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Effective algorithm of spectral estimation for extended realizations with any frequency entry point

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The raise of efficiency for information process engineerings is an actual problem. In the paper, the effective algorithm of digital handling of signals is presented. The given algorithm can be used in many applications of the spectral analysis.

Among a large number of spectral estimation algorithms the most common is an algorithm of the fast Fourier transform (FFT), such as the Kuly-Tuyki algorithm [1, 2]. Currently the FFT algorithm remains "hors concours" in view of its practical application. Such a scope of application is due to its high efficiency at the expense of the maximal minimization of operations of complex multiplication, which explains its speedy response. The FFT algorithm is so densely packed in its interior structure, whose basic element is an algorithm "butterfly", that it does not allow a differentiated entry into the frequency domain of the results obtained (any frerquency entry point). The second essential disadvantage of the conventional FFT algorithms is impossibility of their application for processing in the real-time mode.

As a frequency entry point we understand the following. The N-point realization, generally a time function, serves as input information for the N-point FFT. The FFT procedure yields a result as N-point realization of spectral estimation. At the digital handling a frequency band is determined by discretization frequency and is equal to F_d. A frequency step is equal to F_d/N. Thus, after fulfilling the FFT procedure we obtain N reports with a frequency step F_d/N. If we want to consider a frequency band from F_1 to F_2, this circumstance does not facilitate the FFT procedure. In this case the FFT algorithm should calculate all N reports.

Thus, as a frequency entry point for the FFT procedure the author understands either the points F_1 and F_2, or a set of such points. The interest towards separate segments of the spectrum is obvious. Based on the abovesaid it is possible to make a conclusion that conventional algorithms of the FFT do not allow calculations in the real-time mode with any frequency entry point.

Such a possibility is provided by algorithms with a flexible structure and, obviously, the "rigid" FFT algorithm in this case is not appropriate.

What is to be done in this situation? The FFT algorithms are based on the discrete Fourier transform (DFT). The DTF procedure has a possibility of an arbitrary choice of frequency entry points in the algorithm of handling. However it is a well-known fact that the effectiveness of the DTF is not compatable to the fast FFT at a great number of points of processing.

The author considers the fast algorithm of the FFT, which in effectiveness is compatable to the FFT algorithm, but has rather a flexible algorithm in view of the amount of frequency entry points in the spectral estimation procedure.

It is well-known to experts in digital handling that there is an expression to describe the discrete Fourier transform:

$$X(k) = \sum_{n=0}^{N-1} x(n) W^{nk}, \quad W = e^{-2\pi j/N}, \tag{1}$$
$$n = 0, 1, \ldots, N-1, \quad k = 0, 1, \ldots, N-1.$$

resulting in N frequency reports uniformly located along the whole frequency axis. Disadvantages connected with realization of this expression on a software or a hardware level, are mentioned above.

In many areas of science and technology such as radio-electronics, automatic control theory, acoustics the logarithmic scale for representation of spectral estimations is used, that allows selection and detection of characteristic segments of the spectrum describing and connected with major natural phenomena: resonance, integration, differentiation, qualitative picture of noise: pink, poorly painted, white noise. At a logarithmic scale there is no need to obtain uniformly distributed frequency estimations by formula (1).

As a basic expression for the Fourier transform it is proposed to use the following:

$$X(k) = \sum_{n=0}^{N-1} x(n) W^{(n,k)}, \quad W^{(n,k)} = e^{-2\pi n j/(N-k)}, \tag{2}$$
$$n = 0, 1, \ldots, N-1, \quad k = 0, 1, \ldots, N-1.$$

A close comparison of expressions (1) and (2) reveals a minor difference of one formula from the other. This difference consists in the choice of the index k. Formula (1) brings about spectral estimations, uniformly distributed on the frequency axis. Formula (2) results in k spectral estimations, non-uniformly distributed on the linear frequency axis. When using a logarithmic frequency axis for expression (2) we obtain uniformly distributed spectral estimations. Using expression (2) for calculation of spectral estimations, it is possible to construct an effective algorithm to calculate such estimations. Figure 1 presents a block-scheme of the algorithm illustrating

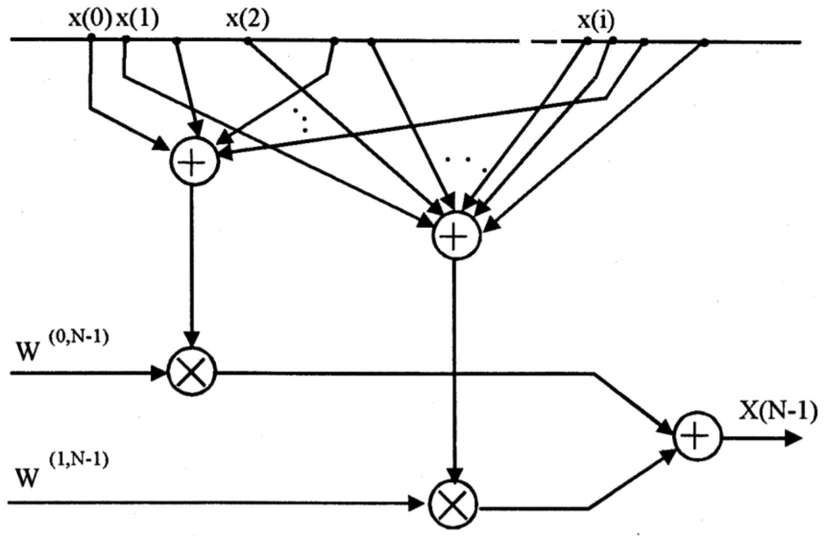

Figure 1

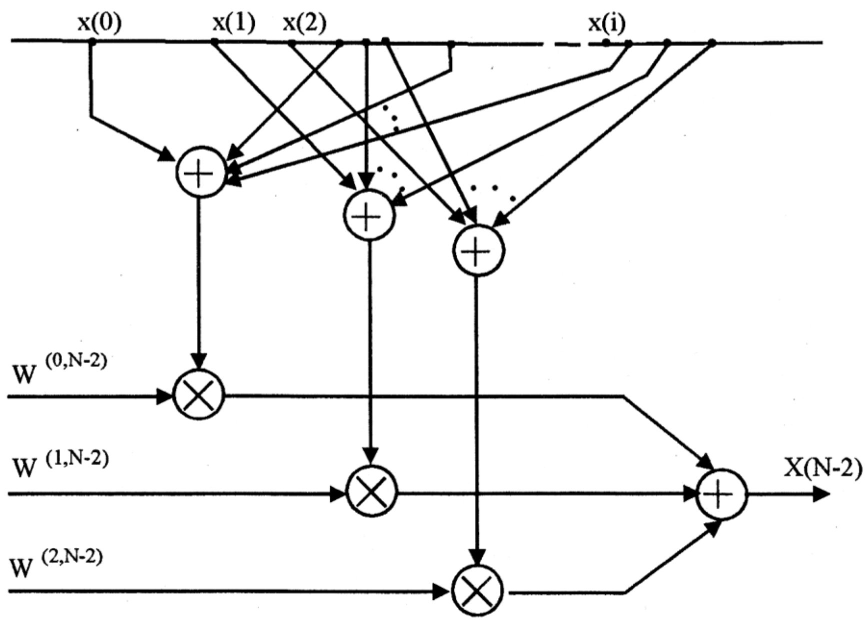

Figure 2

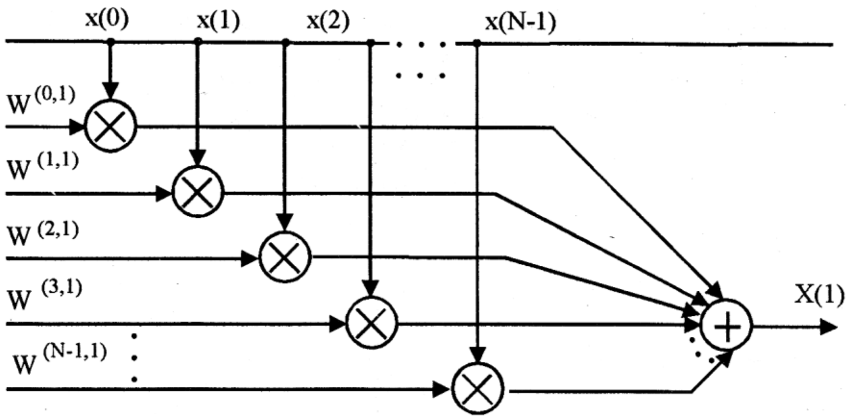

Figure 3

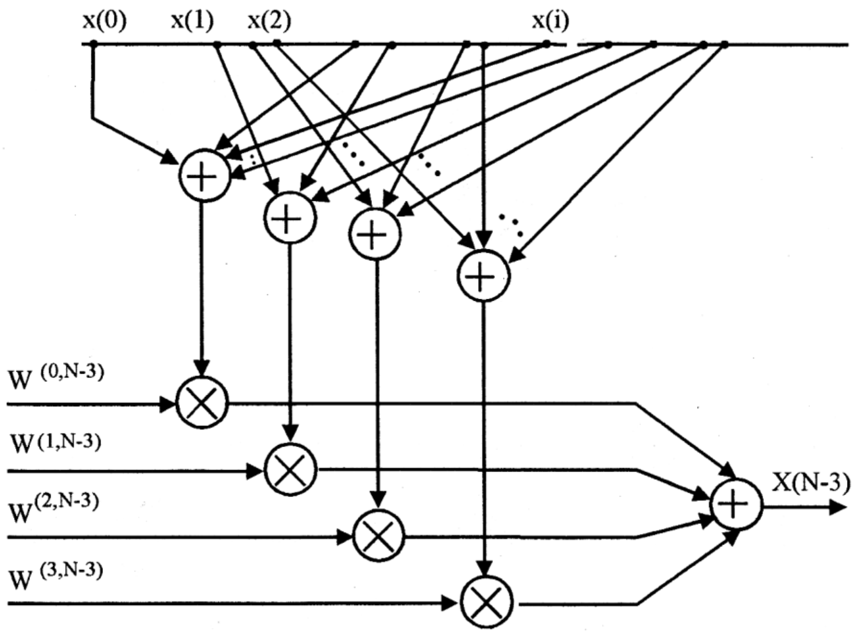

Figure 4

the basic concept of the given calculation and implying the resultant possibilities of optimization of calculations in view of the amount of complex multiplications.

Figures 2–4 present procedures of calculation of the spectral estimations $X(N-1)$, $X(N-2)$, $X(N-3)$, respectively. Here the algorithm of further calculation of the spectral estimations $X(k)$ is explicitly expressed. The procedure calculating the spectral estimation $X(1)$ degenerates to the procedure of the calculation presented in Figure 1.

Analysis of the given algorithm shows that the amount of complex multiplications does not exceed

$$\Pi = \frac{N \ln N}{2}, \tag{3}$$

where N is the amount of points of calculation.

As compared to the conventional algorithm of the fast Fourier Tuyki–Kulie transform, we obtain the following comparative evaluation:

$$S = \frac{N \ln N}{N \log_2 N}. \tag{4}$$

Expression (4) shows the expediency of application of the given algorithm to many problems, connected with calculation of spectral extimations in the real-time mode, in particular, with seismic data processing.

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