Bull. Nov. Comp. Center, Math. Model. in Geoph., 1 (1994), 41–48

Numerical solution of one inverse problem with an unknown source wavelet*

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The problem of the simultaneous determination of the sounding signal form and wave propagation velocity in a vertically-inhomogeneous medium is considered in the paper.

It should be noted that usually the form of the sounding signal is either unknown or given only approximately, although its accurate estimate is necessary for the practical solution of many inverse problems.

We propose an interactive numerical algorithm enabling simultaneous determination of the wave propagation velocity and the sounding signal form in a vertically-inhomogeneous medium. This algorithm is based on the optimization approach, i.e., on minimization of the data misfit functional of observed and calculated data.

Results of numerical experiments are given to illustrate the efficiency of the method.

1. Introduction

The paper considers simultaneous determination of the sounding signal form and wave propagation velocity in a vertically-inhomogeneous medium. The solution of this problem is sought as the minimum point of the data misfit functional. The data misfit functional is defined as quadratic deviation of the wave field that is recorded at the free surface, from the field calculated for "experimental" values of velocity and input signal. Advantages of this approach over the other numerical methods are its universality and the capability to take into account *a priori* information on the structure of the medium under study at each step of the process of the functional minimization. The approach is described in many papers, including those of [1, 2].

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Similar statements of these problems were studied theoretically in [3, 4]. Unique solvability of the given problem to some finite depth was shown

*This work was supported in part by the Russian Fund of Fundamental Research under Grant 93–05–8638 and by the International Science Foundation under Grant NP8000.

and uniqueness theorem for the linearized statement was proved in these publications.

More complete bibliography on numerical solution of problems of simultaneous determination of the medium structure and the input signal form is given in [5]. Of particular interest is the paper [6], in which least-squares inversion is proposed for the solution of problems of this type. However, this work does not contain any numerical results.

The given paper describes an interactive numerical algorithm enabling simultaneous determination of the wave propagation velocity and the sounding signal form in a vertically-inhomogeneous medium. This algorithm is based on the optimization approach, i.e., on minimization of the data misfit functional of observed and calculated data. Results of numerical experiments are given to illustrate the efficiency of the method.

1. Wave propagation in a vertically inhomogeneous medium is described by the following initial boundary value problem:

$$\frac{\partial^2 U}{\partial t^2} = c_*^2(z)\Delta U, \tag{1}$$

$$U|_{t<0} \equiv 0, \tag{2}$$

$$\left.\frac{\partial U}{\partial z}\right|_{z=0} = f(t)g(x,y), \tag{3}$$

where $c_*(z)$ is piecewise-continuous function, characterizing wave propagation velocity in a vertically inhomogeneous medium, $f(t)$ and $g(t)$ describe, correspondingly, influence in time and space distribution of sources on the free surface. Let $z_1, \dots, z_n$ be boundaries of layers with the velocities $c_1, \dots, c_n$. For this model it is necessary to add the conditions at $z = z_i$ for relations (1)–(3):

$$[U]_{z=z_i} = \left[\frac{dU}{dz}\right]_{z=z_i} = 0, \quad i = 1, \dots, N.$$

For the system (1)–(3) let us consider the following inverse problem:

It is necessary to determine the functions $c(z)$ and $f(t)$ using the information

$$U|_{z=0} = U_0(x,y,t);\ 0 < t < T; \quad (x,y) \in S \subset \{(x,y,z):\ z = 0\} \tag{4}$$

on the oscillation regime of the observation surface $z = 0$.

Solution of this problem will be sought as the minimum point of the following data misfit functional:

$$\Phi[c^2(z)] = \int_0^T dt \int_S \{U_0(x,y,t) - B[c^2(z), f(t)](x,y,t)\}^2 dxdy, \tag{5}$$

where $B[c^2(z),\ f(t)]$ is the operator, transforming the functions $c(z)$ and $f(t)$ into the solution of the direct problem (1)–(3) at $z = 0$ (wave field on the set S).

2. Semi-analytical method, described in [7], was chosen for the solution of the direct problem (1)–(3).

Using this method, the initial statement in (1)–(4) will be rewritten in the following form:

$$\frac{d^2U}{dz^2} = \nu^2 U, \tag{6}$$

$$\left.\frac{dU}{dz}\right|_{z=0} = F(\omega), \tag{7}$$

where

$$\nu^2 = k^2 - \omega^2/c^2(z), \quad F(\omega) = \int_0^T f(t)\exp(-i\omega t)dt,$$

and for the unique solution it is supposed that the principle of limiting absorption is fulfilled, i.e.,

$$U(z,k,\omega) = \lim_{\varepsilon\to+0} U(z,k,\omega - i\varepsilon),$$

where $U(z,k,\omega - i\varepsilon) \to 0$ as $z \to \infty$.

As it was shown in [1], realization of the search process of the minimum point of the data misfit functional in the frequency range (k,ω) enables considerable reduction of computational resources, necessary for the multiple solution of the direct problem. Moreover, it is possible to make detailed analysis of spectra of the wave fields at each step of calculations.

In this connection, assuming that the recording time T and the size of the observation domain S on the free surface are sufficiently large, we rewrite the functional (5) in the equivalent form

$$\Phi[n^2(z)] = \int_{\omega_1}^{\omega_2} d\omega \int_{k_1}^{k_2} \left|U_0(k,\omega) - \hat{B}[n^2(z), F(\omega)](k,\omega)\right|^2 kdk, \tag{8}$$

where (ω_1,ω_2) and (k_1,k_2) are time and space frequency ranges, determined by the spectral composition of the sounding signal $F(\omega)$ and the size of the observation domain S. $\hat{B}[n^2(z),\ F(\omega)]$ is the operator transforming the functions $n^2(z) = c^{-2}(z)$ ($c(z)$ is current wave propagation velocity) and $F(\omega)$ (spectrum of the sounding signal) into the solution of the direct problem (6), (7) at $z = 0$.

3. The set of functions $\{n^2(z), F(\omega)\}$, where

$$\begin{aligned}
&0 < \delta \le n^2(z) \le N < \infty, \quad \lim_{z\to\infty} n^2(z) = n_0^2,\\
&n^2(z) - n_0^2 \in L_2(0,\infty) \cap C^1(0,\infty),\\
&F(\omega) \in L_2(\omega_1, \omega_2),
\end{aligned}$$

will be called the set of models M.

Image of the set M under the action of the operator $\hat{B}$ will be called the set of acceptable data D ($D = \hat{B}(M)$).

After introducing scalar products on M and D, the data misfit functional (8) can be rewritten in the following form:

$$\Phi[n^2(z), F(\omega)] = \left(U_0 - \hat{B}[n^2(z), F(\omega)], U_0 - \hat{B}[n^2(z), F(\omega)]\right)_D.$$

Using the method of the conjugate Green function [8] we obtain expressions for the gradients of the data misfit functional (8) over velocity and impulse:

$$\begin{aligned}
\nabla_n^2 \Phi[n^2(z), F(\omega)](\xi) = -2Re \int_{\omega_1}^{\omega_2} (\omega + i\varepsilon)^2 \bar{F}(\omega) d\omega \\
\times \int_{k_1}^{k_2} \left[U_0(k,\omega) - \hat{B}[n^2(z), F(\omega)](k,\omega)\right] \bar{G}^2(\xi, k, \omega) k dk,
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\nabla_F \Phi[n^2(z), F(\omega)](\xi) =\\
&\quad -2Re \int_{k_1}^{k_2} \left[U_0(k,\omega) - \hat{B}[n^2(z), F(\omega)](k,\omega)\right] G(\xi, k, \omega) k dk\\
&\quad - 2i \cdot Im \int_{k_1}^{k_2} \left[U_0(k,\omega) - \hat{B}[n^2(z), F(\omega)](k,\omega)\right] G(\xi, k, \omega) k dk,
\end{aligned} \tag{10}$$

where $G(\xi, k, \omega)$ is the solution of the problem (6), (7) at $F(\omega) = 1$.

4. First-order optimization methods of descent were used for the realization of the search process of the minimum point of the data misfit functional:

a) method of steepest descent (for the impulse spectrum):

$$F_{j+1}(\omega) = F_j(\omega) - \alpha_j \nabla_F \Phi[n^2(z), F_j(\omega)],$$
$$\alpha_j = \arg\min_{a \geq 0} \Phi \left[F_j(\omega) - \alpha \nabla_F \Phi[n^2(z), F_j(\omega)]\right];$$

b) method of conjugate gradients (for velocity):

$$n_{j+1}^2(z) = n_j^2(z) - \alpha_j P_j(z),$$
$$\alpha_j = \arg\min_{a \geq 0} \Phi \left[n_j^2(z) - \alpha P_j(z)\right],$$
$$P_0(z) = \nabla_n^2 \Phi \left[n_0^2(z)\right], \quad P_j(z) = \nabla_n^2 \Phi \left[n_j^2(z)\right] - \beta_j P_j(z), \ j \geq 1,$$
$$\beta_j = \left(\nabla_n^2 \Phi[n_j^2(z)], \nabla_n^2 \Phi[n_{j-1}^2(z)] - \nabla_n^2 \Phi[n_j^2(z)]\right).$$

The step α_j in the both methods was chosen using the method of "golden section".

These methods proved themselves to be very efficient in the solution of a wide range of problems.

Program package GEOMIN [9] was used for the calculations. Program blocks were written for it that make it possible to organize the iterative process of simultaneous determination of the sounding signal form and wave propagation velocity in a vertically-inhomogeneous medium.

The chosen velocity model of the medium is shown in Figure 1. The impulse in the spectral domain was given in the following form:

$$F(\omega) = \left[\exp\left(-\left(\frac{\omega - 2\pi f}{\pi f}\right)^2\right) + \exp\left(-\left(\frac{\omega + 2\pi f}{\pi f}\right)^2\right)\right] \times \exp(-i \cdot 1.75\omega/f). \tag{11}$$

Initial approximation for the impulse was chosen to be zero and for the velocity it was chosen in the form of the linear function (Figure 1).

All calculations were made for the time frequency range from 10 to 40 Hz, typical for seismic prospecting.

The method, described in [1], was used, i.e., space frequency ranges were chosen that provide the best convergence of the iterative process to the minimum point of the data misfit functional.

The first approximation for the velocity and impulse is shown in Figures 2–4. The final results of calculations are shown in Figures 5–7.

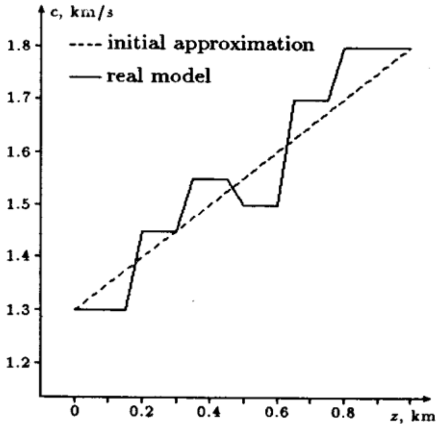

Figure 1

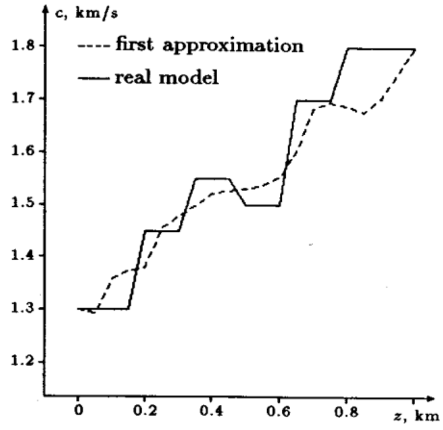

Figure 2

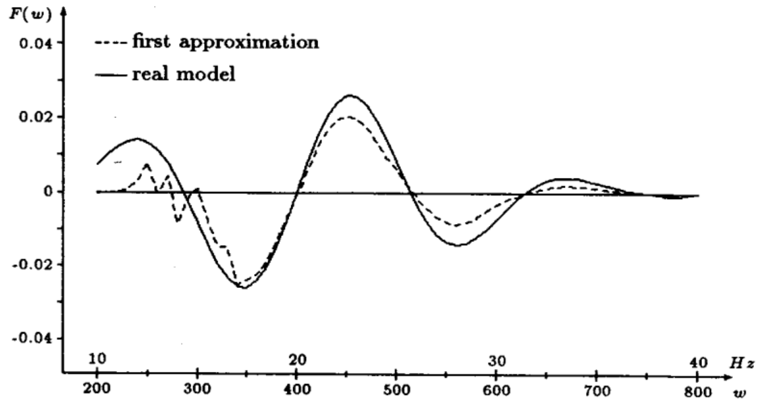

Figure 3. Real part of the wavelet spectrum

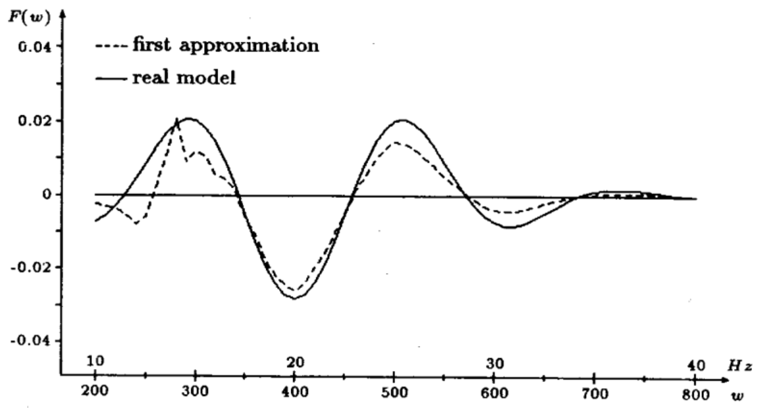

Figure 4. Imaginary part of the wavelet spectrum

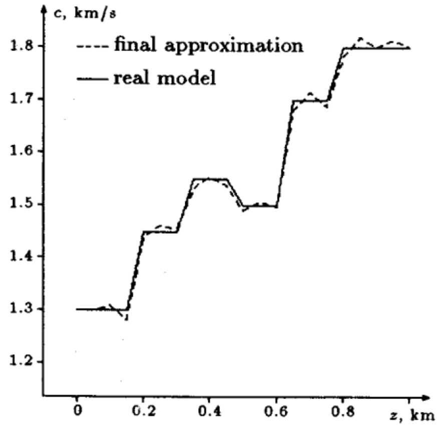

Figure 5

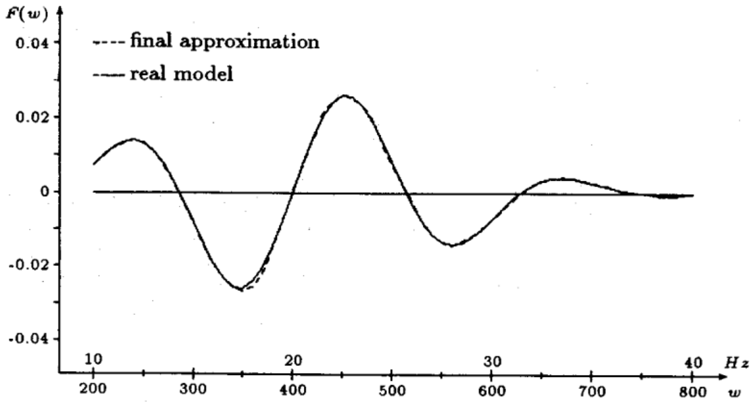

Figure 6. Real part of the wavelet spectrum

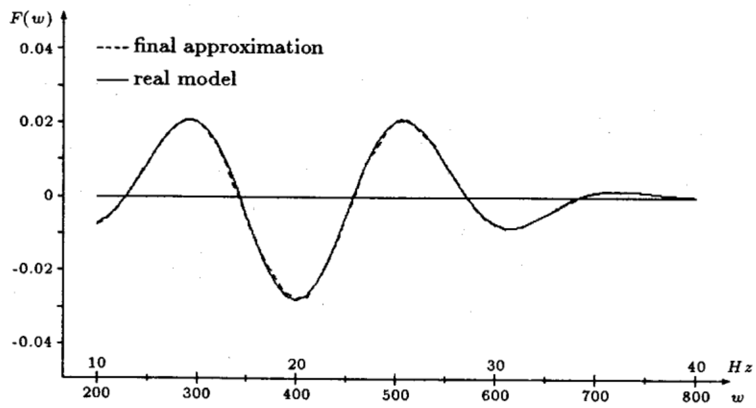

Figure 7. Imaginary part of the wavelet spectrum

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