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On the surface hitting in the receivers by the ray method in 3D-layered homogeneous media

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1. Introduction

Optimization methods of the solution of inverse kinematic problems, have gained importance in the last few years. Such methods demand fast algorithms of ray tracing from a given position of the source M_0 to a given series of the points M, forming a set (generally speaking, two-dimensional) of the receivers M_i, $i = 1, 2, \dots, n$, on the surface of observation.

In particular, when solving the 3D-seismotomography problems in the class of piecewise-constant curvilinear-layered media, which require to quickly carry out hitting with respect to trajectories, consisting of rectilinear pieces, satisfying at the boundaries $S_1, \dots, S_n$, the refraction–reflection laws of geometrical seismics. When solving the fast hitting problem it is natural to use the smoothness of the coordinates $x(M)$, $y(M)$, $z(M)$ of a point M on the surface of observations as functions of the ray parameters θ and φ, specifying the polar and the azimuth angles of a ray going the source at the point M_0. For this purpose it is required that the algorithms of calculation of an arrival curve included the procedure of calculation of partial derivatives vector-functions $\vec{r}(M) \equiv (x(M), y(M), z(M))$ on S by the parameters θ and φ.

In the case a considered model of media, these partial derivatives have explicit expressions, discussed in the present paper. In addition, this paper deals with the algorithm of constructing the areal first arrival curve based on the solution of the initial problem for the ray equation.

In this paper, we have overcome, though obvious in its sense, but a necessary stage of transition to calculations of the 3D seismic fields on the basis of the ray seismic formulas.

2. Model of a medium

The most important fact for the further calculations is that a ray when moving from one boundary to another is in media with constant velocity. In this sense, it is not important how a model of a medium is arranged on the whole – block or stratified. Here, as usual, under the block of the model we understand, such in which inside the considered mathematical bar

$$0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le H, \quad H > 0,$$

the crossing of interfaces of media is admitted and in the layered model, the interfaces do not cross. Here (x, y, z) are rectangular Cartesian coordinates, z is the depth.

In numerical experiments we consider the layered model, whose boundaries S_i are set by the explicit equations:

$$z = f_i(x, y), \quad 0 \le x \le a, \quad 0 \le y \le b, \quad i = 0, 1, \dots, n.$$

Here S_1 is the day's surface and the velocity of elastic waves v_i between the boundaries S_i and S_{i+1} is constant.

Clearly, the curvilinear-layered model is a special case of the block model and it is a natural generalization of the horizontal layered model admitting the same tree of possible ways of passage of rays through the interfaces.

3. Geometrical divergence of rays

In this section, the elementary derivation of the recurrent formulas of the calculation of geometrical divergence of rays along the given surface in the 3D block-homogeneous medium lengthways is obtained. Certainly, these formulas can be obtained as a special case from those already available (see, for example, [1, 2, 6, 9] and modern version [3, 7, 8]) presenting more general expressions for the block-gradient media.

We have found it expedient to give an independent derivation, first, because in transition to more general 3D models it is useful to have reliable formulas for testing, and, secondly, because the offered scheme of derivation can be, in turn, generalized to the block-gradient medium, as it was done for the 2D medium in [4], which represents a methodological interest.

Finally, the expediency of publication of this paper is justified, also, by the fact that, we propose the formulas suitable for the direct programming.

Let us pass to exact definitions and statements of the problem. Let us fix an arbitrary point M_0 of the medium and consider a ray L^* going from M_0, meeting successively $(n-1)$ boundaries $S_{k_1}, \dots, S_{k_{n-1}}$, on which there is a reflection or a refraction (with an exchange or without exchange) and coming to the surface observations S at a point M^*. The sequence $k_1, \dots, k_{n-1}$ together with attributes specifying the character of passing the borders (reflection/refraction) and the type of exchange (PP, SS, PS, SP) is the code of the ray L^*. Further boundaries will be designated by S_i, $i = 1, 2, \dots, n-1$. Let us recall that the ray parameters at any point M lying on the ray L, going from the point M_0 are the tern (φ, θ, s) or the tern (φ, θ, τ). Here φ and θ are the azimuthal and the polar angles of outgoing of the ray from the point M_0 (the source), s is the length of a part (segment) of the ray from its beginning M_0 up to M, τ is the time propagation along the ray from M_0 up to M.

Let us designate by ϕ_*, θ_*, and s_* the value of ray parameters at the point M^*. The ray L^* is assumed to be prospective regular (according to the given code) in the sense that for any (ϕ, θ) essentially close to (ϕ_*, θ_*) there is a ray L with the parameters ϕ and θ in the source M_0 going to some point M on the observations surface S and having the same code as the ray L^*.

For regularity of the ray L^* it is enough to require the absence of zero angles of sliding of the ray with boundaries and surface of observations, and also, sufficient smoothness of these surfaces should be not lower C^2 for boundaries and not lower C^1 for the surface of observations.

Further, ϕ, θ, and s are ray parameters of the point M lying on the ray L, going from M_0, and coming to S with the ray code L^*, and the radius is the vector of the point M with the following ray parameters:

$$\vec{r} \equiv \vec{r}(s, \phi, \theta) \equiv \begin{pmatrix} x(s,\phi,\theta) \\ y(s,\phi,\theta) \\ z(s,\phi,\theta) \end{pmatrix}.$$

We assume that the function $\vec{r}(s, \phi, \theta)$ is determined on the open set U of the form

$$U = \Big\{(\phi, \theta, s) \,\Big|\, (\phi, \theta) \in (\phi_* - \delta, \phi_* + \delta) \times (\theta_* - \delta, \theta_* + \delta), \;\; 0 < s < s_* + \delta\Big\},$$

where $\delta > 0$ is a small enough number.

We consider that the ray L^* and the rays L which are close enough to it, can in case of necessity be continued beyond S along the ray in the same medium in which they approached S. This remark is important when S coincides with the interface of media.

Finally, let us write down, the equation of the surface of observations S near to the point M^*:

$$\Phi(x, y, z) = 0. \tag{1}$$

By virtue of the theorem of an implicit function, from the condition of inequality to zero of the angle of sliding of the ray L^* with the surface S follows $\nabla\phi(M^*) \cdot \vec{r}(s_*, \phi_*, \theta_*) \neq 0$.

Equation (1), written down in the ray parameters:

$$\Phi(\vec{r}(s, \phi, \theta)) = 0 \tag{2}$$

defines the parameter s close to the tern (s_*, ϕ_*, θ_*) as function of ϕ and θ:

$$\grave{s} \equiv \grave{s}(\phi, \theta). \tag{3}$$

Now we are able to give a few definitions of objects be dealt with.

Definition 1. Let us name the vectors $\vec{l}_\phi$ and $\vec{l}_\theta$ determined from the equalities

$$\begin{aligned} \vec{l}_\phi &\equiv \vec{l}_\phi(M^*) \equiv \vec{l}_\phi(L^*; M_0, M^*; S_n) \stackrel{\text{def}}{=} \frac{\partial}{\partial \phi}(\vec{r}(s(\phi,\theta),\phi,\theta))\,(\phi_*,\theta_*), \\ \vec{l}_\theta &\equiv \vec{l}_\theta(M^*) \equiv \vec{l}_\theta(L^*; M_0, M^*; S_n) \stackrel{\text{def}}{=} \frac{\partial}{\partial \theta}(\vec{r}(s(\phi,\theta),\phi,\theta))\,(\phi_*,\theta_*) \end{aligned} \tag{4}$$

by the *azimuthal and polar divergences* of the central ray field with the centre at M_0 along the ray L^* at the point M_* along the surface S.

Remark. Further, we omit the index "$*$" with all these terms only for the convenience of description of the conditions of existence and differentiability of the vector-functions $\vec{r}(s(\phi,\theta),\phi,\theta)$.

Let us formulate the problem, which solving is the main objective of the given section.

Let the ray L (with the ray code L^*) pass the boundaries S_i, $i = 1,2,\ldots,n-1$, at the points M_i. Let S be denoted by S_n, and M_n is the point of arrival of the ray to the surface S_n.

Problem. *It is required to construct formulas for finding the vectors of the azimuthal and the polar divergences at the points M_i on the surfaces S_i, $i = 1,2,\ldots,n$, respectively:*

$$\vec{l}_{\phi i} := \vec{l}_\phi(L, M_0, M_i, S_i), \quad \vec{l}_{\theta i} := \vec{l}_\theta(L, M_0, M_i, S_i). \tag{5}$$

For solving this problem it is required to introduce one more class of objects, the so-called angular divergences of the considered central ray field.

Definition 2. Let L be any ray considered in the problem of the central rays field. Let M be a point of this ray with ray parameters s, ϕ, θ (M may not coincide with M_i, $i = 1,2,\ldots,n$), and S be a smooth surface passing through M under non-zero angle to L (S may not coincide with S_i, $i = 1,2,\ldots,n$).

Let us first assume that S is not a boundary, and let $\vec{\tau}(M) \equiv \vec{\tau}(s,\phi,\theta)$ be the unit vector, tangential to a ray at the point M:

$$\vec{\tau}(M) = \vec{r}'_s(s,\phi,\theta) \equiv \dot{\vec{r}}(s,\phi,\theta). \tag{6}$$

The angular azimuthal and polar divergences along L at the point M along the surface S will be called, accordingly, the vectors $\vec{u}_\phi$ and $\vec{u}_\theta$, determined by the equalities

$$\begin{aligned}\vec{u}_\phi &\equiv \vec{u}_\phi(M) \equiv \vec{u}_\phi(L; M_0, M; S) := \frac{\partial}{\partial\phi}(\vec{\tau}(s(\phi,\theta),\phi,\theta)),\\ \vec{u}_\theta &\equiv \vec{u}_\theta(M) \equiv \vec{u}_\theta(L; M_0, M; S) := \frac{\partial}{\partial\theta}(\vec{\tau}(s(\phi,\theta),\phi,\theta)),\end{aligned} \tag{7}$$

where $s(\phi,\theta)$, as well as in (3), is the length of the ray $L = L(\phi,\theta)$ from M_0 up to $M \in S$.

If S coincides with one of the boundaries S_i, $i = 1, 2, \dots, n-1$, we will consider the angular divergences $\vec{u}_\phi^-$, $\vec{u}_\theta^-$ on the part of the ray falling on S and the angular divergences $\vec{u}_\phi^+$, $\vec{u}_\theta^+$ on the leaving part of the reflected or the refracted part of a ray. These values are determined by the formulas obtained from (7) by attributing the marks "$-$" and "$+$" above the letters "u" and "τ". Thus $\vec{\tau}^-(M)$ and $\vec{\tau}^+(M)$ are tangential vectors at the point M falling on S and leaving parts of the ray L.

3.1. Recalculation of angular divergences through the surface interface

For the sake of simplicity of calculations, omit the index "i" of the boundary S_i, as well as in all the objects connected with this boundary, and designate this boundary through S. All the data concerning falling on S at the point M of the ray L parts will be denoted by "$-$", and the data, concerning the leaving part by "$+$".

In Definition 2, we have introduced the tangential vector to the rays $\vec{\tau}^-(M)$, $\vec{\tau}^+(M)$. Let $\vec{n} \equiv \vec{n}^\pm(M)$ be the unit normal vector to S at the point M directed such that $(\vec{n}^-, \vec{\tau}^-) > 0$ and $(\vec{n}^+, \vec{\tau}^+) > 0$.

Write down in the vector form the Snellius law of passage of the ray L through S at the point M:

$$\vec{\tau}^+ - (\vec{n}^+, \vec{\tau}^+)\vec{n}^+ = \gamma \cdot (\vec{\tau}^- - (\vec{n}^-, \vec{\tau}^-)\vec{n}^-). \tag{8}$$

Here

$$\gamma = \frac{V^+(M)}{V^-(M)}. \tag{9}$$

Using the scalar form of the Snellius law let us express the cosine angle of the reflection or refraction $(\vec{n}^+, \vec{\tau}^+)$ through the cosine of the angle of incidence $(\vec{n}^-, \vec{\tau}^-)$. We have

$$\begin{aligned}(\vec{n}^+, \vec{\tau}^+) &= \cos\beta^+ = \sqrt{1 - \sin^2\beta^+} = \sqrt{1 - \gamma^2 \sin^2\beta^-}\\ &= \sqrt{(1-\gamma^2) + \gamma^2(\vec{n}^-, \vec{\tau}^-)^2}.\end{aligned} \tag{10}$$

Then, taking into account that

$$\vec{n}^+ = (\vec{n}^+, \vec{n}^-)\vec{n}^-, \tag{11}$$

we can write down

$$\vec{\tau}^{+} = \gamma\vec{\tau}^{-} + \lambda\vec{n}^{-}, \tag{12}$$

where

$$\lambda = (\vec{n}^{+},\vec{n}^{-})\sqrt{(1-\gamma^2)+\gamma^2(\vec{n}^{-},\vec{\tau}^{-})^2} - \gamma\cdot(\vec{n}^{-},\vec{\tau}^{-}). \tag{13}$$

Remark. Note, that in the case of reflection

$$\lambda = -(\Delta + \gamma\cdot(\vec{n}^{-},\vec{\tau}^{-})) = -\chi$$

and in the case of refraction

$$\lambda = \Delta - \gamma\cdot(\vec{n}^{-},\vec{\tau}^{-}) = \frac{1-\gamma^2}{\Delta+\gamma\cdot(\vec{n}^{-},\vec{\tau}^{-})} = \frac{1-\gamma^2}{\chi},$$

where $\Delta = \sqrt{(1-\gamma^2)+\gamma^2\cdot(\vec{n}^{-},\vec{\tau}^{-})^2}$.

In order to obtain the required formulas of recalculation it is necessary to differentiate (12) with respect to the parameters ϕ and θ under condition (3). As all the calculations are identical for each of the parameters ϕ and θ, we give them a common name "α".

According to (7) we have

$$\frac{\partial\vec{\tau}^{\pm}}{\partial\alpha} \equiv \frac{\partial\vec{\tau}^{\pm}(s(\phi,\theta),\phi,\theta)}{\partial\alpha} = \vec{u}_{\alpha}^{\pm}. \tag{14}$$

As in (2) $\vec{n}^{-} \equiv \vec{n}^{-}(\vec{r}(s(\phi,\theta),\phi,\theta))$, then according to the rule of differentiation of superpositions and (4) we find that

$$\frac{\partial\vec{n}^{-}}{\partial\alpha} \equiv d\vec{n}^{-}(M)\langle\vec{l}_{\alpha}\rangle, \tag{15}$$

where the right-hand side is the value of differential of a unit normal vector $\vec{n}^{-}$ to S tangential to S on the vector of linear divergence $\vec{l}_{\alpha}$.

Leaving aside the question about the coordinate presentation of the right-hand side of (15), let us write out the result of differentiation of identity (12). We should note, that the functions $\gamma(M)$, $(\vec{n}^{+},\vec{n}^{-})(M)$ are constant. We have

$$\vec{u}_{\alpha}^{+} = \gamma\vec{u}_{\alpha}^{-} + \lambda d\vec{n}^{-}\langle\vec{l}_{\alpha}\rangle + \frac{\partial\lambda}{\partial\alpha}\vec{n}^{-}, \tag{16}$$

$$\frac{\partial\lambda}{\partial\alpha} = -\frac{\gamma(\vec{n}^{+},\vec{n}^{-})}{(\vec{n}^{+},\vec{\tau}^{+})}\lambda\frac{\partial(\vec{n}^{-},\vec{\tau}^{-})}{\partial\alpha}, \tag{17}$$

$$\frac{\partial(\vec{n}^{-},\vec{\tau}^{-})}{\partial\alpha} = (d\vec{n}^{-}\langle\vec{l}_{\alpha}\rangle,\vec{\tau}^{-}) + (\vec{n}^{-},\vec{u}_{\alpha}^{-}). \tag{18}$$

Thus the recalculation formulas have been obtained.

3.2. Recalculation of azimuthal and polar divergences between the interfaces

Let us now establish the connection between $\vec{l}_{\alpha i}$, $\vec{u}^+_{\alpha i}$ and $\vec{l}_{\alpha i-1}$, $\vec{u}^-_{\alpha i-1}$, $i = 1, 2, \dots, n$; $\alpha := \phi, \theta$.

In view of linearity of parts of rays between the points M_{i-1} and M_i the angular divergence is transferred from the point M_{i-1} up to the point M_i without changes, or more precisely

$$\vec{u}^-_{\alpha i} = \vec{u}^+_{\alpha i-1}, \quad i = 1, 2, \dots, n. \tag{19}$$

To recalculate the linear divergence, let us take advantage of the linearity of parts of rays between the points M_{i-1} and M_i.

We will vary the parameter α by the value $d\alpha$. Then the point M_{i-1} will be subject to moving $\overline{M_{i-1}M'_{i-1}}$ on the surface $S_{i-1}(i > 1)$:

$$\overline{M_{i-1}M'_{i-1}} = \vec{l}_{\alpha i-1}d\alpha + o(d\alpha),$$

and the vector $\vec{\tau}^+_{i-1}$ will thus receive the increment $\Delta\vec{\tau}^+_{i-1}$ determined by the formula

$$\Delta\vec{\tau}^+_{i-1} = \vec{u}^+_{\alpha i-1}d\alpha + o(d\alpha).$$

Let us calculate the move $\overline{M_iM'_i}$ of the points M_i caused by variation in the parameter α.

At the first, move the point M_i to the point E on the vector $\overline{M_{i-1}M'_{i-1}}$. It is obvious that

$$|\overline{M'_{i-1}E}| = |\overline{M_{i-1}M_i}|.$$

Then the point E is moved on the vector $|M_{i-1}M_i| \cdot \Delta\vec{\tau}^+_{i-1}$ to the point E'. Obviously, the point E' is on the disturbed ray, and

$$\begin{aligned}\overline{M_iE'} &= \overline{M_{i-1}M'_{i-1}} + |M_{i-1}M_i| \cdot \Delta\vec{\tau}^+_{i-1} \\ &= (\vec{l}_{\alpha i-1} + |M_{i-1}M_i| \cdot \vec{u}^+_{\alpha i-1})d\alpha + o(d\alpha).\end{aligned} \tag{20}$$

Finally, the point M'_i is obtained by mapping the point E' on S_i in a direction $\vec{\tau}^+_{i-1} + \Delta\vec{\tau}^+_{i-1}$ the disturbed ray.

The point M'_i can be taken as projection of the point E' parallel to the undisturbed direction $\vec{\tau}^+_{i-1}$ on the surface tangential to S_i at the point M_i i.e., accurate to the infinitely small value of order higher than $d\alpha$:

$$\overline{M_iM'_i} = \overline{M_iE'} + \mu \cdot \vec{\tau}^+_{i-1} + o(d\alpha),$$

where μ is found from the condition

$$(\overline{M_iE'} + \mu \cdot \vec{\tau}^+_{i-1}, \vec{n}^-_i) = 0.$$

Thus,

$$\overline{M_iM'_i} \equiv \overline{M_iE'} - \frac{(\overline{M_iE'}, \vec{n}^-_i)}{(\vec{n}^-_i, \vec{\tau}^-_i)} \cdot \vec{\tau}^+_{i-1} + o(d\alpha). \tag{21}$$

Taking into account that $\overline{M_i M_i'} = \vec{l}_{\alpha i} d\alpha + o(d\alpha)$, and basing on (20) and (21) we come to the following formulas:

$$\vec{l}_{\alpha i} = \vec{A}_{i-1} - \frac{(\vec{A}_{i-1}, \vec{n}_i^-)}{(\vec{n}_i^-, \vec{\tau}_i^-)} \cdot \vec{\tau}_{i-1}^+, \tag{22}$$

$$\vec{A}_{i-1} = \vec{l}_{\alpha i-1} + |M_{i-1}M_i| \cdot \vec{u}_{\alpha i-1}, \tag{23}$$

$\alpha := \phi, \theta$, $i = 1, 2, \ldots, n$.

The initial conditions are the following:

$$\vec{l}_{\alpha 0} = 0, \qquad \alpha := \phi, \theta;$$

$$\vec{u}_{\alpha 0}^+ = \begin{cases} (-\sin\phi \sin\theta)\vec{i} + (\cos\phi \sin\theta)\vec{j}, & \alpha := \phi; \\ (\cos\phi\cos\theta)\vec{i} + (\sin\phi\cos\theta)\vec{j} + (-\sin\theta)\vec{k}, & \alpha := \theta. \end{cases}$$

Here ϕ, θ are the angular coordinates of the spherical system which correspond to the rectangular system obtained from the initial system by shifting the origin of coordinates to the point M_0. In this case, $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors of the initial system of coordinates.

It is now necessary to write out the formulas for the so-called Rodrig vector $d\vec{n}\langle\vec{l}\rangle$ in the case of the explicit equation $z = f(x, y)$ of the surface S.

The index "$-$" of the vector $\vec{n}$ and the subscript "α" of the vector $\vec{l}$ are omitted for simplicity.

Let $(\vec{r}_x', \vec{r}_y')$ be the basis coordinate system (x, y) on the surface S at the point $M(x, y, z(x, y))$:

$$\vec{r}_x' = (1, 0, f_x')^T, \qquad \vec{r}_y' = (0, 1, f_y')^T. \tag{24}$$

Then for any vector $\vec{h} = (h_x, h_y, h_z)^T$, which belongs to the tangential plane to S at the point M, the following equality is valid

$$\vec{h} = h_x \cdot \vec{r}_x' + h_y \cdot \vec{r}_y'. \tag{25}$$

The vectors

$$\begin{aligned} \vec{n}_x' &= \frac{\partial \vec{n}(x, y, z(x, y))}{\partial x} \stackrel{\text{def}}{=} d\vec{n}\langle\vec{r}_x\rangle, \\ \vec{n}_y' &= \frac{\partial \vec{n}(x, y, z(x, y))}{\partial y} \stackrel{\text{def}}{=} d\vec{n}\langle\vec{r}_y\rangle \end{aligned} \tag{26}$$

belong to the tangential plane to S at the point M, i.e., $\vec{n}_x', \vec{n}_y' \in T_M S$. These expressions can be obtained, using the known formulas of differential geometry in the case of general parametric equations of a surface (the Weingarten formula [5]).

In conclusion, in the coordinate system (x, y), a unit normal vector $\vec{n} = \vec{n}(x, y, z(x, y))$ is written down in the form

$$\vec{n} = \frac{1}{\sqrt{1 + f_x'^2 + f_y'^2}} \cdot (f_x', f_y', -1)^T \equiv (n_x, n_y, n_z)^T. \tag{27}$$

As a result of differentiation of equation (27) with respect to x and y we obtain

$$\vec{n}'_{x/y} = \vec{b}_{x/y} - (\vec{b}_{x/y} \cdot \vec{n}) \cdot \vec{n}, \tag{28}$$

where

$$\vec{b}_x = -n_z \cdot (f''_{xx}, f''_{yx}, 0)^T, \quad \vec{b}_y = -n_z \cdot (f''_{xy}, f''_{yy}, 0)^T.$$

Taking into consideration the remark on the representation of the tangential vector in the basis $(\vec{r}'_x, \vec{r}'_y)$ (formula (25)), we can write down:

$$\begin{aligned} \vec{n}'_x &= (b_{xx}(n_z^2 + n_y^2) - b_{xy}n_xn_y)\vec{r}'_x + (-b_{xx}n_xn_y + b_{xy}(n_z^2 + n_x^2))\vec{r}'_y, \\ \vec{n}'_y &= (b_{yx}(n_z^2 + n_y^2) - b_{yy}n_xn_y)\vec{r}'_x + (-b_{yx}n_xn_y + b_{yy}(n_z^2 + n_x^2))\vec{r}'_y. \end{aligned} \tag{29}$$

Finally, in view of linearity of the mapping $\vec{l} arrow d\vec{n}\langle l\rangle, l \in T_M(S)$

$$d\vec{n}\langle\vec{l}\rangle = l_x\vec{n}'_x + l_y\vec{n}'_y := v_x\vec{r}'_x + v_y\vec{r}'_y, \tag{30}$$

where

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = BT \begin{pmatrix} l_x \\ l_y \end{pmatrix}. \tag{31}$$

Here B is a matrix of the second square form of a surface $z = f(x, y)$

$$\begin{aligned} B &= -n_z \begin{pmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{pmatrix}, \\ T &= \begin{pmatrix} n_z^2 + n_y^2 & -n_xn_y \\ -n_xn_y & n_z^2 + n_x^2 \end{pmatrix}. \end{aligned} \tag{32}$$

Formulas (30)–(32) at $\vec{l} \equiv \vec{l}_\phi$ within the factor $(\vec{n}^- \cdot \vec{n}^+)$ and the index "i" are the calculation formulas for the vector $d\vec{n}^-\langle\vec{l}_{\phi i}\rangle$.

4. The algorithm of hitting in receivers by the ray method

Let us assume that the surface of observations S is described by the equation

$$z = f(x, y), \quad (x, y) \in [0, L] \times [0, L]$$

with a smooth function f and the two-dimensional grid of points $\{P_{ij}\}$ on S:

$$P_{ij} = (x_i, y_j, f(x_i, y_j)), \quad i = 1, \ldots, N_1, \quad j = 1, \ldots, N_2,$$

where (x_i, y_j) are nodes of a two-dimensional grid located in the plane xOy. It is required to find all the rays of the given code which are leaving the source at the point M_0 and coming to receivers located at the points P_{ij}, $i = 1, \ldots, N_1$, $j = 1, \ldots, N_2$.

Let us assume that the ray to which we are going to hit and thus its ray parameters ϕ_0, θ_0 are found coming to some point $P_0 \in \{P_{ij}\}$. Then as the next point $P_* \in \{P_{ij}\}$ will be the nearest to P_0 grid node $\{P_{ij}\}$ not hit yet.

Describe the algorithm of obtaining the values ϕ_*, θ_* for the point P_* by consecutive determination of the ray parameters ϕ, θ starting with ϕ_0, θ_0.

Consider the field of rays of the given code to be regular in the vicinity of P_0 and the point P_* to be in this vicinity.

Then in the vicinity of the point P_0, the surface of observations can be regularly parameterized by the ray parameters ϕ, θ of the rays of this ray code, crossing the surface near to the point P_0. Let $(\phi, \theta) \to \vec{r}(\phi, \theta)$ be the specified parameterization, and (ϕ, θ) belongs to some vicinity of the point (ϕ_0, θ_0). In the previous section, the calculation formulas for the natural local basis $\frac{\partial \vec{r}}{\partial \phi} := \vec{l}_\phi$, $\frac{\partial \vec{r}}{\partial \theta} := \vec{l}_\theta$ on S in the chosen parameterization be given. Our task consists in finding the ray parameters ϕ_*, θ_* of the points P_* provided that we are able to calculate $\vec{r}(\phi, \theta)$ and the vector $\vec{l}_\phi, \vec{l}_\theta$ for some value (ϕ, θ) close to (ϕ_0, θ_0).

The beginning of the algorithm. Let us map the vector $\overrightarrow{P_0P_*}$ on the tangential plane to S at the point P_0 (in the given version mapping is not orthogonal, but parallel to the axis $0z$). The obtained vector $\overrightarrow{P_0P_*}'$ (already being in a subspace of the vectors $\vec{l}_\phi$, $\vec{l}_\theta$) is decomposed to the vectors $\vec{l}_\phi(\phi_0, \theta_0)$, $\vec{l}_\theta(\phi_0, \theta_0)$:

$$\overrightarrow{P_0P_*}' = \Delta\phi_0 \cdot \vec{l}_\phi(\phi_0, \theta_0) + \Delta\theta_0 \cdot \vec{l}_\theta(\phi_0, \theta_0).$$

Assuming $\phi_1 = \phi_0 + \Delta\phi_0$, $\theta_1 = \theta_0 + \Delta\theta_0$, we calculate the point P_1 on S with the ray parameters ϕ_1, θ_1, that is, $\vec{r}(P_1) = \vec{r}(\phi_1, \theta_1)$, and the vectors $\vec{l}_\phi(\phi_1, \theta_1)$, $\vec{l}_\theta(\phi_1, \theta_1)$.

Continuation and termination of the algorithm. If the point P_1 appears close to P_* with the given accuracy, the solving the problem is finished. If the desired accuracy is not attained, then we pass to the beginning of the algorithm by increasing of unit the subscripts of the letters P, ϕ, θ.

As a result of this iterative procedure the ray parameters (ϕ_n, θ_n) at the points P_n close to P_* are calculated with the given accuracy. These parameters are taken as the sought for values of (ϕ_*, θ_*).

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