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A limited area nonlinear normal mode initialization*

A.A. Fomenko

The paper presents a method of nonlinear normal mode initialization which is used in the operational technology of the regional numerical weather prediction for the Siberian region. This technology has been developed at the Siberian Hydrometeorological Research Institute. The comparison of results of forecasting with the use of the initialization method and without it has been made. The method allows the efficient suppression of the amplitudes of high-frequency oscillations at the initial integration stage, the maximal changes in the field of the surface pressure not exceeding 1–2 hPa.

1. Introduction

The problem of initial adjustment of meteorological fields was formulated in the finite form at the same time with going in the numerical weather prediction from quasi-geostrophic equations to the primitive ones. The previously used adjustment of fields on the basis of geostrophic relations or solutions of the balance equation did not allow one to eliminate or somewhat to essentially suppress amplitudes of high-frequency oscillations, arising at the initial steps of integration of equations. Nonlinearity of a system of equations brings about the fact that gravitational waves filtered at the initial time, arise at the very first integration steps, having an unreally large amplitude.

At present, most generally employed is the method of nonlinear normal mode initialization due to its efficiency and low computer costs. The idea of the normal mode initialization is based on the fact that for sufficiently high equivallent depths, eigenfunctions of linearized shallow water equations can be separated in terms of their eigenfrequencies. Although the normal mode initialization methods were initially formulated for global and hemispheric models, by the present time they have found their application to limited-area models [5, 6].

An alternative of the nonlinear normal mode initialization is the bounded derivative method [2]. It should be noted that the comparison of the two above-mentioned methods indicates to their similarity and to that of

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their algorithm realization, particularly, if one considers the normal mode initialization in terms of the physical space [4].

The essence of the nonlinear normal mode initialization is in that the linearized part of the spatial operator of the problem is expanded in eigenfunctions of the vertical operator. After that, fast and slow modes are distinguished in the normal mode space and tendencies of high-frequency harmonics are suppressed. Further we follow a reverse procedure of going to the original physical space. In principle, it is possible to construct a procedure allowing an effective suppression of high-frequency disturbances without going to the normal mode space [13].

The idea of suppression at the initial time not of gravitational waves themselves but of their tendencies was proposed independently in two publications [3, 12]. Thus, we can construct an algorithm allowing filtering (although incomplete) of gravitational waves and suppression of their amplitudes during integration provided the amplitudes of the slow Rossby waves are conserved.

The present paper proposes an algorithm of the nonlinear normal mode initialization for the regional atmospheric model [11]. This procedure is a part of the operational technology of the short-range weather prediction for Siberia. By present, a version of four-dimensional data assimilation with 12-hour cycle has been implemented. In this case we made use of 12-hour prediction of the regional atmospheric model as the first guess for the analysis [7].

2. Projection of equations onto the vertical mode space

The initial statement of the model and its numerical realization are presented in detail in [7]. The conventional notations used below do not demand any comments and correspond to the ones used in [7]. Let us distinguish in the original semi-discrete system of equations the linear part

$$
\begin{aligned}
&\frac{\partial \boldsymbol{u}}{\partial t} - f_o \bar{\boldsymbol{v}}^{xy} + \frac{1}{m_u}\delta_x(\bar{\boldsymbol{\Phi}}^\sigma + R\boldsymbol{T}_o \ln p_s) = \boldsymbol{N}_u,\\
&\frac{\partial \boldsymbol{v}}{\partial t} + f_o \bar{\boldsymbol{u}}^{xy} + \frac{1}{m'}\delta_y(\bar{\boldsymbol{\Phi}}^\sigma + R\boldsymbol{T}_o \ln p_s) = \boldsymbol{N}_v,\\
&\frac{\partial \boldsymbol{T}}{\partial t} + B\boldsymbol{D} = \boldsymbol{N}_T,\\
&\frac{\partial \ln p_s}{\partial t} + \boldsymbol{\Pi}^T \boldsymbol{D} = \boldsymbol{N}_{p_s},
\end{aligned}
\tag{2.1}
$$

are symmetric finite differences with respect to x and y, m_u, m_v, m' are scale factors in the cartographical coordinates, $\mathbf{\Pi}^T = (\Delta\sigma_1, \ldots, \Delta\sigma_N)$, N is the number of vertical levels of the model. The subscripts u, v denote the points on C-grid to which refer the corresponding expressions, the overbar denotes the finite difference averaging in the respective variable, $\bar{\mathbf{\Phi}}^\sigma = \mathbf{\Phi}_s + R G \boldsymbol{T}$. $\boldsymbol{N}_u$, $\boldsymbol{N}_v$, $\boldsymbol{N}_T$, $\boldsymbol{N}_{p_s}$ include nonlinear terms of the corresponding equations, f_o is the mean value of the Coriolis parameter, while its deviations are also included in the right-hand sides of system (2.1).

Properties of the finite difference scheme in vertical are fully defined by the matrix of the quasi-statistics G and the matrix of divergence contribution in the tendency of the temperature B, and the vertical structure of oscillations will be described by the finite difference operator

$$C = RGB + R\boldsymbol{T}_o\mathbf{\Pi}^T, \tag{2.2}$$

$\boldsymbol{T}_o$ is the column-vector of a certain characteristic temperature profile, in our case $\boldsymbol{T}_o = \text{const}$. Eigenvectors of the matrix C are the vertical normal modes of the numerical model with whose help the original three-dimensional model can be reduced to a series (with respect to the number of levels N) systems of shallow water equations with equivalent depths d_n which are equal to eigenvalues of the operator C.

The introduction of the operator C makes it possible to rewrite the two latter equations (2.1) in the form

$$\frac{\partial \mathbf{\Phi}}{\partial t} + C\boldsymbol{D} = \boldsymbol{N}_\Phi, \tag{2.3}$$

where $\mathbf{\Phi} = \bar{\mathbf{\Phi}}^\sigma + R\boldsymbol{T}_o \ln p_s$.

The use of the transformation

$$\psi^{-1} C \psi = \text{diag}(d_1, \ldots, d_N), \tag{2.4}$$

where column-vectors of the matrix ψ represent the vertical modes to which correspond the eigen-numbers d_n, allows us by multiplication of systems (2.1), (2.3) from the left by ψ^{-1} to go to a series of N shallow water equations with equivalent depths d_n

$$\begin{aligned}
&\frac{\partial u_n}{\partial t} - f_o \bar{v}_n^{xy} + \frac{1}{m_u}\delta_x \Phi_n = N_{u_n}, \\
&\frac{\partial v_n}{\partial t} + f_o \bar{u}_n^{xy} + \frac{1}{m'}\delta_y \Phi_n = N_{v_n}, \\
&\frac{\partial \Phi_n}{\partial t} + d_n D_n = N_{\Phi_n}.
\end{aligned} \tag{2.5}$$

Further on, we will omit the index n for the sake of simplicity.

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The vertical modes, the corresponding equivalent depths and phase velocities of the gravitational waves for the 15-level atmosphere model are presented in [15].

3. Projection onto the horizontal mode space

Using the operators **rot** and **div** we can readily reduce the obtained system to the one of the following forms:

$$\begin{aligned} &\frac{\partial D}{\partial t} - f_o\bar{Q}^{xy} + \nabla_u^2\Phi = N_D, && \frac{\partial \bar{D}^{xy}}{\partial t} - f_o Q + \nabla_v^2\bar{\Phi}^{xy} = N'_D, \\ &\frac{\partial \bar{Q}^{xy}}{\partial t} + f_o D = N'_Q, && \frac{\partial Q}{\partial t} + f_o\bar{D}^{xy} = N_Q, \\ &\frac{\partial \Phi}{\partial t} + dD = N_\Phi, && \frac{\partial \bar{\Phi}^{xy}}{\partial t} + d\bar{D}^{xy} = N'_\Phi, \end{aligned} \tag{3.1}$$

here

$$\begin{aligned} Q &= \frac{1}{m_v}\Big(\delta_x v - \frac{1}{m'}\delta_y m_u u\Big), \\ \nabla_u^2 &= \frac{1}{m_u}\Big(\frac{1}{m_u}\delta_x^2 + \frac{1}{m'^2}\delta_y m_v \delta_y\Big), \\ \nabla_v^2 &= \frac{1}{m_v}\Big(\frac{1}{m_v}\delta_x^2 + \frac{1}{m'^2}\delta_y m_u \delta_y\Big). \end{aligned} \tag{3.2}$$

The corresponding nonlinear terms are still concentrated in the right-hand sides. Two systems are simultaneously realized due to the use of C-grid in terms of Arakawa. Note that in principle we can restrict ourselves to realization of one system, the first one, for example, and to take into account the shift of functions with respect to the grid points when recalculating Q in regard to $\bar{Q}^{xy}$ [8, 9].

The approaches to performing calculations without loss of generality will be demonstrated on system (3.1). In this case the indices x, y will be in the sequel omitted for simplicity.

Let us transform the first equation of system (3.1) to the form

$$\frac{\partial D}{\partial t} - f_o\bar{Q} + \Lambda\Phi + \Phi_B = N_D, \tag{3.3}$$

where Λ is the Laplace operator on the spherical rectangle with zero Dirichlet conditions at the lateral boundaries. The boundary conditions for Φ are included in Φ_B. Let Ψ be a matrix whose columns represent eigenvectors of the operator Λ, $-\lambda_{kl}$ are eigen-numbers corresponding to these vectors $(k = 1, \ldots, K; l = 1, \ldots, L)$. Then, by affecting system (3.1) from the left

$$\begin{aligned} &\frac{\partial D_{kl}}{\partial t} - f_o\bar{Q}_{kl} - \lambda_{kl}\Phi_{kl} + \Phi_{B_{kl}} = N_{D_{kl}}, \\ &\frac{\partial \bar{Q}_{kl}}{\partial t} + f_o D_{kl} = N'_{Q_{kl}}, \\ &\frac{\partial \Phi_{kl}}{\partial t} + dD_{kl} = N_{\Phi_{kl}}. \end{aligned} \tag{3.4}$$

Denote

$$\boldsymbol{X} = \begin{pmatrix} D_{kl} \\ \bar{Q}_{kl} \\ \Phi_{kl} \end{pmatrix}. \tag{3.5}$$

Then (3.4) in the vector form can be represented as

$$\frac{d\boldsymbol{X}}{dt} + \mathrm{i}A\boldsymbol{X} + \boldsymbol{X}_B = \boldsymbol{F}, \tag{3.6}$$

where

$$A = \begin{pmatrix} 0 & \mathrm{i}f_o & \mathrm{i}\lambda_{kl} \\ -\mathrm{i}f_o & 0 & 0 \\ -\mathrm{i}d & 0 & 0 \end{pmatrix}, \quad \boldsymbol{X}_B = \begin{pmatrix} \Phi_{B_{kl}} \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{F} = \begin{pmatrix} N_{D_{kl}} \\ N'_{Q_{kl}} \\ N_{\Phi_{kl}} \end{pmatrix}. \tag{3.7}$$

Equation (3.6) is projected onto the normal mode space using representations

$$T^{-1}AT = M = \mathrm{diag}(\mu_1, \mu_2, \mu_3), \tag{3.8}$$

$$T = \begin{pmatrix} 0 & \frac{\mathrm{i}}{2\sigma_{kl}} & -\frac{\mathrm{i}}{2\sigma_{kl}} \\ \frac{\lambda_{kl}}{\sigma_{kl}^2} & \frac{f_o}{2\sigma_{kl}^2} & \frac{f_o}{2\sigma_{kl}^2} \\ -\frac{f_o}{\sigma_{kl}^2} & \frac{d}{2\sigma_{kl}^2} & \frac{d}{2\sigma_{kl}^2} \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 0 & d & -f_o \\ -\mathrm{i}\sigma_{kl} & f_o & \lambda_{kl} \\ \mathrm{i}\sigma_{kl} & f_o & \lambda_{kl} \end{pmatrix}. \tag{3.9}$$

$\mu_1 = 0$ is frequency of the Rossby stationary mode,
$\mu_{2,3} = \pm\sigma_{kl} = \pm\sqrt{\lambda_{kl}d + f_o^2}$ are frequencies corresponding to inertially-gravitational oscillations.

Using (3.6) and performing transformations

$$\begin{aligned} &\boldsymbol{Y} = T^{-1}\boldsymbol{X}, && \boldsymbol{N} = T^{-1}\boldsymbol{F}, \\ &\boldsymbol{Y} = (Y_1, Y_2, Y_3)^T, && \boldsymbol{N} = (N_1, N_2, N_3)^T, \end{aligned} \tag{3.10}$$

we arrive at the dynamic equations for the mode coefficients Y_1, $Y_{2,3}$, connected with corresponding eigenfrequencies

$$\frac{dY_1}{dt} = N_1, \qquad \frac{dY_{2,3}}{dt} \pm \mathrm{i}\sigma_{kl}Y_{2,3} \mp \mathrm{i}\sigma_{kl}\Phi_{B_{kl}} = N_{2,3}. \tag{3.11}$$

Introducing the notation

$$\tilde{\boldsymbol{Y}} = \boldsymbol{Y} - \boldsymbol{Y}_B, \tag{3.12}$$

where $\boldsymbol{Y}_B = (0, \Phi_{B_{kl}}, \Phi_{B_{kl}})^T$, we can present equations for tendencies of gravitational mode amplitudes at the initial time in the form

$$\frac{d\tilde{Y}_{2,3}}{dt} \pm \mathrm{i}\sigma_{kl}\tilde{Y}_{2,3} = N_{2,3} - \frac{dY_{B_{2,3}}}{dt}, \tag{3.13}$$

which allows us to take into account explicitly the effect of the boundary conditions on the initialization. In addition, in this case it is easy to apply a technique, commonly used in the global models [1, 14], to a limited model.

4. Nonlinear normal mode initialization

As an initial step for performing the nonlinear initialization we take a linear balance state

$$Y_1^{(o)} = Y_1, \qquad Y_{2,3}^{(o)} = \Phi_{B_{kl}}. \tag{4.1}$$

Since

$$\boldsymbol{Y} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = T^{-1}\boldsymbol{X} = \begin{pmatrix} d\bar{Q}_{kl} - f_o\Phi_{kl} \\ -\mathrm{i}\sigma_{kl}D_{kl} + f_o\bar{Q}_{kl} + \lambda_{kl}\Phi_{kl} \\ \mathrm{i}\sigma_{kl}D_{kl} + f_o\bar{Q}_{kl} + \lambda_{kl}\Phi_{kl} \end{pmatrix}, \tag{4.2}$$

we obtain the expressions for the linear balance state in terms of expansion coefficients of solving the original system (3.1) in eigenvectors of the Laplace operator on the spherical rectangle

$$\begin{aligned} D_{kl}^{(o)} &= 0, \\ \bar{Q}_{kl}^{(o)} &= \bar{Q}_{kl} - \frac{f_o}{\sigma_{kl}^2}(f_o\bar{Q}_{kl} + \lambda_{kl}\Phi_{kl} - \Phi_{B_{kl}}), \\ \Phi_{kl}^{(o)} &= \Phi_{kl} - \frac{d}{\sigma_{kl}^2}(f_o\bar{Q}_{kl} + \lambda_{kl}\Phi_{kl} - \Phi_{B_{kl}}). \end{aligned} \tag{4.3}$$

Introducing the vector $\boldsymbol{\phi}$ resulted from the operator Ψ^{-1} acting on $\nabla^2\Phi$ from the left makes it possible to rewrite the result of linear initialization in the final form

$$\begin{aligned} D_{kl}^{(o)} &= 0, \\ \Delta Q_{kl}^{(o)} &= -\frac{f_o}{\bar{\sigma}_{kl}^2}(f_oQ_{kl} - \bar{\phi}_{kl}), \\ \Delta\Phi_{kl}^{(o)} &= -\frac{d}{\sigma_{kl}^2}(f_o\bar{Q}_{kl} - \phi_{kl}). \end{aligned} \tag{4.4}$$

Here the shift of functions with respect to each other has been already taken into account.This shift is due to the use of C-grid. It is easy to show that the inverse transformation to the physical space can be interpreted as a geostrophic balance of linearized equations (3.1)

$$D^{(o)} = 0, \qquad f_o\tilde{Q}^{(o)} = \nabla^2\Phi^{(o)}, \tag{4.5}$$

where

$$H\Delta\Phi^{(o)} = d(\nabla^2\Phi - f_oQ), \qquad H = f_o^2 - d\nabla^2. \tag{4.6}$$

The main idea of nonlinear initialization is in the demand for the time tendency of gravitational mode amplitudues to be equal to zero at the initial time under assumption that nonlinear terms of equations slowly change with respect to time. This is valid for small Rossby's numbers. This demand brings about the iterative scheme to define correctness of the original values.

Thus, the gravitational components of the original fields are not rejected as in the case of linear initialization, but are transformed so as to be stationary for the given model. This method of nonlinear initialization brings about a somewhat balanced low-frequency state of the original fields which correspond to the solution on a slow variety. Turning to system (3.13) following [1, 12] it is easy to construct an iterative process for determining the correctness of the balanced state

$$\Delta Y_1^{(\nu)} = 0, \quad \Delta\tilde{Y}_{2,3}^{(\nu)} = \pm\frac{1}{\mathbf{i}\sigma_{kl}}\frac{d\tilde{Y}_{2,3}^{(\nu-1)}}{dt}. \tag{4.7}$$

Hence

$$\Delta Y_{2,3}^{(\nu)} = \pm\frac{1}{\mathbf{i}\sigma_{kl}}\Big(\frac{dY_{2,3}^{(\nu)}}{dt} - \frac{d\Phi_B}{dt}\Big). \tag{4.8}$$

In terms of the expansion coefficients of the solution of system (3.1) in eigenvectors of the Laplace operators, on the spherical rectangle we have

$$\Delta \boldsymbol{X}^{(\nu)} = \begin{pmatrix} \Delta D_{kl}^{(\nu)} \\ \Delta Q_{kl}^{(\nu)} \\ \Delta\Phi_{kl}^{(\nu)} \end{pmatrix} = \begin{pmatrix} \frac{f_o\dot{Q}_{kl}^{(\nu-1)} - \dot{\Phi}_{kl}^{(\nu-1)}}{\sigma_{kl}^2} \\ -\frac{f_o\dot{D}_{kl}^{(\nu-1)}}{\tilde{\sigma}_{kl}^2} \\ -\frac{d\dot{D}_{kl}^{(\nu-1)}}{\sigma_{kl}^2} \end{pmatrix}, \tag{4.9}$$

which in the original physical space of system (3.1) corresponds to

$$\begin{aligned} H\Delta D^{(\nu)} &= f_o\dot{\tilde{Q}}^{(\nu-1)} - \nabla^2\dot{\Phi}^{(\nu-1)}, \\ H\Delta\Phi^{(\nu)} &= -d\dot{D}^{(\nu-1)}, \\ \Delta\dot{\tilde{Q}}^{(\nu)} &= \frac{f_o}{d}\Delta\Phi^{(\nu)}. \end{aligned} \tag{4.10}$$

which in the original physical space of system (3.1) corresponds to

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Thus, when conducting nonlinear initialization the effect of boundary conditions in the limited model is explicitly taken into account. The horizontal velocity components are reconstructed on the basis of the solution of equations with respect to D and Q

$$\begin{aligned} \nabla_u^2 m_u u &= \delta_x D - \frac{1}{m' m_u}\delta_y m_v^2 Q, \\ \nabla_v^2 m_v v &= \delta_x Q + \frac{1}{m' m_v}\delta_y m_u^2 D. \end{aligned} \tag{4.11}$$

5. Numerical experiments

The developed method of nonlinear initialization was used in the operational technology of numerical short-range weather prediction for the Siberian region. The technological line is based on the model having 15 non-uniform, vertically distributed σ-levels. The longitude-latitude resolution is $1.66^o \times 1.25^o$. The domain of integration is included in the spherical rectangle $40^o - 146.6^o$ E and $40^o - 80^o$ N.

The initialization procedure can be performed directly in the physical space based on relations (4.5), (4.6), (4.10). However, if we select the method of expansion in eigenfunctions as a method of inversion of the Laplace operator, then the solution of the above-mentioned systems is, in fact, equivalent to the procedure of using relations (4.4), (4.9) on whose realization an operative version of nonlinear initialization is based. In addition, numerical experiments have shown that the initialization of the first three vertical modes at three iterations is sufficient.

Figure 1 shows the time-dependent behavior of the surface pressure value with initialization or without it. The data of the operative analysis during 1200 GMT of August 4, 1992, was taken as the initial data. The boundary conditions were formed from the geopotential data in the GRID-code. The temperature was reconstructed from the hydrostatic equation, the values of horizontal components of wind speed at the boundaries being assumed geostrophic. Figure 1 is an illustration of the fact that the initialization effect is dramatic not more than 12 hours, which is proved by formal statistical estimations of the prediction quality.

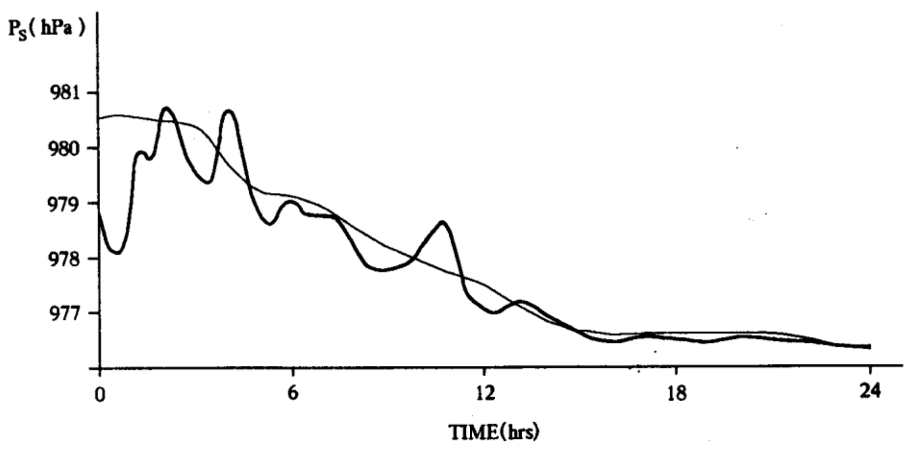

Figure 1. Surface pressure-time plot for grid point at 55 N, 81 E, starting at 1200 GMT 4 August 1992. Before normal-mode initialization (heavy line) and after normal-mode initialization (thin line) of three vertical modes with three iterations.

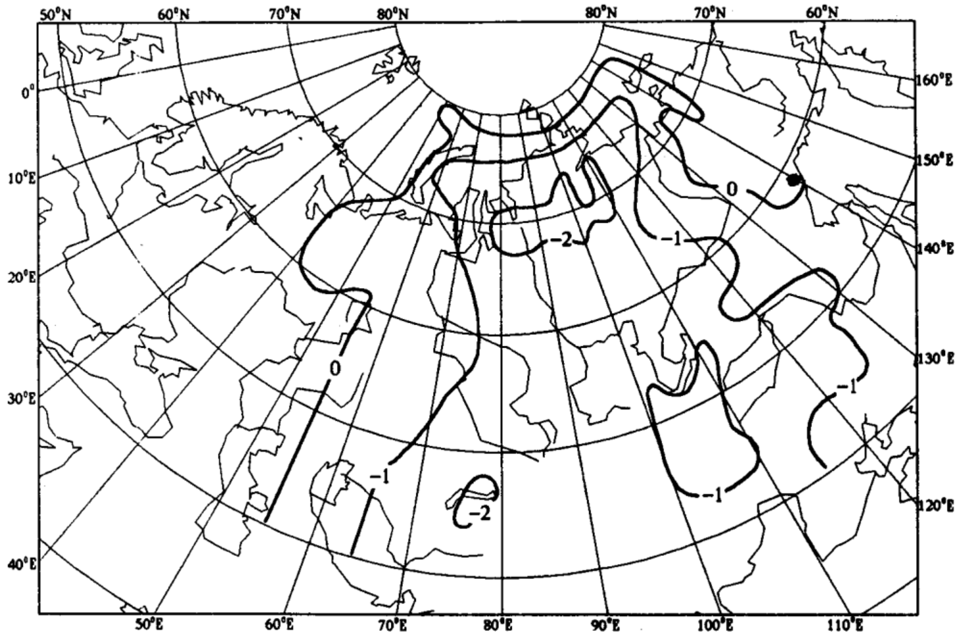

Figure 2. Changes of sea level pressure (hPa) at 1200 GMT, 4 August 1992, by initialization of three vertical modes with three iterations.

6. Conclusion

The paper presents the normal mode nonlinear initialization method for the regional atmospheric model. The procedure is used in the operational technology of the short-range weather prediction for the Siberian region since January, 1992. The method allows one to suppress efficiently amplitudes of high-frequency perturbations at the initial integration stage by inessential changes of the original meteorological elements.

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