*Bull. Nov. Comp. Center*, Math. Model. in Geoph., 5 (1999), 149–161


# Joint inverse dynamic problem for the 1D heterogeneous isotropic medium*

S.N. Yermolayev

The present paper deals with the "joint" inverse dynamic seismic problem. An attempt is made to increase precision of the medium structure reconstruction. The problem is solved for medium having absorption, using complementary information on real structure of the medium.

## 1. Introduction

Precision and timeliness of forecasting the stratum properties in the borehole have a considerable influence upon effectiveness of boring (increase of speed, decrease of cost and accident rate). In this connection, the necessity in algorithms which can more exactly reconstruct the medium in the borehole arises. It is natural to assume that, having additional information of the medium structure along the borehole (obtained, for example, by means of the vertical seismic profiling (VSP) method or of the stratum property analysis in the course of boring), we can get more stable and consequently more accurate seismic methods.

The present paper deals with the inverse dynamic seismic problem. An attempt is made to increase precision of the medium structure reconstruction. The problem is solved for media having absorption. The absorption of the medium is simulated as coefficient of the first-order derivative with respect to the time co-ordinate in the general system of dynamic elasticity theory equations, as done in the paper by Biot [9].

The purpose of this paper is to build an algorithm, based on the given model of elastic wave propagation in the media having absorption, using the given information on the structure along certain intervals of the borehole, for a more detailed reconstruction of the medium structure in the vicinity of the borehole.

In this paper, the solutions of the direct and the inverse problems are presented, provided that the medium absorption function is known. The numerical algorithms for solving the direct and inverse problems as well as the results of experiments are given.

*Supported by the program "Integration" under Grant 247.

## 2. Statement of the problem

Let the whole space $\boldsymbol{R}^3$ be filled with an elastic isotropic medium having the Lame parameters $\lambda$, $\mu$, and the density $\rho$. We assume $\lambda$, $\mu$, and $\rho$ to be twice continuously differentiable functions, and they take constant values in the half-space $z < 0$. The wave propagation in such a medium is described by the general system of dynamic elasticity theory equations

$$(\lambda + \mu)\,\mathrm{grad}\,\mathrm{div}\,U + \mu\Delta U + (\mathrm{grad}\,\lambda)\,\mathrm{div}\,U + (\mathrm{grad}\,\mu)E = \rho U_{tt}.$$

Here $U = (X, Y, Z)$ is the displacement vector, $E = (U' + U'^T)/2$ is the deformation tensor. Further we assume the displacement vector $U$ to be parallel to the axes $Oy$, that is, $X = Z = 0$, in other words, $U = (0, Y, 0)$; the medium filling the half-space $z > 0$ to be heterogeneous with respect to the co-ordinate $z$; the functions $\lambda$, $\mu$, $\rho$ – to depend only on the co-ordinate $z$. Under the assumptions made, the system of equations, which describes the wave process in the medium, is reduced to:

$$\begin{aligned}
&(\lambda + \mu)(Y_y)_x = 0\\
&(\lambda + \mu)(Y_y)_y + \mu\Delta Y + \mu_z Y_z = \rho Y_{tt}\\
&(\lambda + \mu)(Y_y)_z + \lambda_z Y_y = 0.
\end{aligned} \tag{1}$$

In the half-space $z < 0$, system (1) may be written down as follows:

$$\begin{aligned}
&(\lambda_0 + \mu_0)(Y_y)_x = 0\\
&(\lambda_0 + \mu_0)(Y_y)_y + \mu\Delta Y = \rho_0 Y_{tt}\\
&(\lambda_0 + \mu_0)(Y_y)_z = 0.
\end{aligned} \tag{2}$$

A particular solution of system (2) is the wave

$$Y(x, y, z, t) = Y(x, z, t) = \phi\Big(t - \frac{z\cos\alpha + x\sin\alpha}{v_0}\Big).$$

Further we assume that $\phi \in \boldsymbol{D}^2(-\infty, \infty)$ and $\phi(\xi) = 0$ for all $\xi < 0$. Finally, let us suppose that a plain wave falls from the half-space $z < 0$ onto the half-space $z > 0$, forming an angle $\alpha$ between its front and the negative half-axis $z < 0$:

$$Y(x, z, t) = \phi\Big(t - \frac{z\cos\alpha + x\sin\alpha}{v_0}\Big),$$

thus generating the wave process in the half-space $z > 0$. Let us note, that the falling wave does not depend on the coordinate $y$, thus $Y_y = 0$ when $z < 0$. So, as follows from the last equation of system (1), $Y = Y(x, z, t)$ does not depend on the co-ordinate $y$ and, consequently,

$$\mu(Y_{xx} + Y_{zz}) + \mu_z Y_z = \rho Y_{tt},$$
$$Y(x, z, t) = \phi\Big(t - \frac{z\cos\alpha + x\sin\alpha}{v_0}\Big), \qquad z < 0, \quad t < 0.$$

The wave process $Y(x, z, t)$ may be written down as $Y(x, z, t) = u(z, \tau)$, for the displacement along the axis $Ox$ to be equivalent to the time lag $\tau = t - x\sin\alpha/v_0$:

$$\mu u_{zz} + \mu_z u_z = \Big(\rho - \frac{\mu\sin^2\alpha}{v_0^2}\Big)u_{\tau\tau}.$$

The equation proves to remain hyperbolic when the angle $\alpha$ is slightly changed.

Passing over to a new variable, in order to study solvability of the problem, we represent it in a more convenient form

$$x = \int_0^z \frac{1}{c(\xi)}\,d\xi, \quad \text{where} \quad c^2(\xi) = \frac{\mu(\xi)}{r(\xi)}.$$

Thus we obtain the problem

$$u_{xx} + [\ln\sigma(x)]_x u_x = u_{\tau\tau}, \quad \text{where} \quad \sigma(x) = \sqrt{r(x)\mu(x)}, \tag{3}$$
$$u(x, \tau) = \phi(\tau - x), \qquad \tau < 0. \tag{4}$$

Performing the corresponding transformations, we obtain the problem

$$u_{xx} + [\ln\sigma(x)]_x u_x = u_{tt}, \tag{5}$$

in the half-space having the boundary and the initial conditions

$$u_x|_{x=0} = \delta(t); \qquad u|_{t<0} \equiv 0. \tag{6}$$

In connection with (5) and (6), the direct and the inverse problems are usually considered. The direct problem is the problem of determining the wave field $u(x, t)$ by a source and by the characteristics of the medium $\lambda(z)$, $\mu(z)$, $\rho(z)$ given. The inverse problem is the problem of determining the medium characteristics $\lambda(z)$, $\mu(z)$, $\rho(z)$ (or some of their functionals) by the initial data, a source and the given mode of vibrations of the boundary points $G(t) = u(0, t)$.

Further, we shall understand the inverse problem as a problem of determining the coefficient $\sigma(x)$ by the initial data, by a source and by the given mode of vibrations of the boundary points.

The direct and inverse problems for (5), (6) have been already well studied [2–4]; the algorithms of their numerical solutions have been presented

[2, 3, 5]. However, in practice, the precision of the medium structure determination is not high enough, for the absorption, taking place in the real medium, is not taken into consideration.

Attempts have been already made to increase the precision of the algorithms for the inverse problems [6, 8]. For example, in paper [6], Balts's model of the wave propagation for non-elastic media is considered; in paper [8], the authors use property of the low-frequency components of the wave field at the surface for reconstructing the high-frequency components of the wave field, which undergo the absorption to greater extent.

In this paper, an attempt is made to take into account absorption of the medium in the model of the elastic wave propagation. By analogy with Biot's model [9], let us simulate the absorption of the medium as coefficient of $du/dt$ in the general system of dynamic elasticity theory equations. We also assume that the absorption coefficient $\alpha(x)$ is a known continuous function. We obtain the problem:

$$u_{xx} + [\ln \sigma(x)]_x u_x = u_{tt} + \alpha(x) u_t, \tag{7}$$

$$u|_{x=0} = G(t), \qquad u_x|_{x=0} = \delta(t), \qquad u|_{t<0} \equiv 0. \tag{8}$$

Passing over to the new function

$$u(x,t) = \frac{v(x,t)}{\sqrt{\sigma(x)}},$$

$$v_{xx} + B(x)v = v_{tt} + \alpha(x)v_t, \qquad B(x) = -\Big[\big[\ln\sqrt{\sigma(x)}\big]_{xx} + \big[\ln\sqrt{\sigma(x)}\big]_x^2\Big],$$

$$v|_{x=0} = G(t), \qquad v_x|_{x=0} = \delta(t) - \frac{\sigma'(0)}{2\sigma(0)} G(t), \qquad v|_{t<0} \equiv 0,$$

we, finally, obtain the following problem

$$v_{xx} + B(x)v = v_{tt} + \alpha(x)v_t, \tag{9}$$

$$v|_{x=0} = G(t), \qquad v_x|_{x=0} = \delta(t), \qquad v|_{t<0} \equiv 0. \tag{10}$$

Now we are able to formulate the inverse problem: *let absorption of the medium $\alpha(x)$ be a given and continuous function, $\sigma(0)$ and $\sigma'(0)$ be known. Moreover, the mode of vibrations of boundary points is known. It is necessary to determine $\sigma(x)$.* Further, the conditions of solvability of the problem will be formulated.

## 3. Direct problem

Let us take into account problem (9), (10). Continue the solution of this problem evenly into the half-space $x < 0$. Let us note that the continued function $u(x,t)$ will be the solution of the Cauchy problem

$$u_{xx} + B(x)u = u_{tt} + \alpha(x)u_t$$

with the Cauchy data

$$u_t|_{t=0} = -2\delta(t); \qquad u|_{t=0} = 0.$$

Supposing $f(x,t) = -B(x)u(x,t) + \alpha(x)\partial u/\partial t$ to be known, we can write the solution of this problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + f(x,t),$$

owing to the d'Lember formula

$$u(x,t) = -\epsilon(t-|x|) + \frac{1}{2}\iint\limits_{\Delta(x,t)} B(\xi)u(\xi,\tau)\,d\xi\,d\tau - \frac{1}{2}\iint\limits_{\Delta(x,t)} \alpha(\xi)u_\tau(\xi,\tau)\,d\xi\,d\tau,$$

$$\Delta(x,t) = \{(\xi,\tau) \mid t - x + \tau \le \xi \le t + x - \tau\}.$$

It is easy to prove existence and uniqueness of the solution of the direct problem. By analogy of the proof for the similar problem in [7], we can easily obtain the estimation, which gives us the uniqueness of the solution for the direct problem,

$$U(t) \le t_1 B\int_0^t U(\tau)\,d\tau + \alpha\int_0^t U(\tau)d\tau = C\int_0^t U(\tau)\,d\tau,$$

where

$$U(t) = \max_{x_1-(t_1-t)\le x\le x_1+(t_1-t)} |u(x,t)|, \qquad B = \max_{t-t_1\le x\le t+t_1} |B(x)|.$$

To show that the solution $u(x,t)$ of the integral equation is really the solution of our direct problem, we have to verify the continuity of every derivative of $u(x,t)$ up to the second order. The method of this proof is well described in [7] and is applicable to our case.

Finally, let us note some properties of the solution:

$$u(x,|x|) = \lim_{t\to|x|+0} u(x,t) = -1,$$

$$u_t(x,|x|) = \lim_{t\to|x|+0} u_t(x,t) = -\frac{1}{2}\int_0^x B(\xi)\,d\xi - \frac{1}{2}\int_0^x \alpha(\xi)u_t(\xi,\xi)\,d\xi.$$

## 4. Inverse problem

Based on (9), (10), let us note that $u = v_t$ satisfies the equation

$$u_{xx} + B(x)u = u_{tt} + \alpha(x)u_t \tag{11}$$

with the boundary and the initial data

$$u|_{x=0} = G'(t) = g(t), \qquad u_x|_{x=0} = \delta'(t), \qquad u|_{t<0} \equiv 0. \tag{12}$$

Assuming $B(x)$ to be known, let us write down the solution of the Cauchy problem (11), (12) by means of the d'Lember formula:

$$u(x,t) = \frac{1}{2}(g(t+x) + g(t-x)) - \frac{1}{2}\iint\limits_{\Delta(x,t)} B(\xi)u(\xi,\tau) + \alpha(\xi)u_\tau(\xi,\tau)\,d\xi\,d\tau$$

for all $(x,t)$ such that $t > x \geq 0$ , where

$$\Delta(x,t) = \{(\xi,\tau) \mid t - x + \xi \leq \tau \leq t + x - \xi\}.$$

$$\begin{aligned} u(x,t) \;=\; & \frac{1}{2}(g(t+x) + g(t-x)) - \\ & \frac{1}{2}\iint\limits_{\Delta(x,t)} B(\xi)u(\xi,\tau)\,d\tau\,d\xi + \\ & \frac{1}{2}\int\limits_0^x \alpha(\xi)(u(\xi,t+x-\xi) - u(\xi,t-x+\xi))\,d\xi \end{aligned} \tag{13}$$

$$\begin{aligned} u_t(x,t) \;=\; & \frac{1}{2}(g'(t+x) + g'(t-x)) - \\ & \frac{1}{2}\int\limits_0^x B(\xi)(u(\xi,t+x-\xi) - u(\xi,t-x+\xi))\,d\xi + \\ & \frac{1}{2}\int\limits_0^x \alpha(\xi)(u_\tau(\xi,t+x-\xi) - u_\tau(\xi,t-x+\xi))\,d\xi \end{aligned} \tag{14}$$

$$\begin{aligned} u_x(x,t) \;=\; & \frac{1}{2}(g'(t+x) - g'(t-x)) - \\ & \frac{1}{2}\int\limits_0^x B(\xi)(u(\xi,t+x-\xi) + u(\xi,t-x+\xi))\,d\xi + \\ & \frac{1}{2}\int\limits_0^x \alpha(\xi)(u_\tau(\xi,t+x-\xi) + u_\tau(\xi,t-x+\xi))\,d\xi \end{aligned}$$

Then we can derive

$$\frac{d}{dx}u(x,x) = g'(2x) - \int_0^x B(\xi)u(\xi, 2x-\xi)\,d\xi + \int_0^x \alpha(\xi)u_\tau(\xi, 2x-\xi)\,d\xi, \quad (15)$$

where $\frac{d}{dx}u(x,x)$ is considered to be a limit from the domain $t > x \geq 0$. In addition,

$$\lim_{t\to x+0} u(x,t) = -\frac{1}{2}\int_0^x B(\xi)\,d\xi - \frac{1}{2}\int_0^x \alpha(\xi)u(\xi,\xi)\,d\xi. \quad (16)$$

From (15),(16) we come to

$$g'(2x) - \int_0^x (B(\xi)u(\xi, 2x-\xi) + \alpha(\xi)u_\tau(\xi, 2x-\xi))\,d\xi$$
$$= -\frac{1}{2}B(x) - \frac{1}{2}\alpha(x)u(x,x). \quad (17)$$

Equations (13), (14), (17) form a close system of the integral equations according to $u(x,t)$, $u_t(x,t)$, $B(x)$. Denote

$$\phi_1 = u(x,t), \qquad \phi_2 = u_t(x,t), \qquad \phi_3 = B(x),$$
$$\phi_1^0 = \frac{1}{2}(g(t+x) + g(t-x)), \quad \phi_2^0 = \frac{1}{2}(g'(t+x) + g'(t-x)), \quad \phi_3^0 = -2g'(2x).$$

Now the system of the integral equations (13), (14), (17) can be written down as an operator $\phi = A\phi$, where $\phi_i = A_i\phi$:

$$A_1\phi = \phi_1^0 - \frac{1}{2}\int_0^x \int_{t-x+\xi}^{t+x-\xi} \phi_3(\xi)\phi_1(\xi,\tau)\,d\tau\,d\xi +$$
$$\frac{1}{2}\int_0^x \alpha(\xi)(\phi_1(\xi, t+x-\xi) - \phi_1(\xi, t-x+\xi))\,d\xi$$

$$A_2\phi = \phi_2^0 - \frac{1}{2}\int_0^x \phi_3(\xi)(\phi_1(\xi, t+x-\xi) - \phi_1(\xi, t-x+\xi))\,d\xi +$$
$$\frac{1}{2}\int_0^x \alpha(\xi)(\phi_2(\xi, t+x-\xi) - \phi_2(\xi, t-x+\xi))\,d\xi$$

$$A_3\phi = \phi_3^0 + 2\int_0^x \phi_3(\xi)\phi_1(\xi, 2x-\xi)\,d\xi - 2\int_0^x \alpha(\xi)\phi_2(\xi, 2x-\xi)\,d\xi -$$
$$\alpha(x)\phi_1(x,x).$$

One can easily prove that for $X > 0$ sufficiently small, the transformation $A$ is squeezing in the domain $\Delta(X^*) = \Delta(X^*, X^*)$, where

$$X^* = \min\Big\{ \frac{\sqrt{\alpha^2 + 2\|\phi_0\|} - \alpha}{2\|\phi_0\|}, \frac{\sqrt{\alpha^2 + 8\|\phi_0\|} - \alpha}{4\|\phi_0\|}, \frac{2\|\phi_0\|}{16\|\phi_0\|^2 + 5\alpha + \alpha^2}, \frac{2}{16\|\phi_0\| + 5\alpha + \alpha^2} \Big\}.$$

The proof of this fact is performed according to the scheme of the similar statement in [7]. Due to the Banach theorem for squeezing transformation, the equation $\phi = A\phi$ determines a unique continuous solution in the domain $\Delta(X^*)$.

It is not difficult to show the stability of solution (13), (14), (17), according to the initial data. It directly follows from the estimations

$$U(x) \le u_0^1 + \int_0^x (M_1 x_0 K(\xi) i + M_2 x_0 U(\xi) + \alpha U(\xi))\, d\xi,$$

$$V(x) \le u_0^2 + \int_0^x (M_1 K(\xi) + M_2 U(\xi) + \alpha V(\xi))\, d\xi,$$

$$K(x) \le u_0^3 + \int_0^x (2M_1 K(\xi) + 2M_2 U(\xi) + 2\alpha V(\xi))\, d\xi + \int_0^x (\frac{\alpha}{2} K(\xi)\, d\xi + 2\alpha V(\xi))\, d\xi,$$

where

$$U(x) = \max_{(\xi,\tau)\in T(x)} |u(\xi,\tau)|, \qquad V(x) = \max_{(\xi,\tau)\in T(x)} |u_t(\xi,\tau)|,$$

$$K(x) = |B(x)|, \quad \text{and} \quad T(x) = \{(\xi,\tau) \mid 0 \le \xi \le x,\ \xi \le \tau \le t - \xi\}.$$

## 5. Joint problem

In practice, some information of the medium structure proves to be known (for example, in the course of boring one can obtain the exact structure of the medium along the borehole using the VSP method or analyzing the strata). Besides, the input data of the inverse problem are measured "roughly" enough (its precision is about 10–20%). But having the medium structure down to some depth to be known, it is possible to reconstruct the real wave field at the surface of the Earth up to a certain time moment. In this connection, the following "non-classical" statement of the inverse problem, further called as the "joint dynamic seismic problem", arises. The meaning of such a definition will become clear later.

The joint statement of the problem: *let the absorption $\alpha(x)$ of the medium be a given continuous function, $\sigma(x)$ be known and continuous inside the interval $[0,h]$. Moreover, the mode of the boundary points' vibrations is known. It is necessary to determine a continuous function $\sigma(x)$ being the coefficient of equation* (7) *inside the interval* $[0,H]$ (*where* $h < H$). "Joint" means that in addition to the wave field known at the surface, we have information of the medium itself inside certain interval $[0,h]$. It is clear that such an overdetermined statement of the classical inverse dynamic seismic problem adds some stability to computations.

## 6. Numerical algorithm

Let us define a set of some special constructions [4] in the plane of independent variables $(x,t)$ and substitute the integrals by the quadratic formulas in (13), (14), (17).

$$u_k^n = \frac{1}{2}(g_k + g_n) - \frac{h^2}{4}B_0\sum_{i=n+1}^{k}(u_i^i + u_{i-1}^{i-1}) - \frac{h^2}{2}\sum_{i=1}^{k-n-1}B_i\sum_{j=n+1}^{k-i}(u_{i+j}^j + u_{i+j-1}^{j-1}) + \frac{h}{2}\sum_{i=n}^{k-1}\alpha_{i-n}(u_k^{n+k-i} - u_i^n) + O(h^2),$$

$$v_k^n = \frac{1}{2}(g_k' + g_n') - \frac{h}{2}\sum_{i=n}^{k-1}(B_{i-n}(u_k^{n+k-i} - u_i^n) + \alpha_{i-n}(v_k^{n+k-i} - v_i^n)) + O(h^2),$$

$$B_k = -\left(2g_k' - 2h\sum_{i=0}^{k-1}B_iu_k^{k-i} + 2h\sum_{i=0}^{k-1}\alpha_iv_k^{k-i}\right) - \alpha_ku_k^0 + O(h).$$

Let us take into account the system without residuals $O(h)$ and $O(h^2)$. Such a system is equivalent to the following system of algebraic equations:

$$\begin{aligned}
&u_k^{n-1} + u_{k-1}^n = (u_k^n + u_{k-1}^{n-1})\left(1 - \frac{h^2}{2}B_{k-n}\right) + \frac{h}{2}\alpha_{k-n}(u_k^n - u_{k-1}^{n-1}),\\
&2u_k^{k-1} = \left(1 - \frac{h^2}{2}B_0\right)(u_{k-1}^{k-1} + u_k^k) + \frac{h}{2}\alpha_0(u_{k-1}^{k-1} + u_k^k),\\
&(v_k^n + v_{k-1}^{n-1}) + \frac{h}{2}\alpha_{k-n}(v_k^n - v_{k-1}^{n-1}) = v_k^{n-1} + v_{k-1}^n + \frac{h}{2}B_{k-n}(u_k^n + u_{k-1}^{n-1}),\\
&v_k^{k-1} = \frac{1}{2}(g_{k-1}' - g_k') - \frac{1}{2}B_0(u_{k-1}^{k-1} + u_k^k) + \frac{1}{2}\alpha_0(v_k^k + v_{k-1}^{k-1}),\\
&B_k = -\left(2g_k' - 2h\sum_{i=0}^{k-n}B_iu_k^{k-i} + 2h\sum_{i=0}^{k-1}\alpha_iv_k^{k-i}\right) - \alpha_ku_k^0.
\end{aligned} \tag{18}$$

System (18) allows us to find $B_k$ when $B_0$ and $u_k^k$ are known. In addition, two first equations of system (18) give the solution of the direct problem when $B_k$ are given.

Consider the convergence of the numerical solution of system (18) to the precise solution of system (13), (14), (17). Let $\alpha_k^n := u(kh-nh, kh+nh)-u_k^n$, $\beta_k^n := v(kh-nh, kh+nh) - v_k^n$, $\gamma_k := B(kh) - B_k$, $a_i := \alpha_i$.

$$\begin{aligned}
\alpha_k^n &= -\frac{h^2}{2}\sum_{i=1}^{k-n-1} B_i \sum_{j=n+1}^{n-i} (\alpha_{i+j}^j + \alpha_{i+j-1}^{j-1}) - \frac{h^2}{2}\sum_{i=1}^{k-n-1} \gamma_i(u_{i+j}^j - u_{i+j-1}^{j-1}) + \\
&\quad \frac{h}{2}\sum_{i=n}^{k-1} a_{i-n}(\alpha_k^{n+k-1} + \alpha_i^n) + O(h^2), \\
\beta_k^n &= -\frac{h}{2}\sum_{i=n}^{k-1} B_{i-n}(\alpha_k^{n+k-1} + \alpha_i^n) - \frac{h}{2}\sum_{i=n}^{k-1} \gamma_i(u_k^{n+k-i} - u_i^n) + \\
&\quad \frac{h}{2}\sum_{i=n}^{k-1} a_{i-n}(\beta_k^{n+k-1} + \beta_i^n) + O(h^2), \\
\gamma_k &= +2h\sum_{i=0}^{k-1} B_i\alpha_k^{k-i} + 2h\sum_{i=0}^{k-1}\gamma_i u_k^{k-i} - 2h\sum_{i=0}^{k-1} a_i\beta_k^{k-i} + \\
&\quad a_k\frac{h^2}{2}\sum_{i=1}^{k-1} B_i\sum_{j=1}^{n-j}(\alpha_{i+j}^j + \alpha_{i+j-1}^{j-1}) + a_k\frac{h^2}{2}\sum_{i=1}^{k-1}\gamma_i\sum_{j=1}^{n-j}(u_{i+j}^j + u_{i+j-1}^{j-1}) - \\
&\quad a_k\frac{h}{2}\sum_{i=0}^{k-1} a_i(\alpha_k^{k-i} - \alpha_i^0) + O(h).
\end{aligned}$$

Consider the system

$$\begin{aligned}
\xi_m &= h^2\sum_{i=1}^{m-1}(m-i)(M_2\xi_i + M_1\eta_i) + ha\sum_{i=n}^{m-1}\xi_i + O_1, \\
\zeta_m &= h\sum_{i=1}^{m-1}(M_2\xi_i + M_1\eta_i) + ha\sum_{i=n}^{m-1}\zeta_i + O_2, \\
\eta_m &= 2h\sum_{i=1}^{m-1}(M_2\xi_i + M_1\eta_i) + 2ah\sum_{i=1}^{m-1}\zeta_i + \\
&\quad ah^2\sum_{i=1}^{m-1}(m-i)(M_2\xi_i + M_1\eta_i) + a^2h\sum_{i=1}^{m-1}\xi_i + O_3.
\end{aligned}$$

Let us note that $|\alpha_{m+j}^j| \le \xi_m$, $|\beta_{m+j}^j| \le \zeta_m$, $|\gamma_m| \le \eta_m$.

Denote $\psi_m = \max\{\psi_m, \zeta_m, \eta_m\}$. It is easy to obtain the following estimation:

$$\psi_m \le hC \sum_{i=1}^{m-1} \psi_i + o, \quad \text{where} \quad o = \max\{O_1, O_2, O_3\}.$$

This inequality yields the convergence of the numerical solutions for the direct and the inverse problems to the precise solution of system (13), (14), (17), where the order of convergence equals $O(h)$.

## 7. Numerical experiments

The results obtained in the previous sections allow us to build the algorithm solving the direct and the inverse problems. The algorithm built by recursive relations (18) shows sufficiently high precision of reconstructing the medium for the test examples. The author's program (in Pascal) was used for carrying out the test examples.

Figure 1 shows the profile of acoustic stiffness $\sigma(x)$: (1) is a real medium, (2) is the medium reconstructed by taking absorption into consideration, and (3) is the medium reconstructed by absorption not taking into consideration. As earlier, $x$ means the propagation time of the wave from the surface to the current point. The scales of the values are real: seconds and kilometers. The given examples visually illustrate the precision increase of the reconstruction of the medium having absorption. The test examples were carried out for the medium with the constant absorption averaged for the frequency 100 Hz.

Figure 2 shows the graph of the acoustic stiffness $\sigma(x)$: (1) is a real medium, (2) is the medium reconstructed without complementary information along the borehole taken into account, and (3) is the medium reconstructed with complementary information along the borehole taken into account. The scales of the values are real. The absorption of the medium was chosen as the mean for the frequency 100 Hz. The depth of the borehole

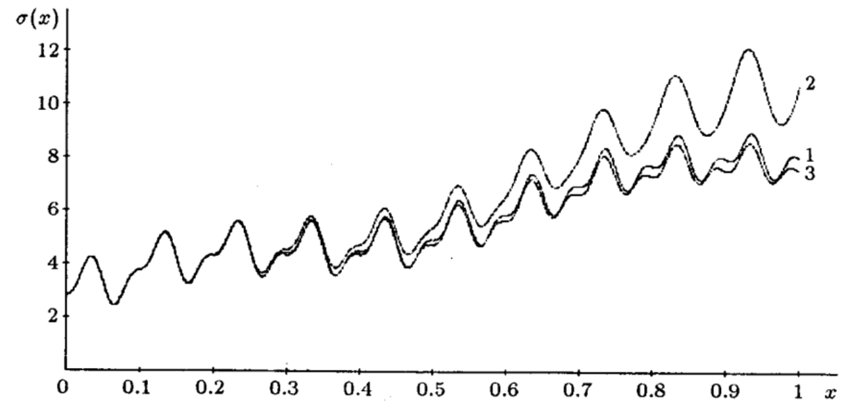


**Figure 1**

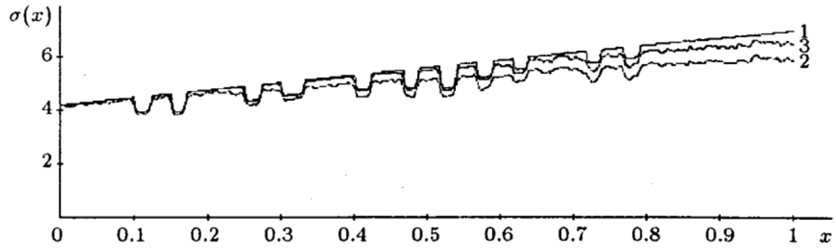


**Figure 2**

was 0.6 s. The given example shows increasing the stability, hence increasing the precision of the medium reconstruction for the algorithm making use of the complementary information of the medium along the borehole. All examples were performed with "white noise" (15%) being introduced to the input data of the problem. The test examples demonstrate rather a high accuracy and noise-stability of the algorithm.

# 8. Conclusion

Let us note that the proposed statement of the inverse problem (when in addition to the wave field of the medium at the surface there is a complementary information of the real structure of the medium) has been never considered before. The results of the test examples have demonstrated that such an overdetermined statement of the problem increases the precision of the medium reconstruction.

*Acknowledgements.* The author takes pleasure in thanking Prof. A.S. Alekseev for advice and help in stating the problem.

# References

[1] Alekseev A.S. Some inverse problems of the wave propagation theory I, II // Izv. AN SSSR. Ser. Geophyz. – 1962. – № 11. – P. 1514–1531.

[2] Alekseev A.S. Inverse problems of geophysics // Some Methods and Algorithms of Geophysical Data Interpretation. – Nauka, 1967. – P. 9–84.

[3] Alekseev A.S., Mikhailenko B.G., Tcheverda V.A., Numerical methods of solving direct and inverse problems of wave propagation theory // Actual Problems of Computational and Applied Mathematics. – Novosibirsk, 1983. – P. 165–172.

[4] Alekseev A.S., Dobrinsky V.I. Some questions of practical use of inverse dynamic seismic problems // Mathematical Problems of Geophysics. – Novosibirsk, 1975. – Vol. 2, № 6 (in Russian).

[5] Dobrinsky V.I. Convertible difference schemes for calculating fundamental solutions of two hyperbolic equations // Mathematical Problems of Geophysics. – Novosibirsk, 1977. – P. 64–75 (in Russian).

[6] Dobrinsky V.I. On some problems of scattering of plain SH-waves in heterogeneous media // Mathematical Problems of Geophysics. – Novosibirsk, 1977. – P. 56–63 (in Russian).

[7] Romanov V.G. Inverse Problems for Differential Equations: Special Course for the Students of NSU. – Novosibirsk: NSU, 1973 (in Russian).

[8] Tsetsokho V.A., Belonosov A.S., Belonosova A.V. On peculiarities of the inverse filtration method in the problems of reconstructing the real motion ground by its seismogram data // Theory and Methods of Solving Incorrect Stated Problems.

[9] Biot M.A. Theory of propagation of elastic waves in a fluid-saturated porous solid. I // J. Acoustic. Soc. America. – 1956. – Vol. 28. – P. 168–178.