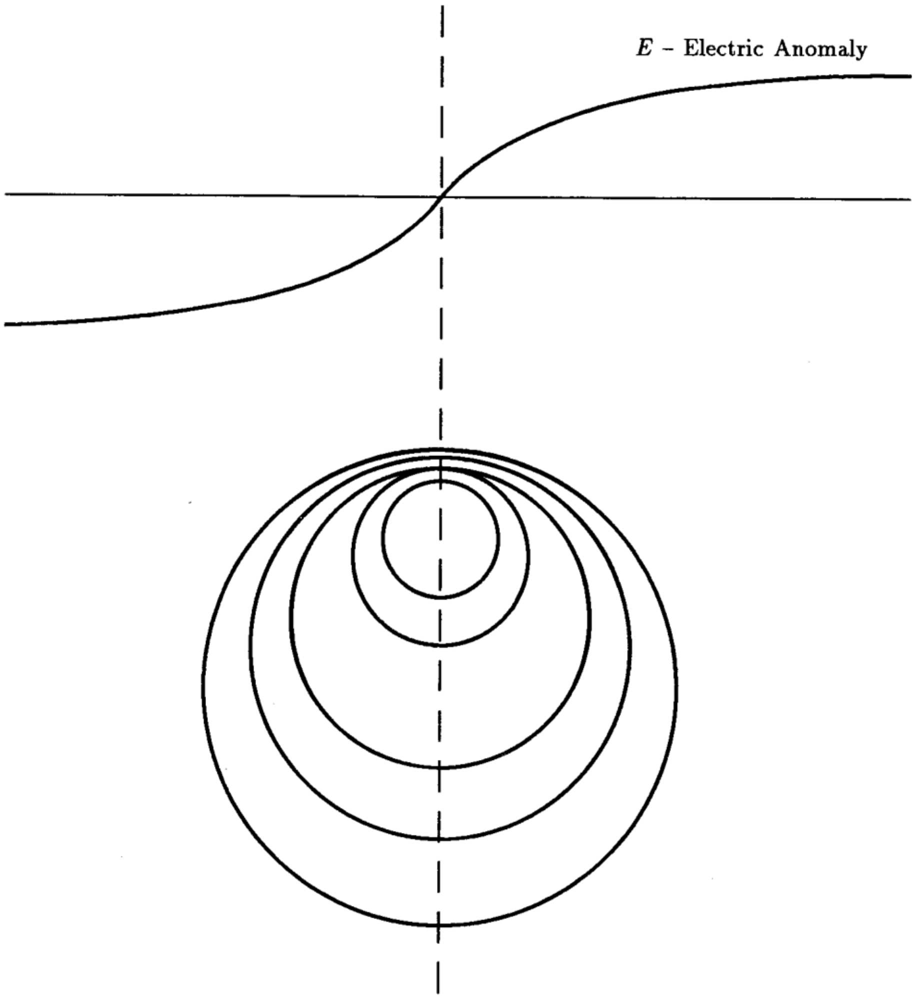
(according to I.P. Nedialkov)



# Appendix J

## Electric family of cylinders equivalent to the given anomaly $E$

(according to I.P. Nedialkov)



# Appendix K

## Equations of the multidisciplinary inverse problem for the determination of integral precursor

**1.** Equations of elasticity for the compressible gravitating medium (according to A. Love, 1911).

**1.1.** For the determination of $V_p/V_{s_1}$, $V_p/V_{s_2}$:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + g\rho\theta\delta_{i3} + \rho\frac{\partial V}{\partial x_i} = \rho\frac{\partial^2 U_i}{\partial t^2}, \qquad \sigma_{ij} = c_{ik}(\nu_s, k_s, k_f, e)e_{kj}.$$

$\sigma_{ij}$ – stresses,

$e_{kj}$ – deformations,

$\theta = e_{11} + e_{22} + e_{33}$ – elastic volume deformation,

$\rho$ – initial density of a medium,

$g$ – acceleration of gravity,

$V$ – gravitational potential,

$c_{ik}$ – parameters of the Hooke law for the initially stressed cracky medium (according to J.D. Eshelby, 1957),

$\nu_s$ – Poisson coefficient for the containing medium,

$k_s$ – Young module of volume deformation for the embedding medium,

$k_f$ – Young module for the fluid,

$e$ – volume density of cracks.

**1.2.** Equations of elastic static deformation (for the determination of stressed-deformed state in the interval $T_i$):

$$\frac{\partial \sigma_{ij}}{\partial x_j} + g\rho\theta\delta_{i3} + \rho\frac{\partial V}{\partial x_i} = 0, \qquad \sigma_{ij} = c_{ik}e_{kj}.$$

**2.** Equation of gravitational potential

$$\Delta V = 4\pi\rho G\theta.$$

$G$ is gravitational constant.

**It is necessary to find $\rho\theta$.**

**3.** Equation of electric conductivity for the case of moist cracks

$$\Delta U + (\operatorname{grad} U \operatorname{grad} \sigma) + \frac{\omega^2}{c^2}\Big(\bar{E} - i\frac{4\pi\sigma}{\omega})U = 0.$$

$U$ – electric potential,
$\sigma(x, y, z)$ – conductivity,
$\bar{E}$ – dielectric penetrability.

**It is necessary to find $\sigma(x, y, z) \approx K_e\theta$**

with an empirical coefficient $K_e$.

**Remark.** We have

$$\varphi_1 \equiv \frac{V_p}{V_{s_1}} = F_1(\nu_s, k_s, k_f, e),$$
$$\varphi_2 \equiv \frac{V_p}{V_{s_2}} = F_2(\nu_s, k_s, k_f, e),$$
$$\varphi_3 \equiv \theta = F_3(\nu_s, k_s, k_f, e)$$

where $\varphi_k$ are measured or computed and $F_k$ are computed.

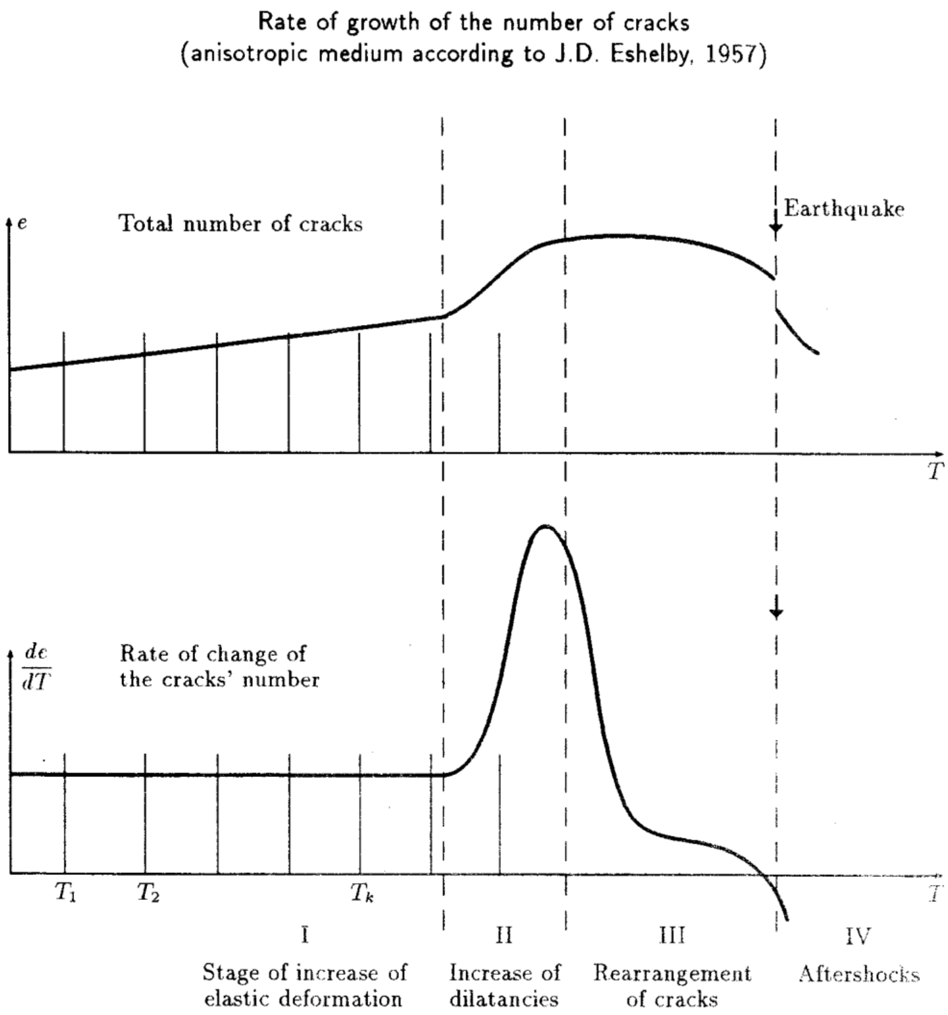
**It is necessery to find $e(T_i)$**

from

$$J = \min_e \sum_k [\varphi_k - F_k(e)]^2.$$

# Appendix L

## Integral precursor ($\dot{e}$-variant)

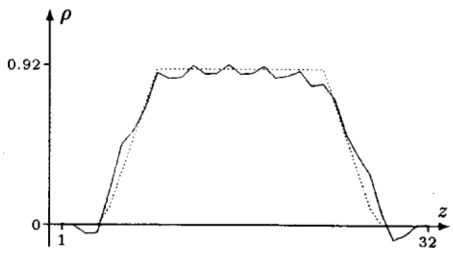


# Appendix M

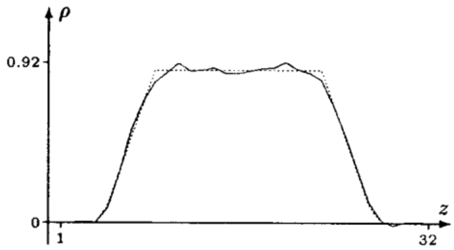
## Cooperative inversion of seismic and gravitational data

(in comparision with seismic inversion only)

Time frequencies $0 \div 5$ Hz

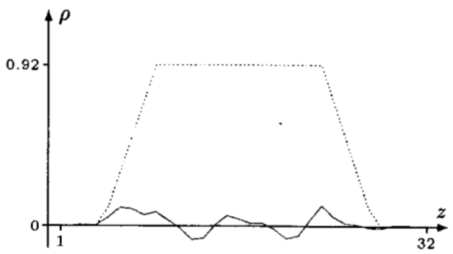



Seismic inversion

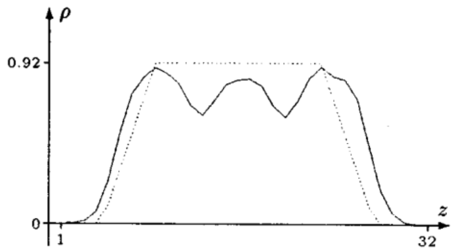



Cooperative inversion

Time frequencies $2 \div 5$ Hz




Seismic inversion




Cooperative inversion