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Numerical simulation of various scenarios of nonlinear evolution in a beam-plasma system*

E.A. Berendeev, A.A. Efimova, V.A. Vshivkov

Abstract. This paper is aimed at studying the efficiency of the electromagnetic radiation generation in various nonlinear processes occurring during the beam-plasma interaction. The beam and plasma parameters were chosen close to the parameters in the experiment on the GOL-3 facility (BINP SB RAS). The research was conducted by means of the numerical model based on the particles-in-cell (PIC) method. The model of collisionless plasma is described by the system of the Vlasov–Maxwell equations, boundary conditions being periodic. Various scenarios of the nonlinear evolution in a beam-plasma system with a beam of low density were studied. It is shown how these scenarios change under the influence of an external magnetic field. There is a transfer of energy from one unstable mode to another.

1. Introduction

The efficiency of generation of electromagnetic radiation in various nonlinear processes occurring during the beam-plasma interaction has been studied based on the computer simulation. This problem is urgent both for laboratory experiments on turbulent plasma heating in open traps, and for the interpretation of various phenomena in space plasma (the solar and gamma flares, radiation in magnetospheres of planets, generation of high-energy cosmic rays). Open magnetic traps are one of the directions in solving of the controlled thermonuclear fusion problem, and one of advantages of these systems against the closed configurations consists in the possibility of introducing into plasma of high power electron beams. In particular, the injection of a weakly relativistic beam into plasma on the GOL-3 facility (the Budker Institute of Nuclear Physics, SB RAS) (see [1]) leads to the excitation of strong Langmuir turbulence and consequent plasma heating up to the temperature of 2–3 keV for a few microseconds. It was found that a significant part of the energy lost by a beam can be spent on the generation of electromagnetic radiation.

In this paper, the 2D numerical model based on the PIC-method is described. In this model the kinetic description of plasma and an electron

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beam allowing the exploration of the dynamics of the studied processes from uniform positions is used. The system of equations consists of the Vlasov and Maxwell's kinetic equations. The beam and plasma parameters were chosen to be close to the parameters in the experiment on the GOL-3 facility. Various scenarios of the nonlinear evolution in a beam-plasma system, transfer of energy from one unstable mode to another, and the influence of an external magnetic field in the case of a low-density beam have been studied. The convergence of the solutions of calculating parameters was investigated. A good agreement of the numerical and the analytical solutions was obtained.

2. The problem statement

A most comprehensive investigation of the plasma processes can be carried out only through a complex approach that combines both experimental studies and research computing procedures, which adequately describe these processes. It is necessary to construct an adequate mathematical model of the physical processes in order to avoid simplifications and to obtain a qualitatively correct physical picture. The model uses the collisionless plasma approximation [2]. The plasma is described by the system of the Vlasov–Maxwell equations:

$$\frac{\partial f_k}{\partial t} + (\vec{v}, \nabla) f_k + q_k \left(\vec{E} + \frac{1}{c}[\vec{v} \times \vec{B}] \right) \frac{\partial f_k}{\partial \vec{p}} = 0,$$

$$\operatorname{rot} \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}, \qquad \operatorname{div} \vec{B} = 0,$$

$$\operatorname{rot} \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \qquad \operatorname{div} \vec{E} = 4\pi\rho,$$

and

$$\vec{j} = \sum_k q_k \int \vec{v} f_k(\vec{p}, \vec{r}, t)\, d\vec{p}, \qquad \rho = \sum_k q_k \int f_k(\vec{p}, \vec{r}, t)\, d\vec{p}, \tag{1}$$

where f_k is the particle distribution function of the species k (beam electrons, plasma electrons and ions), $\vec{B}$ is the magnetic field, $\vec{E}$ is the electric field, c is the speed of light, $\vec{v}$ is the velocity of the particles, ρ is the electric charge density, $\vec{j}$ is the electric current density and q_k is the charge of the particle of the species k.

Equations (1) define the current and the charge densities by the particle distribution functions. It is assumed that all magnitudes depend on the spatial Cartesian coordinates (x, y), i.e. a 2D non-stationary problem is solved. For the transition to a dimensionless form as the main variables the following was used: the speed of light c, the electron mass m_e, the time $t = \omega_{pe}^{-1}$, where ω_{pe} is the plasma electron frequency.

At the initial time, there is a hydrogen plasma and an electron beam in the rectangular box decision area: $x \in [0, L]$, $y \in [0, L_\perp]$.

The initial distribution of the beam electron velocities is the Maxwellian:

$$f(v) = \frac{1}{\Delta v\sqrt{2\pi}} \exp\left(-\frac{(v-v_0)^2}{2\Delta v^2}\right),$$

where Δv is the particle velocity dispersion ($T_b = \Delta v^2$), v_0 is the mean velocity. All particles in the area have a uniform distribution. The boundary conditions are periodic, i.e. $F|_{x=0} = F|_{x=l_x}$, $F|_{y=0} = F|_{y=l_y}$, where F is any of the following quantities $\vec{E}$, $\vec{B}$, f_k, $\vec{j}$, ρ.

3. The main equations solution

The PIC-method (see [2, 3]) is used for solving the Vlasov equation. In this method, the plasma is simulated by a set of separate particles, each being characterized by the motion of many physical particles. The characteristics of the Vlasov equation describe the trajectories of the particle motion. The equations of these characteristics are described as

$$\frac{d\vec{p}_{i,e}}{dt} = q_k(\vec{E} + [\vec{v_{i,e}}, \vec{B}]), \qquad \frac{d\vec{r}_{i,e}}{dt} = \vec{v}_{i,e}, \qquad \vec{p}_{i,e} = \frac{\vec{v}_{i,e}}{\sqrt{1-\vec{v}^{\,2}_{i,e}}}.$$

To solve these equations, the following lip-frog scheme is used.

The density is calculated according to the position of a particle, the currents— by the continuity equation as shown in [4].

Maxwell's equations are solved with the Langdon–Lazinsky scheme [5]:

$$\frac{B^{m+1/2} - B^{m-1/2}}{\tau} = -\operatorname{rot}_h E^m,$$
$$\frac{E^{m+1} - E^m}{\tau} = -j^{m+1/2} + \operatorname{rot}_h B^{m+1/2}.$$

In this scheme, the electric and magnetic fields are calculated on the grids displaced as related to each other with respect to time and space that allows attaining the second order of accuracy: B is calculated at a fractional time step, and E is calculated at the whole time step.

At the first (Lagrangian) phase, the velocities and coordinates of particles are calculated with a lip-frog scheme. So, components of the current density $j^{m+1/2}$ and the charge density ρ^{m+1} are obtained. At the second (Euler) step, Maxwell's equations are solved, i.e. the values of $B^{m+1/2}$ and E^{m+1} grid nodes are determined. The details of solving the basic equations are described in [6].

4. Simulation results

The objective of this research is to create a numerical model and a program to adequately describe various scenarios of the nonlinear evolution in a beam-plasma system.

The simulation parameters were chosen close to the parameters in the experiment on the GOL-3 facility: the density ratio $\frac{n_b}{n_p} = 0.002$; the beam temperature $T_b = 10$ eV; the ratio of the beam speed to the speed of light $\frac{v_b}{c} = 0.382$; the plasma electron temperature $T_e = 60$ eV; the plasma ion temperature $T_i = 0$ eV; the magnetic field $\Omega = 0,\ 0.5,\ 2$, where $\Omega = \frac{w_c}{w_p}$, $w_c = \frac{eB}{m_e c}$, $w_p = \left(\frac{4\pi e^2 n_p}{m_e}\right)^{1/2}$; the area size $L_x \times L_y = 7.2 \times 7.68\ \frac{c}{w_p}$; the wave number $k_x = \frac{6\pi}{L_x} = \frac{w_p}{v_b}$; and the ion mass to the electron mass ratio $m_i/m_e = 1836$.

The PIC-method requires a large number of the model particles to simulate the plasma instabilities. To reproduce the resonant interaction of a relativistic electron beam with plasma, a sufficiently detailed spatial grid must be taken. The monoprocessor memory and time resources are insufficient to perform the numerical simulations with a large number of particles and a small spatial step. The efficient scalable parallel algorithm with the mixed Euler–Lagrangian decomposition [7] was developed. A group of processors is associated with each subdomain, and the particles are divided among the processors group within each subdomain. A group of processors solves the equations for the fields only in the subdomain. In this case, there is an exchange of the boundary field values between the groups. Also, the group should exchange with the particles, which move to other subdomains. It is necessary to exchange the current density values within a group.

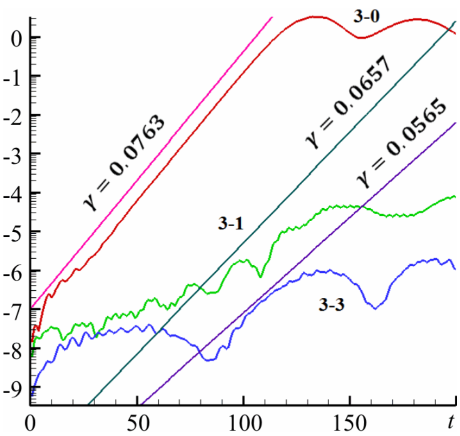

Figure 1. The Fourier harmonics time history for $\Omega = 0.5$ in the logarithmic scale

The characteristic parameters of the calculations on a supercomputer: 4 subdomains, 128 processor cores, 360×384 grid, the grid steps $\Delta x = \Delta y = 0.02$; the number of particles in a cell is 1,000–2,500; the time step $\tau = 0.01$; and the number of time steps from 10^5 to $8 \cdot 10^5$.

Figure 1 shows a good agreement between the numerical and the analytical results for the rate of growth of the field amplitude. In this figure, the straight lines correspond to the analytical value of the increment.

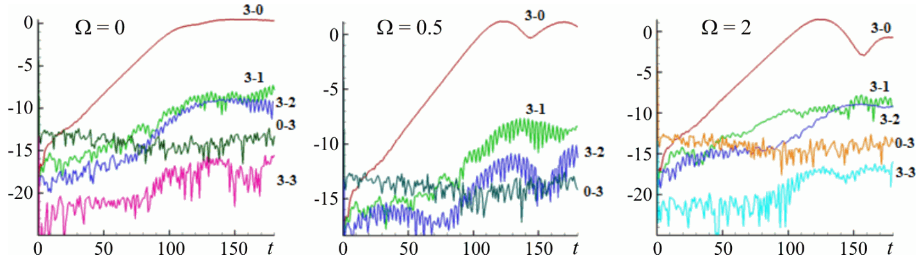

Figure 2. The Fourier harmonics time history for different values of the initial magnetic field

At the initial stage ($t < 200w_p^{-1}$), which corresponds to an increase in the amplitude of the field, the main contribution to the solution making one particular harmonic (3-0 mode) for different values of the initial magnetic field is shown in Figure 2.

Then, in the course of time, the energy is transferred from the main unstable mode to other ones. At $t > 200w_p^{-1}$, the values of the mode 3-0 reduces, and the values of other modes eventually increase (Figure 3).

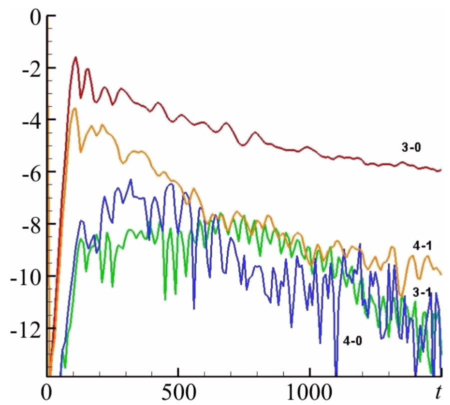

Figure 3. The Fourier harmonics time history for $\Omega = 0.5$

5. Conclusion

In this paper, the developed model and the program for the simulation of nonlinear effects in plasma is proposed. The considered problem is sufficiently resource-intensive, but the developed scalable parallel algorithm providing a uniform loading of cores allows one to perform complex calculations. By means of the developed program it was succeeded to simulate various problems whose solution requires using a large area and a huge number of particles in a cell. The simulation of different scenarios of the nonlinear evolution in beam-plasma system with a low density beam has been carried out. It appeared possible to reproduce the spectral energy transfer process from the main unstable mode to another. A good agreement between the calculated and the analytical results has been obtained.

Calculations were performed at the Siberian Supercomputer Center of the Institute of Computational Mathematics and Mathematical Geophysics SB RAS and at the Supercomputer Center of the Lomonosov Moscow State University.

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