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Numerical models of the electrostatic shock waves*

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Abstract. The self-consistent evolution of the ion-acoustic collisionless shock with shock-reflected ions is numerically studied with kinetic simulations. We want to compare different approaches to the shock simulation based on the PIC-method. The results of one-dimensional fully kinetic simulation for both plasma components with the real ion-to-electron mass ratio are compared with those obtained for two different forms of the electron density distribution in terms of the wave electrostatic potential. One distribution corresponds to the Boltzmann approach, the other — to the approach of the adiabatically trapped electrons.

Introduction

The origin of cosmic rays has a significant influence on the processes in the near-Earth space. By now it has not been precisely determined and still is one of the main questions in cosmology. Observational astronomical data suggest that the source of these rays is the supernova explosions and the acceleration on the front of a collisionless shock wave can be the generation mechanism. The difficulty of confirming this assumption is complicated due to the recent observations made with the use of modern complex equipment, which raised a number of new questions and the need to revise generally accepted hypotheses and theories. The formation of shock waves excluding collision has been investigated since the 60s of the 20th century [1, 2]. However, our understanding of collisionless shocks, including their structure and mechanisms whereby they accelerate particles, remains incomplete.

The possibility of using supercomputers enabled one to carry out more detailed simulations of shock waves. Mathematical modeling makes possible to study in detail the mechanism of acceleration of cosmic rays on the front of the collisionless shock wave and its dependence on environmental parameters.

Here we present the results of numerical modeling of an ion-acoustic collisionless shock based on the one-dimensional kinetic approximation for ions. The reflection effictiveness, the velocity distribution of reflected particles and the shock electrostatic structure are studied in terms of the shock parameters.

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The most complete description is possible with the Vlasov kinetic equations for electrons and ions and Maxwell's equations. Nevertheless, the problems in numerical implementation of this model, associated with essential differences in spatial and temporal scales for electron and ion plasma components are complicated even for calculations with modern computer systems. We take into account two different cases of electron spatial distribution. The first case is the Boltzmann distribution [1]. The second case is the adiabatically trapped electrons [3, 4]. These two cases are similar in that they both predict the steadily propagating solitons that have limiting Mach numbers M.

1. Statement of the problem

We consider the classical problem of the decay of the initial density discontinuity. The self-consistent evolution of an ion-acoustic collisionless shock with shock-reflected ions is numerically studied by kinetic simulations. The density profile has the shape of a step. We measure the density of fully ionized hydrogen plasma components in terms of the unperturbed density n_0, coordinates — in terms of the Debye length $\lambda_D = \left(\frac{T_e}{4\pi n_0 e^2}\right)^{1/2}$; the velocity — in terms of the electron thermal speed $u_0 = \left(\frac{T_e}{m_e}\right)^{1/2}$, the time $\frac{1}{\omega_{pe}} = \left(\frac{m_e}{4\pi n_0 e^2}\right)^{1/2}$ and the electric field — in terms of $E_0 = (4\pi n_0 T_e)^{1/2}$.

Initial and boundary conditions with $T_i = 0$ are the following:

$$
\begin{aligned}
t = 0: \quad & f_e = \exp\left(\frac{-m_e(v_e - v_0)^2}{2kT_e}\right), \quad v_i = 0, \quad \vec{E} = 0, \\
& n(x,0) = \begin{cases} n_1, & 0 \le x \le x_0, \\ n_2, & x_0 \le x \le L, \end{cases} \\
x = 0: \quad & v_e = v_i = 0, \quad \vec{E} = 0, \\
x = x_{\max}: \quad & v_e = v_i = 0, \quad \vec{E} = 0,
\end{aligned}
$$

where x_0 is the break position, L is the area size.

In the PIC-method, we ensure a high enough resolution to resolve low-density regions, sharp gradients and the dynamics of both electrons and ions by taking at least 100 particles per cell and the spatial resolution better than $0.05\lambda_D$, where λ_D is the Debye length.

The results of the one-dimensional fully kinetic simulations for the ion plasma component are compared with two different forms of the electron density distribution in terms of the wave electrostatic potential. The electron distribution corresponds to:

- the kinetic approach,
- the Boltzmann approach, and
- the adiabatically trapped electrons.

The kinetic approach. In order to consider the ions that are reflected and escape from the shock, to incorporate the ion reflection into the global shock structure and to investigate its effect on the shock itself, we use the kinetic plasma description. The plasma dynamics is governed by the Vlasov equations for the distribution functions of the plasma components:

$$\frac{\partial f_{i,e}}{\partial t} + \vec{v}\frac{\partial f_{i,e}}{\partial \vec{r}} + \frac{q_{i,e}}{m_{i,e}}\vec{E}\frac{\partial f_{i,e}}{\partial \vec{v}} = 0 \tag{1}$$

and Poisson's equation for the electrostatic field:

$$\operatorname{div}\vec{E} = -\Delta\varphi = 4\pi\rho, \tag{2}$$

where $f_{i,e}$ is the particle distribution function of plasma electrons and ions, $\vec{E}$ is the electric field, v is the velocity of the particles, ρ is the electric charge density, and $q_{i,e}$ is the charge of particles. The electric charge density ρ is defined by equation

$$\rho = e\int (f_i - f_e)\, d\vec{v}.$$

The hybrid approach. In this approach, the kinetic equation for the ion distribution function is used:

$$\frac{\partial f_i}{\partial t} + \vec{v}\frac{\partial f_i}{\partial \vec{r}} + \frac{q_i}{m_i}\vec{E}\frac{\partial f_i}{\partial \vec{v}} = 0.$$

Poisson's equation (2) is used to describe the electrostatic field.

The charge density is calculated by the integration for ions and from the potential for electrons:

$$\rho = q_i\int f_i\, d\vec{v} - \rho_e(\varphi).$$

For this problem we want to compare two different approaches for electron density. The first one is the well-known Boltzmann approach (see [1]). In this case, the charge density is described by the Boltzmann function:

$$\rho_e = \exp\varphi_0, \quad \varphi_0 = \frac{e\varphi}{kT}. \tag{3}$$

In the second case, the electron charge density is described by the Gurevich function [3]. The corresponding analytical expression for the Gurevich function was obtained by Malkov [4]:

$$\rho_e = \frac{1}{1+\sqrt{1.5\varphi_0}} + 1.96\sqrt{\frac{\varphi_0}{\pi}}. \tag{4}$$

2. The solution to governing equations

The PIC-method (see [5, 6]) is used to solve the Vlasov equation. In this method, plasma is simulated by a set of separate particles, each characterized by the motion of many physical particles. The characteristics of the Vlasov equation describe the trajectories of particles. The equations of these characteristics for the particle j are described as

$$\frac{dx_j}{dt} = v_j, \quad \frac{dv_j}{dt} = \frac{q_j}{m_j} E(x_j).$$

The electrostatic field is obtained by the direct integration of the charge density $\frac{dE}{dx} = 4\pi\rho$, which, in turn, is calculated from the individual particle positions.

For solving the equations of motion of particles, an organic time-reversible and second order leapfrog algorithm is applied. The spatial second order accuracy is obtained by calculating the electric field and the charge density on staggered grids.

3. The results of simulation

Let us compare two different statements of the problem in question. In the first case, the electron density is described by (3). In the last case, it is described by (4).

The Mach number of the shock wave is an important parameter for determining the character of the plasma flows, $M = V_i/C_s$, V_i is the shock wave velocity. The charge density (Figure 1a) has the soliton structure and corresponds to the laminar regime.

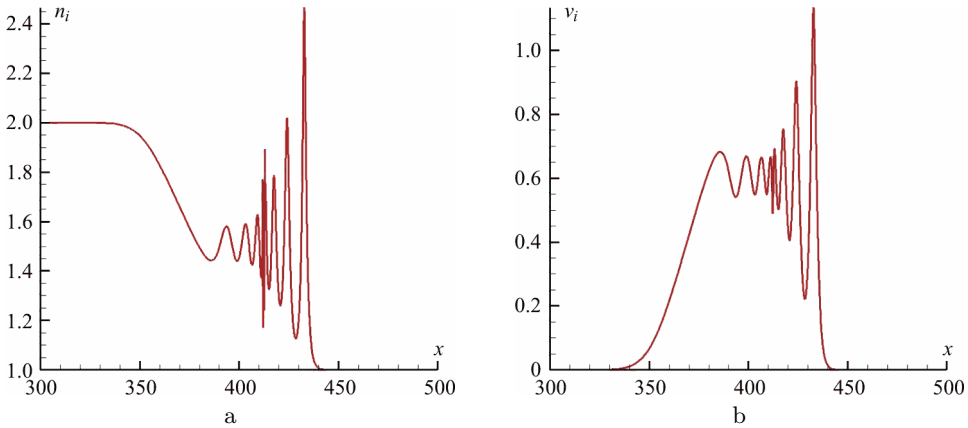

Figure 1. The charge density (a) and the phase space (b) for the initial maximum density $n_1/n_2 = 2$ and the time step 20 w_{pi}^{-1}

From Figure 2, we see that if the density decay increases, then the soliton begins to reflect the upstream ions. These results are obtained for the Gurevich approach. The results of both approaches have a similar structure, but the obtained Mach number is equal to 3.2 for the Gurevich approach 1.6 for the Boltzmann approach.

Figure 3 shows the phase space graphics for the initial maximum density $n_1/n_2 = 5$. We see that the particle emission for the Gurevich approach has a different structure than with the use of the Boltzmann approach.

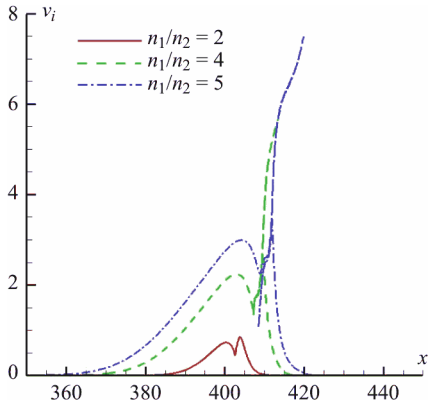

Figure 2. The phase space for different initial maximum densities

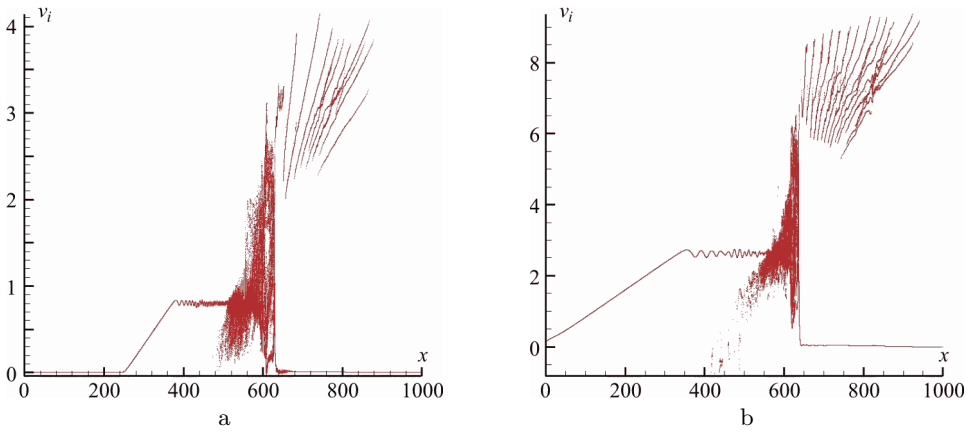

Figure 3. The phase space for a) Boltzmann approach and b) Gurevich approach

The Mach number, at which the wave is tipping is equal to 3.2.

The ions reflected from the shock are clearly visible on the phase plane. The ions are reflected at a single point, where the potential reaches its maximum and the electric field has a jump. The number of ions reflected from the shock increases with the growth of the flow velocity. It is clearly seen that when the ions begin to reflect from the soliton tip, the classical single solution bifurcates into a more complex structure that comprises the leading soliton, the periodic wave train is downstream of it and the foot is occupied by the reflected ions. This foot is supported by the reflected ions and also accelerates them somewhat further.

In Figure 4, there is a time history of the maximum ion density and the number of emitted particles. We can see fluctuations in the graph, and

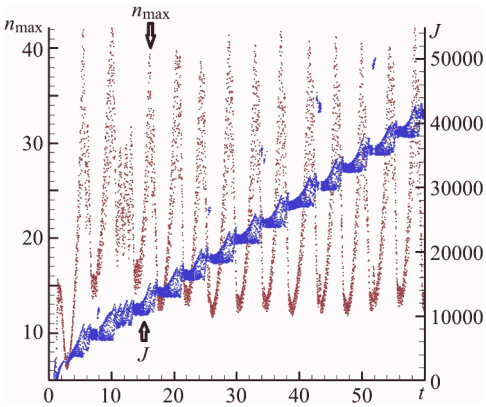

Figure 4. The time history of the maximum density n_{max} and the number of emitted particles J

it can be seen that the density maxima correspond to a maximum emission of particles.

The influence of the number of particles per cell on the solution has been observed. In the fully kinetic approach, we set the velocity spread for both electrons and ions. As is shown in our previous studies [7, 8], the number of particles per cell is a very important parameter for the PIC-simulation. It is the phase space with the time step equal to 74, and with 10 and 500 particles per cell (Figure 5). One can see that these pictures

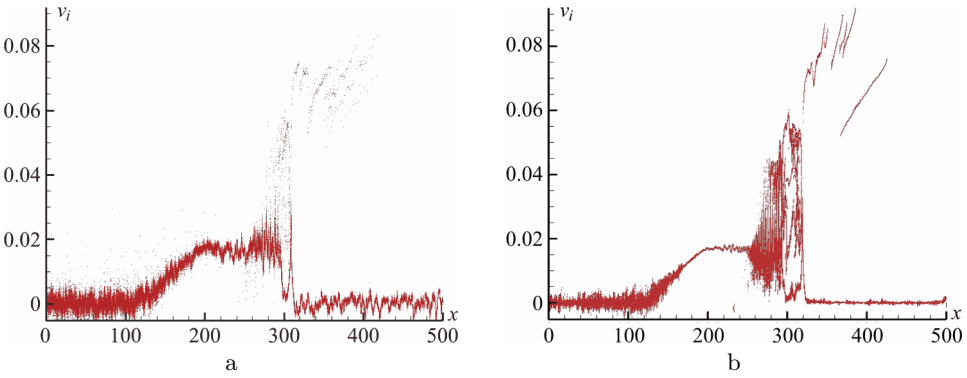

Figure 5. The phase space for 10 (a) and 500 (b) particles per cell

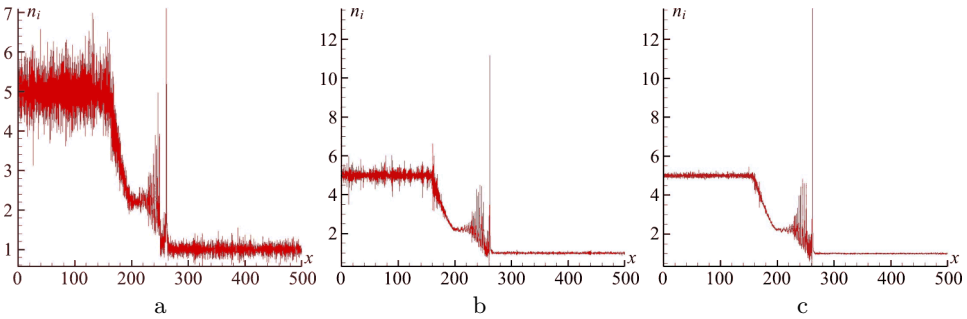

Figure 6. The charge density for 10 (a), 250 (b), and 500 (b) particles per cell

differ in significant ways from each other. In Figure 6, the charge density is shown for the number of particles per cell from 50 to 500. The result is similar, but maximum amplitudes are different, and we have to choose more than 100 particles per cell for calculations.

Conclusion

Numerical simulations of the ion-acoustic collisionless shock are performed using 1D kinetic approximation both for electrons and ions with the real mass ratio as well as within a hybrid approach. The reflection effectiveness, velocity distribution of reflected particles, and shock electrostatic structure are studied in terms of the shock parameters. The solution extends a classic soliton solution beyond the critical Mach number $M = 1.6$ for the Boltzmann approach and $M = 3.2$ for the Gurevich approach, where the soliton ceases to exist because of the upstream ion reflection.

These purely electrostatic simulations are relevant to the microphysics of particle reflection of the shock front including the cosmic ray loaded shocks. The influence of the number of particles per cell on the solution has been investigated and its optimal value has been determined.

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