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Compositional methods in characterization of timed event structures*

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Abstract. In this paper we use compositional methods for construction of a characteristic formula for the timed testing preorder in a model of timed event structures with discrete internal actions.

1. Introduction

Complex systems are not trivial for analysis. One of useful tools for that is the notion of equivalence. As a matter of fact, equivalences are used in specification and verification both to compare two distinct systems and to reduce the structure of a system. Over the past several years, a variety of equivalences have been proposed, and the relationship between them has been quite well-understood (see, for example, [11]).

Among the major equivalences are testing ones presented in [10]. Two processes are considered to be testing equivalent, if there is no test that can distinguish them. A test itself is usually a process applied to another process by computing them together in parallel. A particular computation is considered to be successful, if the test reaches a designated successful state, and the process passes the test if every computation is successful. This notion has led to a well-developed mathematical theory of processes that ties together the equivalences and preorders. Testing decision procedures are usually based on reduction of testing to bisimulation [8]. These equivalences have been considered for synchronous and asynchronous formal system models without time delays [1], [8], [7], [10], [12].

Testing equivalences have also been developed for models with time (see, for example, [4], [5], [9], [13], [14], [16]). Papers [9] and [14] have treated timed testing for discrete time transition models. The alternative characterization of timed testing given in these papers uses a notion similar to that of an acceptance set in the testing theory. Paper [13] has investigated timed testing for the discrete time process algebra. In [5], time testing relations have been developed for asynchronous timed Petri nets. In [17], testing equivalences are considered for the model of determinized timed automata.

In paper [4], a framework for testing preorders and equivalences in the setting of timed event structures has been developed.

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In that model, a time interval associated with an event means the interval, during which the event can occur. Occurrence of the event does not take any time. The model is a timed generalization of Winskel's prime event structures [18]. The alternative characterization of the timed testing relations is given.

In [6], the problem of decidability of timed *must*-equivalences is reduced to the model-checking one. As a basic logic, we take the timed logic L_ν. This logic has been defined in [15] and used for construction of a characteristic formula for a timed automaton up to the timed bisimilarity and, as a consequence, for reduction of the timed bisimilarity decidability problem to the model-checking one. It is known that the latter problem is decidable ([2], [3]). In [6], a characteristic formula up to the timed testing preorders is constructed. We do it for timed event structures with discrete internal actions. The characteristic formula consists of formulas for each class of the class graph. Each subformula is modelling a possible transition from the class and contain conditions on the formula clocks.

Usually, complex systems consist of subsystems. In the case when events of different subsystems are in the same relation – partial order, conflict or concurrency – we say that the system is a composition of subsystems.

So, it is interesting to construct characteristic formulas for the whole system using only similar formulas for subsystems. In such way we can avoid construction of region and class graphs, algorithms for which are exponential. According to a usual structure of the characteristic formula, we construct its subformulas using the formulas for classes of substructures.

The rest of the paper is organized as follows. In Section 2, we remind the basic notions concerned with timed event structures and timed testing. The timed modal logic L_ν is described in Section 3. In Section 4, we obtain a class graph from the state-space. In Section 5, we construct a formula which characterizes a timed event structure up to the timed testing preorders. In Section 6, the characteristic formula for a timed event structure is constructed on the basis of the formulas for its substructures. Conclusion is given in Section 7. In Appendix, we consider an auxilary construction of composition of the class graphs.

2. Timed event structures

In this section, we remind a model of timed event structures that is a real-time extension of Winskel's model of prime event structures [18] by equipping events with time intervals.

We first recall the notion of an event structure. The main idea behind event structures is to view the distributed computations as action occurrences, called events, together with the notion of causal dependence between events (which are reasonably characterized via a partial order). Moreover, to model nondeterminism, there is a notion of conflicting (mutually incompatible) events. A labelling function determines which action corresponds to an event.

Let Act be a finite set of visible actions and τ be an internal action. Then $Act_\tau = Act \cup \{\tau\}$.

Definition 1. *A* (labelled) event structure *over* Act_τ *is a 4-tuple* $S=(E, \leq, \#, l)$*, where*

- E *is a countable set of events;*
- $\leq \subseteq E \times E$ *is a partial order (the* causality relation*) satisfying the* principle of finite causes*:* $\forall e \in E \ . \ \{e' \in E \mid e' \leq e\}$ *is finite;*
- $\# \subseteq E \times E$ *is a symmetric and irreflexive relation (the* conflict relation*) satisfying the* principle of conflict heredity*:* $\forall e, e', e'' \in E \ . \ e \# e' \leq e'' \ \Rightarrow \ e \# e''$*;*
- $l : E \to Act_\tau$ *is a labelling function.*

For pairs of events are neither in causality nor in conflict relations, we define the *concurrency relation* as $\smile = (E \times E) \setminus (\leq \cup \geq \cup \#)$.

Let $C \subseteq E$. Then C is *left-closed* iff $\forall e, e' \in E \ . \ e \in C \ \wedge \ e' \leq e \Rightarrow e' \in C$; C is *conflict-free* iff $\forall e, \ e' \in C \ . \ \neg(e \# e')$; C is a *configuration of* S iff C is left-closed and conflict-free. Let $Conf(S)$ denote the set of all configurations of S. For $C \in Conf(S)$, we define the set of events enabled in C as $En(C) = \{e \in E \mid C \cup \{e\} \in Conf(S)\}$.

In the following, we will consider only finite event structures, i.e. the structures whose sets of events are finite.

Before introducing the concept of a timed event structure, we need to propose some auxiliary notations. Let $\mathbf{N}_0$ be the set of natural numbers with zero, $\mathbf{R}^+$ be the set of positive real numbers, and $\mathbf{R}_0^+$ be the set of nonnegative real numbers. For any $d \in \mathbf{R}_0^+$, $\{d\}$ denotes its fractional part, $\lfloor d \rfloor$ and $\lceil d \rceil$ – its smallest and largest integer parts, respectively.
Define the set $Interv(\mathbf{R}_0^+) = \{[d_1, d_2] \subset \mathbf{R}_0^+ \mid d_1, d_2 \in \ \mathbf{N}_0\}$.

We are now ready to introduce the concept of timed event structures.

Definition 2. *A* (labelled) timed event structure *over* Act_τ *is a pair* $TS = (S, D)$*, where*

- $S = (E, \leq, \#, l)$ *is a (labelled) event structure over* Act_τ*;*
- $D : E \to Interv(\mathbf{R}_0^+)$ *is a timing function such that* $D(e) = [d, d]$ *for some* $d \in \mathbf{N}_0$ *for all* e *with* $l(e) = \tau$*.*

In a graphic representation of a timed event structure, the corresponding action labels and time intervals are drawn close to events. If no confusion arises, we will use action labels instead of the event identifiers to denote events. The $<$-relations are depicted by arcs (omitting those derivable by transitivity), and conflicts are depicted by "#" (omitting those derivable by the conflict heredity). Following these conventions, a trivial example of a labelled timed event structure is shown in Fig. 1.

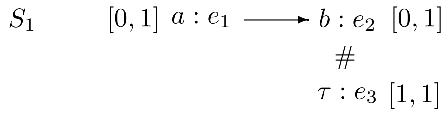

Figure 1. A simple example

Let $\mathcal{E}_\tau$ denote the set of all labelled timed event structures over Act_τ. For convenience, we fix timed event structures $TS = (S = (E, \leq, \#, l), D)$, $TS' = (S' = (E', \leq', \#', l'), D')$ from the class $\mathcal{E}_\tau$ and work with them further.

TS is called *conflict-free*, if E is conflict-free. TS' is called *a substructure* of TS, if $E' \subset E$, $\leq' \subseteq \leq|_{E'}$, $\#' \subseteq \# \,|_{E'}$, $l' = l\,|_{E'}$, $D' = D\,|_{E'}$.

A *state* of TS is a pair $M = (C, \delta)$, where $C \in Conf(S)$ and $\delta : E \to \mathbf{R}_0^+$. The *initial state* of TS is $M_{TS} = (C_0, \delta_0) = (\emptyset, 0)$. A state $M = (C, \delta)$ is said to be *terminated*, if $En(C) = \emptyset$. Let $ST(TS)$ denote the set of all states of TS.

A timed event structure progresses through a sequence of states in one of two ways given below.

Let $M_1 = (C_1, \delta_1), M_2 = (C_2, \delta_2) \in ST(TS)$ such that M_1 is a non-terminated state. An event $e \in En(C_1)$ *may occur* in M_1 (denoted as $M_1 \stackrel{e}{\to}$) if $\delta_1(e) \in D(e)$ and $\forall e' \in En(C_1)\ \exists d \in \mathbf{R}_0^+ \ .\ \delta_1(e') + d \in D(e)$. We write $M_1 \stackrel{a}{\to}$, if $M_1 \stackrel{e}{\to}$ and $l(e) = a$. The occurrence of e in M_1 *leads* to M_2 (denoted as $M_1 \stackrel{e}{\to} M_2$), if $M_1 \stackrel{e}{\to}$, $C_2 = C_1 \cup \{e\}$ and

$$\delta_2(e') = \begin{cases} 0, & \text{if } e' \in En(C_2) \setminus En(C_1) \\ \delta_1(e'), & \text{otherwise.} \end{cases}$$

We write $M_1 \stackrel{a}{\to} M_2$, if $M_1 \stackrel{e}{\to} M_2$ and $l(e) = a$.

A time $d \in \mathbf{R}^+$ *may pass* in M_1 (denoted as $M_1 \stackrel{d}{\to}$), if $\forall e \in En(C_1)\ \exists d' \in \mathbf{R}_0^+ (d' \geq d)\ .\ \delta_1(e) + d' \in D(e)$. The passage of d in M_1 *leads* to M_2 (denoted as $M_1 \stackrel{d}{\to} M_2$), if $C_2 = C_1$ and $\delta_2(e) = \delta_1(e) + d$ for all $e \in E$.

The *weak leading relation* $\Rightarrow$ *on the states of* TS is the largest relation defined by: $\stackrel{\epsilon}{\Rightarrow} \iff \stackrel{\tau}{\to}^*$ and $\stackrel{x}{\Rightarrow} \iff \stackrel{\epsilon}{\Rightarrow}\stackrel{x}{\to}\stackrel{\epsilon}{\Rightarrow}$, where $\stackrel{\tau}{\to}^*$ is the reflexive and transitive closure of $\stackrel{\tau}{\to}$ and $x \in Act \cup \mathbf{R}^+$. We consider the relation $\stackrel{d}{\Rightarrow}$ as possessing the time continuity property: $M \stackrel{d_1+d_2}{\Longrightarrow} \iff M \stackrel{d_1}{\Rightarrow}\stackrel{d_2}{\Rightarrow}$ for some $d_1, d_2 \in \mathbf{R}^+$.

From now on, we will use the following notions and notations. Let $Act(\mathbf{R}_0^+) = \{a(d) \mid a \in Act \wedge d \in \mathbf{R}_0^+\}$ be the set of *timed actions* of Act over $\mathbf{R}_0^+$. Then $(Act(\mathbf{R}_0^+))^*$ is the set of finite *timed words* over $Act(\mathbf{R}_0^+)$. The function $\triangle : (Act(\mathbf{R}_0^+))^* \to \mathbf{R}_0^+$ measuring the *duration* of a timed word is defined by: $\triangle(\epsilon) = 0$, $\triangle(w.a(d)) = \triangle(w) + d$. The domain for real-time languages is denoted by $Dom(Act,\ \mathbf{R}_0^+) = \{\langle w, d\rangle \mid w \in (Act(\mathbf{R}_0^+))^*,\ d \in \mathbf{R}_0^+,\ d \geq \triangle(w)\}$.

The weak leading relation $\Rightarrow$ is extended to timed words from $(Act(\mathbf{R}_0^+))^*$ and $Dom(Act,\ \mathbf{R}_0^+)$ as follows. Let $d \in \mathbf{R}_0^+$, $d' \in \mathbf{R}^+$, $a \in Act$ and $w \in (Act(\mathbf{R}_0^+))^*$. Then

$$\text{if } M \stackrel{a}{\Rightarrow} M', \text{ then } M \stackrel{a(0)}{\Rightarrow} M'; \text{ if } M \stackrel{d'}{\Rightarrow}\stackrel{a}{\Rightarrow} M', \text{ then } M \stackrel{a(d')}{\Rightarrow} M';$$
$$\text{if } M \stackrel{w}{\Rightarrow}\stackrel{a(d)}{\Rightarrow} M', \text{ then } M \stackrel{w.a(d)}{\Longrightarrow} M'; \text{ if } M \stackrel{w}{\Rightarrow} M', \text{ then } M \stackrel{\langle w,\ \triangle(w)\rangle}{\Longrightarrow} M';$$
$$\text{if } M \stackrel{\langle w,d\rangle}{\Longrightarrow}\stackrel{d'}{\Rightarrow} M', \text{ then } M \stackrel{\langle w,\ d+d'\rangle}{\Longrightarrow} M'.$$

The set $L(TS) = \{\langle w, d\rangle \in Dom(Act, \mathbf{R}_0^+) \mid M_{TS} \stackrel{\langle w,d\rangle}{\Longrightarrow}\}$ is the *language* of TS. For instance, for the timed event structure TS_1 in Figure 1, we have $L(TS_1) = \{\langle\epsilon, d_1\rangle, \langle\epsilon, 1\rangle,\ \langle a(d_1), d_1 + d_2\rangle,\ \langle a(1), 1\rangle,\ \langle a(d_1)b(d_2), d_1 + d_2\rangle \mid d_1 + d_2 \leq 1\}$.

The timed testing relations may be defined in terms of responses of timed event structures to a collection of tests. We will, however, use an alternative characterization [4]. It turned out that *may*-preorder is characterized by inclusion of languages. For *must*-preoder to exist, inclusion of sets of enabled visible actions and the possibility of time passing in states of two timed event structures reachable by the same timed word are necessary. The formal definition relies on the following notations. Let $M \in ST(TS)$ and $\langle w,\ d\rangle \in Dom(Act,\ \mathbf{R}_0^+)$. Then $S(M) = \{x \in Act_\tau \cup \mathbf{R}^+ \mid M \stackrel{x}{\to}\}$ and $Acc(TS,\ \langle w,\ d\rangle) = \{S(M') \mid M_{TS} \stackrel{\langle w,\ d\rangle}{\Longrightarrow} M',\ M' \stackrel{\tau}{\not\to}\}$ (timed acceptance set). Let $N, N' \subset 2^{Act \cup \mathbf{R}^+}$. Then $N \subset\subset N' \iff \forall S \in N\ \exists S' \in N'\ .\ [(S' \mid_{Act} \subseteq S \mid_{Act}) \wedge (S \mid_{\mathbf{R}^+} = \emptyset \Rightarrow S' \mid_{\mathbf{R}^+} = \emptyset)]$; $N \equiv N' \iff N \subset\subset N' \wedge N' \subset\subset N$.

Definition 3.

- $TS \leq_{must} TS' \iff \forall\langle w,\ d\rangle \in Dom(Act, \mathbf{R}_0^+)\ .\ Acc(TS', \langle w, d\rangle) \subset\subset Acc(TS, \langle w, d\rangle)$;
- $TS \simeq_{must} TS' \iff TS \leq_{must} TS'$ *and* $TS' \leq_{must} TS$.

An example of timed *must*-equivalent structures is shown in Figure 2(a). The timed event structures TS_3 and TS_3' shown in Figure 2(b) are not timed *must*-equivalent. Let us consider the timed word $\langle w, d\rangle = \langle a(0.5), 1.5\rangle \in$

$L(TS_3) \cap L(TS'_3)$. We have $Acc(TS_3, \langle w, d\rangle) = \{\{b, c\} \cup (0, 1]\}$ and $Acc(TS'_3, \langle w, d\rangle) = \{\{b, c\} \cup (0, 1], \{c\}\}$. This means that in TS_3, after executing the action a and passing time 1, the action c or b can be executed or time from $(0, 1]$ can pass. In TS'_3, after executing the same timed word, we get two states. In the first state, also as in TS_3, the action c or b can be executed or time from $(0, 1]$ can pass, but in the other state only the action c can be executed. So, $\neg\big(Acc(TS'_3, \langle w, d\rangle) \subset\subset Acc(TS_3, \langle w, d\rangle)\big)$.

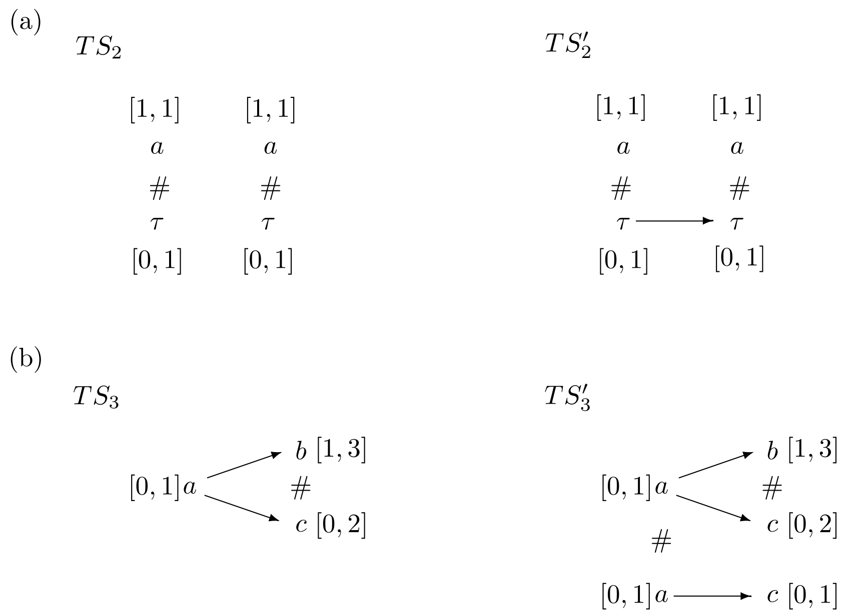

Figure 2. An example of (a)timed *must*-equivalent and (b)non-timed *must*-equivalent timed event structures

3. Timed modal logic

Here we will recall a dense-timed logic L_ν [15] and modify a satisfiability relation for timed event structures. The logic L_ν is a fragment of μ-calculations with maximal recursion. Below we will use formulas of this logic for characterization of timed event structures up to testing equivalences.

Definition 4. *Let K be a finite set of clocks, Id be a set of identifiers and k be an integer. The set of formulas of L_ν over K, Id and k is generated by the abstract syntax with ϕ and ψ ranging over L_ν:*

$$\phi := tt \mid ff \mid \phi \wedge \psi \mid \phi \vee \psi \mid \exists\phi \mid \forall\phi \mid \langle a\rangle\phi \mid [a]\phi \mid x \ in\ \phi \mid x + n \bowtie y + m \mid x \bowtie m \mid Z,$$

where $a \in Act$, $x, y \in K$, $n, m \in \{0, 1, \ldots, k\}$, $\bowtie \in \{=, <, \leq, >, \geq\}$ *and* $Z \in Id$.

The meaning of identifiers from Id is specified by a declaration D that assigns a formula of L_ν to each identifier. When D is clear, we write $Z := \phi$ for $D(Z) = \phi$. The clocks from K are called formula clocks and a formula ϕ is said to be *closed* if every formula clock x occurs in ϕ in the scope of an "x in ..." operator. Given a timed event structure TS, we interpret the formulas from L_ν over *an extended* state $(C, \delta u)$, where (C, δ) is a state of TS and u is a time assignment for K. Transitions between extended states are defined by: $(C, \delta u) \stackrel{\epsilon(d)}{\rightarrow} (C, (\delta + d)(u + d))$ and $(C, \delta u) \stackrel{a}{\rightarrow} (C', \delta' u')$ iff $(C, \delta) \stackrel{a}{\rightarrow} (C', \delta')$ and $u = u'$. Formally, the satisfaction relation between extended states and formulas is defined just as in [15] and here only a part of operators is given.

Definition 5. *Let TS be a timed event structure and D be a declaration. The satisfaction relation $\models_D$ is the largest one that satisfies the following implications:*

$$
\begin{array}{rcl}
(C, \delta u) \models_D tt & \Rightarrow & true; \\
(C, \delta u) \models_D ff & \Rightarrow & false; \\
(C, \delta u) \models_D \phi \wedge \psi & \Rightarrow & (C, \delta u) \models_D \phi \text{ and } (C, \delta u) \models_D \psi; \\
(C, \delta u) \models_D \exists\phi & \Rightarrow & \exists d \in \mathbf{R}_0^+ \ . \ (C, \delta) \stackrel{\epsilon(d)}{\Rightarrow} (C', \delta') \\
 & & \text{and } (C', \delta' u + d) \models_D \phi; \\
(C, \delta u) \models_D [a]\phi & \Rightarrow & \forall (C', \delta') \in ST(TS) \ . \ (C, \delta) \stackrel{a}{\rightarrow}\stackrel{\epsilon}{\Rightarrow} (C', \delta') \\
 & & \text{implies } (C', \delta' u) \models_D \phi. \\
(C, \delta u) \models_D x + m \bowtie y + n & \Rightarrow & u(x) + m \bowtie u(y) + n; \\
(C, \delta u) \models_D x \text{ in } \phi & \Rightarrow & (C, \delta u') \models_D \phi, \text{ where } u' = [\{x\} \to 0]u; \\
(C, \delta u) \models_D Z & \Rightarrow & (C, \delta u) \models_D D(Z).
\end{array}
$$

Any relation that satisfies the above implications is called a satisfiability relation. We say that TS *satisfies* a closed formula ϕ from L_ν and write $TS \models \phi$, when $(C_0, \delta_0 u) \models_D \phi$ for any u. Note that if ϕ is closed, then $(C, \delta u) \models_D \phi$ iff $(C, \delta u') \models_D \phi$ for any $u, u' \in \mathbf{R}_0^{+K}$.

4. From state-space to class graph

For the purpose of constructing a characteristic formula, the infinite state-space is transformed to a finite representation in such a way that states reachable by the same timed word be collected together in one class. We will briefly consider the transformation through this section. As usual, in order to get a discrete representation of the state-space of a timed event structure, the concept of regions (equivalence classes of states) [2] is used. To get a deterministic representation, classes are used.

In the definition of a region, we will use the notion of common states.

Definition 6. *A subset* $\mu \subseteq ST(TS)$ *is called a* common state *of* TS*. The* initial *common state of* TS *is* $\mu_0 = \{M_{TS}\}$.

Sometimes μ is denoted as $(M_1, \ldots, M_n)$ or $(\langle \mathbf{C} \rangle^n, \langle \delta \rangle^n)$, where $M_i = (C_i, \delta_i) \in \mu$ $(1 \le i \le n)$, $\langle \mathbf{C} \rangle^n = (C_1, \ldots, C_n)$, $\langle \delta \rangle^n = (\delta_1, \ldots, \delta_n)$.

Let $n^+ = \{1, \ldots, n\}$. Renaming $\pi(n) : n^+ \to n^+$ is extended to $\langle \mathbf{C} \rangle^n$ as $\pi(n)(\langle \mathbf{C} \rangle^n) = (C_{\pi(n)(1)}, \ldots, C_{\pi(n)(n)})$, in a similar way $\pi(n)(\langle \delta \rangle^n)$ is defined and $\pi(n)(\mu) = (\pi(n)(\langle \mathbf{C} \rangle^n), \pi(n)(\langle \delta \rangle^n))$.

A visible action and time can be executed in a common state only if an internal action is not enabled. So, the relation $\xrightarrow{z}$ is defined on common states as follows:

- $\mu \xrightarrow{\tau} \mu'$ iff $\mu \neq \mu'$ and $\mu' = \{(C', \delta') \mid \exists (C, \delta) \in \mu \, . \, (C, \delta) \xrightarrow{\tau} (C', \delta')\} \cup \mu$;
- $\mu \xrightarrow{z} \mu'$ iff $\mu \not\xrightarrow{\tau}$ and $\mu' = \{(C', \delta') \mid \exists (C, \delta) \in \mu \, . \, (C, \delta) \xrightarrow{z} (C', \delta')\}$ $(z \in Act \cup \mathbf{R}^+)$.

$STC(TS)$ denotes the set of all common states reachable from μ_0. Below we will consider common states only from $STC(TS)$. The leading relation on common states of $STC(TS)$ is extended to timed words from $Dom(Act, \mathbf{R}_0^+)$ just as on the states of $ST(TS)$.

Let $\mu = (C_1, \ldots, C_n, \delta_1, \ldots, \delta_n) \neq \mu' = (C_1', \ldots, C_n', \delta_1', \ldots, \delta_n')$. Then $\mu \simeq \mu'$ iff $(C_1, \ldots, C_n) = (C_1', \ldots, C_n')$ and

(i) $\forall 1 \le i \le m \, . \, \lfloor \delta_1 | \ldots | \delta_n(i) \rfloor = \lfloor \delta_1' | \ldots | \delta_n'(i) \rfloor$;

(ii) $\forall 1 \le i, j \le m$.

- $\{\delta_1 | \ldots | \delta_n(i)\} \le \{\delta_1 | \ldots | \delta_n(j)\} \iff \{\delta_1' | \ldots | \delta_n'(i)\} \le \{\delta_1' | \ldots | \delta_n'(j)\}$,
- $\{\delta_1 | \ldots | \delta_n(i)\} = 0 \iff \{\delta_1' | \ldots | \delta_n'(i)\} = 0$,

where $\delta_1 | \ldots | \delta_n$ is the concatenation of vectors $\bar{\delta}_i$ $(1 \le i \le n)$ and $m = \sum_{1 \le i \le n} | C_i |$.

A set $R = [\mu] = \{\mu' \mid \exists \pi(n) \; \mu \simeq \pi(n)(\mu')\}$ is called a *region* of TS. We define $R_0 = [\mu_0]$.

Let R and R' be regions of TS. Then the leading relation on regions is defined as follows:

- $R \xrightarrow{a} R'$ iff $\exists \mu \in R, \; \mu' \in R' \, . \, \mu \xrightarrow{a} \mu'$ $(a \in Act_\tau)$;
- $R \xrightarrow{\chi} R'$ iff $\exists \mu \in R, \; \mu' \in R' \; \exists d \in \mathbf{R}^+ \, . \, \mu \xrightarrow{d} \mu' \wedge \forall \; 0 < d' < d \;\; \mu \xrightarrow{d'} \widetilde{\mu} \in R \cup R'$.

We will call a partition of $STC(TS)$ into regions *stable* if the following holds:

- if $R \stackrel{a}{\rightarrow} R'$, then $\forall \mu \in R\ .\ \mu \stackrel{a}{\rightarrow} \mu'$ for some $\mu' \in R'$ ($a \in Act_\tau$);
- if $R \stackrel{\chi}{\rightarrow} R'$, then $\forall \mu \in R\ \exists d \in \mathbf{R}^+\ .\ \mu \stackrel{d}{\rightarrow} \mu'$ for some $\mu' \in R'$ and $\mu \stackrel{d'}{\rightarrow} \widetilde{\mu} \in R \cup R'$ for all $0 < d' \leq d$.

So, we can define the notion of a region graph of TS.

Definition 7. A region graph *of TS is a tuple $RG(TS) = (V_{RG}, E_{RG}, l_{RG})$, where the set of vertices V_{RG} is the stable partition of $STC(TS)$, the set of edges E_{RG} is the leading relation on regions of V_{RG} and the labelling function $l_{RG} : E_{RG} \longrightarrow Act_\tau \cup \{\chi\}$ is defined as $l((R, R')) = z \iff R \stackrel{z}{\rightarrow} R'$.*

We define $Der(R,\ z) = \{R' \mid R \stackrel{z}{\rightarrow} R'\}$.

Lemma 1. *Let $R \in V_{RG}$. Then $\forall \mu,\ \mu' \in R\ \forall (C, \delta) \in \mu\ \exists\ (C',\ \delta') \in \mu'\ .\ C = C'\ \wedge\ S((C,\delta)) \mid_{Act} = S((C',\delta')) \mid_{Act}\ \wedge\ S((C,\delta)) \mid_{\mathbf{R}^+} = \emptyset \iff S((C',\delta')) \mid_{\mathbf{R}^+} = \emptyset$.*

4.1. Adding of counters

Let $RG(TS)$ be a region graph and X be a countable set of counters. Let all the regions of $RG(TS)$ get a unique number, then each region R_i is associated with its own counter x_{R_i}. For simplicity, sometimes we will denote x_{R_i} by x_i.

Moreover, a tuple $T = (RC(R),\ \mu_R,\ \sigma_R,\ \Delta_R)$ is associated with each region R, where $RC(R)$ is the set of counters, $\mu_R = (\langle \mathbf{C} \rangle^{n_R}, \langle \delta \rangle^{n_R}) \in R$ is the region representative, the function $\sigma_R : RC(R) \longrightarrow 2^{E \times N}$ associates a pair, an event and a configuration number from μ_R, with each counter of $RC(R)$, $\Delta_R : RC(R) \rightarrow \mathbf{R}_0^+$ is a time assignment of counters.

The counter x_R is added to $RC(R)$, if some event becomes enabled in μ_R and this event is associated with x_R by the function σ_R. Counters for which there is no associated event are deleted from $RC(R)$. The values of counters (Δ_R) depend on the values $\langle \delta \rangle^{n_R}$ of μ_R.

4.2. Class graph

To receive the deterministic representation, the notion of a class [4] as the τ-closure of regions is used.

Let $RG(TS) = (V_{RG}, E_{RG}, l_{RG})$ and $Q \subseteq V_{RG}$. A set $Q^\tau = \{R' \in V_{RG} \mid \exists R \in Q\ .\ R \stackrel{\epsilon}{\Rightarrow} R'\}$ is called a *class* of TS. Define $Q_0 = \{R_0\}^\tau$, and $Der(Q,\ z) = \bigcup_{R \in Q} Der(R,\ z)$.

For classes Q, Q_1 and $z \in Act \cup \{\chi\}$, the *leading relation on classes* is given by: $Q \xrightarrow{z} Q_1$, if $Q_1 = (Der(Q,\ z))^\tau$.

We need the following notations.

$S(Q) = \{z \in Act \cup \{\chi\} \mid\ Q \xrightarrow{z}\}$, $QC(Q) = \bigcup_{R \in Q} RC(R)$.

Definition 8. *A* class graph *of TS is the labelled directed graph $CG(TS) = (V_{CG}, E_{CG}, l_{CG})$. The set of vertices V_{CG} is the set of reachable classes of TS, E_{CG} is the leading relation on the classes of V_{CG} and the labelling function $l_{CG} : E_{CG} \longrightarrow (Act \cup \{\chi\})$.*

5. Formula construction

For each class Q, a formula F_Q is constructed. The characteristic formula of TS consists of the formulas of classes.

In the formula, we use the notations Q_a and Q_χ, if $Q \xrightarrow{a} Q_a$ and $Q \xrightarrow{\chi} Q_\chi$, and we write its optional parts between $\langle\!\langle$ and $\rangle\!\rangle$. In addition, we suppose $\hat{R} \in Q$ such that $\hat{R} \not\xrightarrow{\mathcal{I}}$, $vis(\hat{R}) = \{M \in \mu_{\hat{R}} \mid M \not\xrightarrow{\mathcal{I}}\}$. The clocks $\hat{x}_i$ correspond to counters $x_i \in QC(Q)$, and the clock $\hat{x}$ is used additionally.

$$\begin{aligned} F_Q = \ & \forall\!\!\!\forall \beta(Q) \Rightarrow\ \psi_Q; \\ \psi_Q = \ & \textstyle\bigwedge_{a \notin S(Q)|_{Act}} [a]\mathit{ff}\ \wedge\ \bigwedge_{a \in S(Q)|_{Act}} [a](\langle\!\langle XQ_a\ in \rangle\!\rangle\ F_{Q_a}) \ \wedge \\ & \langle\!\langle F_\chi \rangle\!\rangle\ \wedge\ \langle\!\langle F_{Q_\chi} \rangle\!\rangle\ \wedge\ (ACC(Q)\ \vee\ \langle\tau\rangle \mathit{tt}); \end{aligned}$$

Informally, ψ_Q can be written as:

$$\psi_Q = \begin{bmatrix} \text{part for actions which} \\ \text{can't be run in } Q \end{bmatrix} \wedge \begin{bmatrix} \text{part for actions which} \\ \text{can be run } Q \end{bmatrix} \wedge$$
$$\langle\!\langle\ Q_\chi \text{ doesn't exist} \rangle\!\rangle\ \wedge\ \langle\!\langle\ Q_\chi \text{ exists} \rangle\!\rangle\ \wedge\ [\text{ modeling of } Acc(TS, \langle w, d\rangle)].$$

The conditions $\beta(Q)$ hold for the time assignment of states only from $\hat{R}$.

Below we give the subformulas of ψ_Q and conditions on including them into ψ_Q.

- $XQ_a = \{\hat{x} \mid x \in QC(Q_a) \setminus QC(Q)\}$ is added, if it is not empty;
- $F_\chi = \hat{x}$ in $(\forall\!\!\!\forall \hat{x} > 0 \Rightarrow \bigwedge_{a \in Act_\tau} [a]\mathit{ff})$ is added into ψ_Q, if the class Q_χ does not exist;
- F_{Q_χ} is added into ψ_Q, if there is the class Q_χ;
- $ACC(Q) = \bigvee_{M \in vis(\hat{R})} \big((\bigwedge_{a \in S(M)|_{Act}} \langle a \rangle \mathit{tt})\ \wedge\ \langle\!\langle \chi' \rangle\!\rangle\ \wedge\ \langle\!\langle F_{all} \rangle\!\rangle \big)$
- $F_{all} = \bigvee_{a \in Act} [a]\mathit{tt}$ is added into $ACC(Q)$ for all $M \in \mu_{\hat{R}}$ such that $S(M)) \mid_{Act} = \emptyset$;
- $\chi' = \hat{x}$ in $(\exists\!\!\!\exists \hat{x} > 0 \Rightarrow (\bigvee_{a \in Act_\tau} \langle a \rangle \mathit{tt})$ is added into $ACC(Q)$ for all $M \in \mu_{\hat{R}}$ such that $S(M) \mid_{\mathbf{R}^+} \neq \emptyset$, i.e. time may pass in the state M.

For a timed event structure TS, *a characteristic must-formula* is defined as $F_{TS}^{must} = \hat{x}_0\ in\ F_{Q_0}$.

Theorem 1. *[6] $TS \leq_{must} TS' \iff TS' \models_D F_{TS}^{must}$, where D corresponds to the previous definition of F_Q for each Q from $V_{CG(TS)}$.*

Using the defined identifiers and formulas, a characteristic may-formula is constructed as follows.

$F'_Q = \forall\!\!\!\forall\beta(Q) \Rightarrow\ \phi_Q;$

$\phi_Q = \bigwedge_{a \notin S(Q)|_{Act}} [a]ff\ \wedge\ \bigwedge_{a \in S(Q)|_{Act}} \langle a\rangle(\langle\!\langle XQ_a\ in\rangle\!\rangle\ F'_{Q_a})\ \wedge\ \langle\!\langle F_\chi\rangle\!\rangle\ \wedge\ \langle\!\langle F_{Q_\chi}\rangle\!\rangle.$

Definition 9. *For a timed event structure TS,* a characteristic may-formula *is defined as $F_{TS}^{may} = \hat{x}_0\ in\ F'_{Q_0}$.*

Theorem 2. *[6] $TS \leq_{may} TS' \iff TS' \models_{\mathcal{D}'} F_{TS}^{may}$, where $\mathcal{D}'$ corresponds to the previous definition of F'_Q for each Q from $V_{CG(TS)}$.*

6. Compositional methods

Let us consider timed event structures TS_1 and TS_2 from $\mathcal{E}_\mathcal{T}$ and their characteristic *must*-formulas $F_{TS_1}^{must}$, $F_{TS_2}^{must}$. Suppose that their event sets do not intersect.

Let TS_1 and TS_2 be substructures of TS such that the event set of TS is a union of their event sets. We say TS is constructed from TS_1 and TS_2 using the operator $||$ (; or $\#$) if the events of TS_1 and TS_2 are in the pairwise $\smile$-relation ($\#$-relation or $\leq$-relation, respectively). In the case of the operator ;, TS_1 must be conflict free. Our aim is to construct the characteristic *must*-formula for TS using characteristic *must*-formulas of its substructures without constructing the region and class graphs of TS.

Suppose $TS = TS_1 \alpha TS_2$, where $\alpha \in \{;, \#, \ ||\}$.

Let K_1 and K_2 be non-intersecting sets of clocks and Id_1 and Id_2 be non-intersecting sets of identifiers used in the characteristic *must*-formulas $F_{TS_1}^{must}$ and $F_{TS_2}^{must}$, and their meanings are specified by the declarations $\mathcal{D}_1$ and $\mathcal{D}_2$.

By definition, $F_{TS_1}^{must} = x_0^1\ in\ F_0^1$ and $F_{TS_2}^{must} = x_0^2\ in\ F_0^2$.

6.1. Composition with operator ;

Suppose $TS = TS_1;TS_2$, where TS_1 is a conflict free structure.

In the formula $F_{TS_1}^{must}$, there are subformulas for classes which are lists in the class graph. Denote the set of such identifiers as $LIST(TS_1)$. By construction, $F_0^2 = \forall\!\!\!\forall\beta(F_0^2) \Rightarrow\ \psi(F_0^2)$ and F^1 from $LIST(TS_1)$ are of the form $F^1 = \forall\!\!\!\forall\beta(F^1) \Rightarrow\ \psi(F^1)$.

Let $K = K_1 \cup K_2$ be the set of clocks and $Id = Id_1 \cup Id_2$ be the set of identifiers. Then the declaration $\mathcal{D}$ of Id coincides with $\mathcal{D}_1$ and $\mathcal{D}_2$ on all identifiers except those from $LISTS(TS_1)$. For F^1 from $LIST(TS_1)$, we define $\mathcal{D}(F^1) = x_0^2\ in\ \forall\!\!\!\forall\beta(F^1) \Rightarrow\ \psi(F_0^2)$, i.e. we combine time conditions of the list vertices of the class graph of TS_1 with the main part of the formulas of the initial class of TS_2.

Let $F = x_0^1\ in\ F_0^1$.

Theorem 3. *F is the characteristic must-formula of TS with the declaration $\mathcal{D}$.*

6.2. Composition with operator

Suppose $TS = TS_1 \# TS_2$. Let K be the set of clocks and Id be the set of identifiers, which do not intersect with $K_1 \cup K_2$ and $Id_1 \cup Id_2$.

To connect clocks and identifiers from substructures formulas with those from the structure formula, we will use the functions $\overline{to} : Id_1 \cup Id_2 \to Id$, $to : K_1 \cup K_2 \to K$ and $from : K \to (K_1 \cup \{*\}) \times (K_2 \cup \{*\})$. The clocks $\hat{x}' \in K_1 \cup K_2$ and $\hat{x} \in K$ will be called *synchronized* if $to(\hat{x}') = \hat{x}$.

The main idea of construction of the characteristic *must*-formula for TS is sequential consideration of formulas for classes of TS_1 and TS_2 and composition of parts of these formulas.

Let $F,\ F_0 \in Id$, $\hat{x}_0 \in K$, define $F = \hat{x}_0\ in\ F_0$.

Construct F_0 as a composition $F_0^1 \# F_0^2$, where $F_0^1 \in Id_1$, $F_0^2 \in Id_2$. By definition, the formula for the class F_0 is of the form $\forall\!\!\!\forall\beta(F_0) \Rightarrow\ \psi_{F_0}$. Define $\beta(F_0) = (\hat{x}_0 = 0)$ and construct ψ_{F_0} as a composition $\psi_{F_0^1} \# \psi_{F_0^2}$. Synchronize clocks as follows: $to(\hat{x}_0^1) = to(\hat{x}_0^2) = \hat{x}_0$, $from(\hat{x}_0) = (\hat{x}_0^1, \hat{x}_0^2)$.

ψ_{F_0} will be constructed as a composition $\psi_{F_0^1} \# \psi_{F_0^2}$.

Beginning from F_0, we construct identifiers from K. Let F_m be the current identifier which we construct.

We suppose that $\beta(F_m)$ has been defined already and ψ_{F_m} must be constructed as $\psi_{F^1} \# \psi_{F^2}$ for some identifiers F^1, F^2 from Id_1, Id_2, respectively.

In the following, we need notations for conditions on the clocks from K, which are synchronized with only one clock from $K_1 \cup K_2$. Let B_1 be a condition from the formula of TS_1 and B_2 be a condition from the formula of TS_2. We define $Synch_m(B_1,\ B_2) = \bigwedge\{\{\hat{x}_i\} < \{\hat{x}_j\} \mid \{\hat{x}_i\} < \{\hat{x}_j\} \in \beta(F_m), i \neq j \in \{1,2\}, from(\hat{x}_1) = (\hat{x}, *), from(\hat{x}_2) = (*, \hat{y}), \hat{x} \in B_1, \hat{y} \in B_2\} \wedge \bigwedge\{\{\hat{x}_i\} = \{\hat{x}_j\} \mid \{\hat{x}_i\} = \{\hat{x}_j\} \in \beta(F_m), i \neq j \in \{1,2\}, from(\hat{x}_1) = (\hat{x}, *), from(\hat{x}_2) = (*, \hat{y}), \hat{x} \in B_1, \hat{y} \in B_2\}$.

By construction of the characteristic formulas, F^1 and F^2 areofthe form $F^1 = \forall\!\!\!\forall\beta^1 \Rightarrow\ \psi^1$ and $F^2 = \forall\!\!\!\forall\beta^2 \Rightarrow\ \psi^2$, where

$\psi^1 = \bigwedge_{a \in \overline{S^1}} [a]ff\ \wedge\ \bigwedge_{a \in S^1} [a](\langle\!\langle X_a^1\ in \rangle\!\rangle\ F_a^1)\ \wedge\ \langle\!\langle F_\chi^1 \rangle\!\rangle\ \wedge\ \langle\!\langle F_{Q_\chi}^1 \rangle\!\rangle\ \wedge\ (ACC^1\ \vee\ \langle\tau\rangle tt)$;

$\psi^2 = \bigwedge_{a\in\overline{S^2}} [a]ff \wedge \bigwedge_{a\in S^2} [a](\langle\langle X_a^2\ in\rangle\rangle\ F_a^2) \wedge \langle\langle F_\chi^2\rangle\rangle \wedge \langle\langle F_{Q_\chi}^2\rangle\rangle \wedge (ACC^2 \vee \langle\tau\rangle tt)$
and $S^1, \overline{S^1} \subseteq Act$, $\overline{S^1} = Act \setminus S^1$, $S^2, \overline{S^2} \subseteq Act$, $\overline{S^2} = Act \setminus S^2$.

Then

$\psi_{F_m} = \bigwedge_{a\in\overline{S^1}\cap\overline{S^2}} [a]ff \wedge \bigwedge_{a\in S^1\cup S^2} [a](\langle\langle X_a\ in\rangle\rangle\ F_a) \wedge \langle\langle F_\chi\rangle\rangle \wedge \langle\langle F_{Q_\chi}\rangle\rangle \wedge (ACC \vee \langle\tau\rangle tt)$.

Let us consider how to define parts of ψ_{F_m}.

a) The acceptance set is modeled as $ACC = ACC^1 \vee ACC^2$.

b) If the action $a \in Act$ can not be executed in both substructures, it can not be executed in TS, therefore, we include it into the corresponding part of the formula TS.

c) If theaction $a \in Act$ can be executed in both substructures, then the corresponding part F_a will be found as a composition of subformulas of the substructures formulas $F_a^1 \# F_a^2$.

d) If the action $a \in Act$ can be executed only in the substructure TS_1 (TS_2) (i.e. $a \in S^1 \setminus (S^1 \cap S^2)$ ($a \in S^2 \setminus (S^1 \cap S^2)$, respectively)), then $F_a = \overline{to}(to(F_a^1))$, ($F_a = \overline{to}(to(F_a^2))$, respectively). A composition of functions $\overline{to}(to(F_a^1))$ replace all clocks and identifiers from K_1 and Id_1 by those from K and Id.

e) For each $a \in S^1 \cap S^2$ (which can be executed in both substructures), we define timed conditions for the identifier F_a.

– If X_a^1 and X_a^2 are both non-empty and $\hat{x}^1 \in X_a^1$, $\hat{x}^2 \in X_a^2$, then we add a new clock $\hat{x}_{F_a}$ and synchronize it with the corresponding ones from X_a^1 and X_a^2: $X_a = \{\hat{x}_{F_a}\}$, $to(\hat{x}^1) = \hat{x}_{F_a}$, $to(\hat{x}^2) = \hat{x}_{F_a}$, $from(\hat{x}_{F_a}) = (\hat{x}^1, \hat{x}^2)$.

If one of X_a^i is empty, then $*$ is used in $from(\hat{x}_m)$ instead. If both are empty, then X_a is empty.

– If β is a set of conditions with clocks from $K_1 \cup K_2$, then an extension of the function *to* to β gives the condition β with replacing all clocks from $K_1 \cup K_2$ by the corresponding values of the function *to*.

We construct the condition $\beta(F_a)$ such that it includes the corresponding conditions of the substructures formulas and additional conditions for clocks, for which only one synchronized clock exists K_1 or K_2, namely, if we add a new counter which is synchronized only with the counter from one substructure, we include the relation of the new counter with counters from another substructure. For keeping similar relations defined in the previous steps, we use $Synch_m(\beta(F_a^1), \beta(F_a^2))$. So,

$\beta(F_a) = to(\beta(F_a^1)) \wedge to(\beta(F_a^2)) \wedge rel(x_m, a) \wedge Synch_m(\beta(F_a^1), \beta(F_a^2))$, where $rel(x_m, a) = \bigwedge\{\{x_m\} < y\} \mid y\ from\ to(\beta(F_a^1))$ if $(X_a^1 = \emptyset \ \wedge\ X_a^2 \neq \emptyset)$ and $y\ from\ to(\beta(F_a^2))$ if $(X_a^2 = \emptyset \ \wedge\ X_a^1 \neq \emptyset)\}$.

– Define ψ_{F_a} as a composition $\psi_{F_a^1} \# \psi_{F_a^2}$.

f) – If there is no $F^1_{Q_\chi}$ and $F^2_{Q_\chi}$, then F_χ is included.

– If there is only $F^1_{Q_\chi}$ (or $F^2_{Q_\chi}$), then $F_{Q_\chi} = \overline{to}(to(F^1_{Q_\chi}))$ ($F_{Q_\chi} = \overline{to}(to(F^2_{Q_\chi}))$, respectively).

– Suppose $F^1_{Q_\chi}$ and $F^2_{Q_\chi}$ both exist. By definition, existence of $F^i_{Q_\chi}$ corresponds to the relation $\xrightarrow{\chi}$ on classes and, therefore, to a change of the order on fractional parts and/or to a change of the integral part of counters. So, we need to understand what to prefer: the change of counters of TS_1 or of TS_2. The change in the integral part is perfomed earlier, so, we will find the clocks which are changing in the integral part, and in the case of its absence, we consider which fractional part is larger.

First, propose some auxiliary notations.

Let A_1, A_2, β be conditions on the clocks from K.

$Diff(A_1, A_2) = \{\hat{x} \mid$ there exists an equality $(\hat{x} = c)$ in A_i and there exists an inequality $(b < \hat{x} < b+1)$ in A_j for some $c, b \in \mathbf{N}_0, i \neq j \in \{1,2\}\}$

So, for for each F^i $(i = 1,2)$, the set $Diff(to(\beta^i), to(\beta(F^i_{Q_\chi})))$ includes the clocks which were changed.

Let $i \neq j \in \{1,2\}$, for A_i s.t. $Diff(A_i, \overline{A_i}) \neq \emptyset$ define the relation $\gtrdot_\beta$ as follows: $A_i \gtrdot_\beta A_j$ iff $Diff(A_j, \overline{A_j}) \neq \emptyset$ and there exists $\hat{x}_j \in Diff(A_j, \overline{A_j})$, $\hat{x}_i \in Diff(A_i, \overline{A_i})$ s.t. for some $a \in \mathbf{N}_0$, there exists either $((\{\hat{x}_j\} > \{\hat{x}_i\}) \wedge (\hat{x}_i = c))$ or $((\{\hat{x}_j\} < \{\hat{x}_i\}) \wedge (c < \hat{x}_i < c+1))$ in β.

Now we are ready to construct $\beta(F_{Q_\chi})$:

1. There exist changes on clocks from K which are synchronized with the clocks from both substructures formulas, i.e.
   $Diff(to(\beta^1), to(\beta(F^1_{Q_\chi}))) \cap Diff(to(\beta^2), to(\beta(F^2_{Q_\chi}))) \neq \emptyset$
   Then we take the conditions from $F^i_{Q_\chi}$ and keep relations between counters synchronized only with one counter:
   $\beta(F_{Q_\chi}) = to(\beta(F^1_{Q_\chi})) \wedge to(\beta(F^2_{Q_\chi})) \wedge Synch_m(\beta(F^1_{Q_\chi}), \beta(F^2_{Q_\chi}))[A]$.
   Since the order on fractional parts is changed with changing in integral part, namely, the largest became the smallest, we replace the inequalities for the clocks such that there exists an inequality of the form $b < \hat{x} < b+1$ in $\beta(F_m)$.

$Synch_m[A]$ is $Synch_m$ after replacing $\{\hat{y}\} < \{\hat{x}\}$ by $\{\hat{x}\} < \{\hat{y}\}$, where $\hat{x} \in A$.

$A = \emptyset$, if for some $\hat{x} \in Diff(to(\beta^1), to(\beta(F^1_{Q_\chi})))$ and some $b \in \mathbf{N}_0$, there exists an equality $(\hat{x} = b)$ in β , otherwise $A = Diff(to(\beta^1), to(\beta(F^1_{Q_\chi}))) \cup Diff(to(\beta^2), to(\beta(F^2_{Q_\chi})))$.

Define $\psi_{F_{Q_\chi}}$ as a composition $\psi_{F^1_{Q_\chi}} \# \psi_{F^2_{Q_\chi}}$.

2. There exist changes in the clocks synchronized with the clocks from only one of the substructures formulas, i.e.
$Diff(to(\beta^1), to(\beta(F^1_{Q_\chi}))) \cap Diff(to(\beta^2), to(\beta(F^2_{Q_\chi}))) = \emptyset$
If changes occur in the clocks from formulas of both substructures, then the condition is constructed as in the previous item (1). Else
   - Suppose $\beta^i \gtrdot_{\beta(F_m)} \beta^j$ for $i \neq j \in \{1, 2\}$. Then $\beta(F_{Q_\chi}) = to(\beta(F^i_{Q_\chi})) \wedge to(\beta^j) \wedge Synch_m(\beta(F^i_{Q_\chi}), \beta^j)[A]$.
   $A = \emptyset$, if for some $\hat{x} \in Diff(to(\beta^i), to(\beta(F^i_{Q_\chi})))$ and some $b \in \mathbf{N}_0$ there exists a condition $(\hat{x} = b)$ in $\beta(F_m)$, otherwise $A = Diff(to(\beta^i), to(\beta(F^i_{Q_\chi})))$.
   And $\psi_{F_{Q_\chi}}$ is defined as a composition $\psi_{F^i_{Q_\chi}} \# \psi_{F^j}$.
   - if it is not true that $\beta^1 \gtrdot_{\beta(F_m)} \beta^2$ or $\beta^2 \gtrdot_{\beta(F_m)} \beta^1$, then $\beta(F_{Q_\chi})$ is constructed as in item (1) and $\psi_{F_{Q_\chi}}$ is defined as a composition $\psi_{F^1_{Q_\chi}} \# \psi_{F^2_{Q_\chi}}$.

We would like to show that F is the characteristic *must*-formula of TS. We will construct in a similar way the class graph of TS and show that the characteristic formula $F(Q)$ for each class coincides with some identifier F_i from Id. Since a tuple of attributes T of the region $\hat{R}$ $(\hat{R} \not\xrightarrow{T})$ is used for constructing characteristic formulas, we will also construct a tuple of attributes for $\hat{R}$ of each class.

Let $GC(TS_1)$ and $GC(TS_2)$ be graph classes of TS_1, TS_2. Construct a graph $GC'(TS)$ as a composition $GC(TS_1) \# GC(TS_2)$ (see Appendix).

Lemma 2. *The graph $GC'(TS) = GC(TS_1) \# GC(TS_2)$ is the class graph of TS.*

Proof is given by induction on classes reachable from the initial class Q_0. □

Theorem 4. *The formula F is the must-characteristic formula of TS.*

□

6.3. Composition with operator ||

Suppose $TS = TS_1||TS_2$. Under assumptions on the sets of clocks, identifiers and functions *to* and $from$ of the previous section, we will construct F_0 from Id as a composition $F_0^1||F_0^2$. Define $\beta(F_0) = (\hat{x}_0 = 0)$. We construct ψ_{F_0} as a composition $\psi_{F_0^1}||\psi_{F_0^2}$.

So, beginning from F_0, we will construct the subformulas $F_i \in Id$. Let F_m be a current identifier which we are constructing.

We suppose that $\beta(F_m)$ has been allready defined and ψ_{F_m} must be constructed as $\psi_{F^1}||\psi_{F^2}$ for some identifiers F^1, F^2 from Id_1 and Id_2, respectively.

By definition, ψ_{F_m} has the form:

$\psi_{F_m} = \bigwedge_{a\in \overline{S^1}\cap\overline{S^2}} [a]ff \;\wedge\; \bigwedge_{a\in S^1\cup S^2} [a](\langle\langle X_a\ in\rangle\rangle\ F_a) \;\wedge\; \langle\langle F_\chi \rangle\rangle \;\wedge\; \langle\langle F_{Q_\chi} \rangle\rangle$

a) If X_a^1 and X_a^2 are both non-empty and $\hat{x}^1 \in X_a^1$, $\hat{x}^2 \in X_a^2$, then $X_a = \{\hat{x}_a\}$, $to(\hat{x}^1) = \hat{x}_a$, $to(\hat{x}^2) = \hat{x}_a$, $from(\hat{x}_a) = (\hat{x}^1, \hat{x}^2)$. If both are empty, then X_a is empty.

   For F_a, the condition $\beta(F_a)$ is defined as:
   $\beta(F_a) = to(\beta(F_a^1)) \wedge Synch_m(\beta(F_a^1), \beta(F_a^2)) \wedge to(\beta(F_a^2)) \wedge rel(x_a, a)$,
   $\psi(F_a)$ is defined as a composition $((\psi(F_a^1)||\psi^2)||(\psi^1||\psi(F_a^2))$.

b) If $F^1_{Q_\chi}$ and $F^2_{Q_\chi}$ both exist, then construction is similar to the corresponding case for the operation #.

c) If there is F^i_χ for one of $i \in \{1, 2\}$, then F_χ is included.

Let $F' \in Id$. Define $F' = \hat{x}_0\ in\ F_0$.

Theorem 5. *The formula F' is the must-characteristic formula of TS.*

□

7. Conclusion

The characteristic formula allows us to decide the problem of recognizing the timed *must*-equivalence by reducing it to the model-checking one. This article is concentrated on constructing a characteristic formula for timed event structures which can be represented as a composition of its substructures. We develop the methods of composition of the characteristic *must*-formulas of substructures for operators of causality, concurrency and conflict. It is obvious that identifiers and declarations defined here could be easily used for constructing the characteristic *may*-formulas for all these operators.

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8. Appendix. Construction of the class graph of $TS_1 \# TS_2$

Let $GC(TS_1)$ and $GC(TS_2)$ be the graph classes of TS_1, TS_2. Construct a graph $GC'(TS)$ as a composition $GC(TS_1)\#GC(TS_2)$ Define $to' : QC(TS_1) \cup QC(TS_2) \to QC(TS)$; $from' : QC(TS) \to QC(TS_1) \times QC(TS_2)$.

Let Q_0 be the initial class constructed as $Q_0^1 \# Q_0^2$ with attributes $\mu(Q_0) = \{(\emptyset, \overline{0})\}$, $QC(Q_0) = \{x_0\}$, $\sigma(Q_0)(x_0) = (e_0, 1)$, $\Delta(Q_0)(x_0) = 0$.

Suppose that we construct the class Q_m as a composition $Q^1 \in GC(TS_1)$ and $Q^2 \in GC(TS_2)$. Note that $n^1 = \mid \mu(Q^1) \mid$, $n^2 = \mid \mu(Q^2) \mid$.

Then the attributes are defined as follows: $\mu(Q_m) = \mu(Q^1) \biguplus \mu(Q^2)$, where $\biguplus$ is defined as $C_i = C_i(Q^1)$ for $i = 1, \ldots, n^1$, $C_{n^1+i} = C_i(Q^2)$ for $i = 1, \ldots, n^2$,

$\delta_i \mid_{E_1} = \delta_i^1$, $\delta_i \mid_{E_2} = \delta_i^1(e_0)$, $i = 1, \ldots, n^1$,

$\delta_{n^1+i} \mid_{E_2} = \delta_i^2$, $\delta_{n^1+i} \mid_{E_1} = \delta_i^2(e_0)$, $i = 1, \ldots, n^2$ (i.e. the functions δ_i^1 and δ_i^2 are extended to the set E),

$$\sigma(Q_m)(x) = \begin{cases} (e, l), & \text{if } to(x^1) = x' \wedge \sigma(Q^1)(x^1) = (e, l) \\ (e, n^1 + l), & \text{if } to(x^2) = x' \wedge \sigma(Q^2)(x^2) = (e, l) \end{cases}$$

The actions which are executable in substructures are also executable in TS. Therefore, $S(Q_m) = S(Q_l^1) \cup S(Q_k^2)$.

a) Suppose that $Q^1 \xrightarrow{a} Q_a^1$, $Q^2 \xrightarrow{a} Q_a^2$, then $Q_m \xrightarrow{a} Q_{m'}$, where $Q_{m'}$ is a composition $Q_a^1 \# Q_a^2$.

   If there exist new counters in the classes of substructures $x^i \in QC(Q^i) \setminus QC(Q_a^i)$ $(i = 1, 2)$, then we add a new counter for a class of TS $QC(Q_a) = QC(Q) \cup \{x_a\}$, $to'(x^i) = x_a$, $from'(x_a) = (x^1, x^2)$ and

$$\Delta(Q_{m'})(x) = \begin{cases} \Delta(Q_a^1)(x'), & \text{if } from'(x) = (x', y); \\ \Delta(Q_a^2)(y), & \text{if } from'(x) = (*, y). \end{cases}$$

b) If χ exists only in $S(Q^i)$ $(i = 1, 2\ )$, then $Q_m \xrightarrow{\chi} Q_{m'}$ with $T(Q_{m'}) = T(\hat{R}_\chi^i)$, $\hat{R}_\chi^i \in Q_\chi^i$ and $Q_{m'}$ is a composition $Q_\chi^i \ \# \ *$.

c) Suppose that $Q^1 \xrightarrow{\chi} Q_\chi^1$ and $Q^2 \xrightarrow{\chi} Q_\chi^2$. We have to choose one of these transitions. Analogously to the similar case of formula construction, we find counters with a change in the integral or fractional parts and an order on them.

   So, let Q_1, Q_2 be classes of TS_1 and TS_2, respectively.

   $Diff(Q_1, Q_2) = \{to'(x) \mid x \text{ in } QC(Q_i) \text{ s.t. } \Delta_{Q_i}(x) = c$ and $(b < \Delta_{Q_j}(x) < b + 1) \text{ for some } c, b \in N, i \neq j \in \{1, 2\}\}$

   Suppose that $Diff(Q_1, \overline{Q_1}) \neq \emptyset$ and define the relation $\gtrdot_Q$ as follows: $Q_1 \gtrdot_Q Q_2$ iff $Diff(Q_2, \overline{Q_2}) \neq \emptyset$ and there exist $x_2 \in Diff(Q_2, \overline{Q_2})$

and $x_1 \in Diff(Q_1, \overline{Q_1})$ s.t. for some $b \in \mathbf{N}_0$ either $(\{\Delta(x_j)\} > \{\Delta(x_i)\} \wedge \Delta(x_i) = b)$ or $(\{\Delta(x_j)\} < \{\Delta(x_i)\} \wedge b < \Delta(x_i) < b+1)$ in Q.

If change occurs in the counters which rely with counters from both substructures, then transition $\xrightarrow{\chi}$ is taken from both, else choise depends on $\gtrdot_Q$.

1. $Diff(QC(Q^1), QC(Q^1_\chi))) \cap Diff(QC(Q^2), QC(Q^2_\chi))) \neq \emptyset$.

   Then $Q_m \xrightarrow{\chi} Q_{m'}$, where $Q_{m'}$ is constructed as a composition $Q^1_\chi \# Q^2_\chi$, $QC(Q_{m'}) = to(QC(Q^1_\chi)) \cup to(QC(Q^2_\chi))$.

$$\Delta(Q_{m'})(x) = \begin{cases} \Delta(Q^1_a)(x'), & \text{if } from'(x) = (x', y); \\ \Delta(Q^2_a)(y), & \text{if } from'(x) = (*, y). \end{cases}$$

2. $Diff(QC(Q^1), QC(Q^1_\chi))) \cap Diff(QC(Q^2), QC(Q^2_\chi))) = \emptyset$.

   – $Q^i \gtrdot_{Q_m} Q^j$ for $i \neq j \in \{1, 2\}$. Then $Q_m \xrightarrow{\chi} Q_{m'}$, where $Q_{m'}$ is constructed as a composition $Q^i_\chi \# Q^j$, $QC(Q_{m'}) = to(QC(Q^i_\chi)) \cup to(QC(Q^j))$ and

$$\Delta(Q_{m'})(x) = \begin{cases} \Delta(Q^1_a)(x'), & \text{if } from'(x) = (x', y); \\ \Delta(Q^2_a)(y), & \text{if } from'(x) = (*, y). \end{cases}$$

   – if it is not true that $Q^1 \gtrdot_{Q_m} Q^2$ or $Q^2 \gtrdot_{Q_m} Q^1$, then $Q_{m'}$ is constructed as in item (1).

Suppose that we construct the class Q_m as a composition $Q^1 \in GC(TS_1)$ and $*$. Then all leading relations from Q_m, the classes reachable from Q_m and a tuple of attributes for them are copies of the leading relations and the classes reachable from Q^1 in the graph class TS_1. The tuple of attributes is extended from the tuple for $\hat{R}$ for the corresponding class of TS_1. The extension means that, in the representative state $\mu = (\langle \mathbf{C} \rangle^n, \langle \delta \rangle^n)$, the functions from $\langle \delta \rangle^n$ are extended to E as $\langle \delta \rangle^n(e) = \langle \delta \rangle^n(e_0)$ for $e \notin E_1$.