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Estimates of conditional stability of some combined inverse problems for Maxwell's equations and equations of porous media*

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A combined mathematical model of Maxwell's system of equations and a system of equations for porous media taking into account admixtures is constructed.

1. Introduction

The mathematical studies of combined inverse problems for equations of mathematical physics are comparatively new. The questions of uniqueness of the solutions to the combined inverse problems of mathematical physics are key issues in their investigation. The uniqueness and stability of the solutions of the combined inverse problem of seismic acoustics and gravics were studied for the first time in [1, 2]. The conditional stability and uniqueness of the solutions to the inverse problem for equations of porous media are considered in [3, 4]. Combined one-dimensional inverse problems for Maxwell's equation and the equation of continual filtration theory are discussed in [5, 6]. It is shown that the solutions to the problems being considered are unique: three one-dimensional functions are determined from two one-dimensional functions (by using the information measured at the free boundary of the displacement velocity of a conducting elastic porous body and the electric field intensity). In this case Archie's law [7] is used: $\sigma_l/\sigma = \rho_{0,l}^m$, m is a positive constant.

The statements of combined inverse problems for Maxwell's equations and equations of porous media result from the mathematical modeling of the processes of vibrational treatment of oil strata. Here it is necessary to determine the coefficients of friction, the permeability and viscosity of the conducting liquid, shear module, all-round compression, and others using indirect information about an oil stratum.

The estimates of conditional stability in a class wider than that considered in [3–6] are obtained. Other statements are considered.

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2. One-dimensional inverse problem for transverse waves

Let us consider the following initial boundary value problem [8]:

$$\rho_{0,s}(z)u_{tt} - (\mu(z)u_z)_z + \chi(z)\rho_{0,l}^2(z)(u_t - v_t) = 0, \qquad (t,z) \in R_2^+, \quad (1)$$

$$\rho_{0,l}(z)v_{tt} - \chi(z)\rho_{0,l}^2(z)(u_t - v_t) = 0, \qquad (t,z) \in R_2^+, \quad (2)$$

$$u(0,z) = U_1(z), \;\; u_t(0,z) = U_2(z), \;\; v(0,z) = v_t(0,z) = 0, \qquad z \in R_1^+, \quad (3)$$

$$\mu(0)u_z(t,0) = 0, \qquad t \in R_1. \quad (4)$$

Here $u(t,z)$ and $v(t,z)$ are the horizontal components of the velocity vector of the elastic porous body and liquid with partial densities $\rho_{0,s}(z)$ and $\rho_{0,l}(z)$, respectively, $\chi(z)$ is the friction coefficient,

$$R_2^+ = \Big\{(t,z) \mid -\infty < t < \infty, \; z > 0\Big\}.$$

Problem 1. *Determine the friction coefficient $\chi(z)$ (the other functions $\rho_{0,l}(z)$, $\rho_{0,s}(z)$, and $\mu(z)$ are known) from the relations (1)–(4) by using the information*

$$u(t,0) = u_0(t), \quad t \in [-T,T]. \quad (5)$$

Assuming in (1) that $t = 0$, we obtain, for $\chi(z)$, the formula

$$\chi(z) = \frac{(\rho_{0,s}(z)c_t^2(z)U_{1z})_z - \rho_{0,s}(z)u_{tt}(0,z)}{\rho_{0,l}^2(z)U_2(z)}, \quad (6)$$

where $c_t(z) = \sqrt{\mu(z)/\rho_{0,s}(z)}$.

Let us eliminate the friction coefficient $\chi(z)$ from system (1), (2) using (6). We obtain a Cauchy problem with a nonlocal operator for u, v

$$\begin{aligned} u_{tt} - c_t^2(z)u_{zz} = {} & \frac{(\rho_{0,s}(z)c_t^2(z))_z}{\rho_{0,s}(z)}u_z + \\ & \frac{(\rho_{0,s}(z)c_t^2(z)U_{1z})_z - \rho_{0,s}(z)u_{tt}(0,z)}{\rho_{0,s}(z)U_2(z)}(v_t - u_t), \end{aligned} \quad (7)$$

$$v_{tt} = \frac{(\rho_{0,s}(z)c_t^2(z)U_{1z})_z - \rho_{0,s}(z)u_{tt}(0,z)}{\rho_{0,l}(z)U_2(z)}(u_t - v_t), \quad t \in R, \;\; z \in R_1^+, \quad (8)$$

$$u(t,0) = u_0(t), \quad u_z(t,0) = 0, \quad t \in [-T,T], \quad (9)$$

$$v(0,z) = v_t(0,z) = 0, \quad z \in R_1^+. \quad (10)$$

Let $Q \subset R^2$ be an open set of real variables t, z, and $\|\cdot\|_{(k)}(Q)$ is the norm in the Sobolev space $W_2^k(Q)$. For the real function $\varphi \in C^\infty(Q)$ and the number $\tau \geq 0$, we introduce a family of norms

$$\|u\|_{\tau}^2(Q) = \int_Q e^{2\tau\varphi(t,z)}|u(t,z)|^2 dz\, dt$$

Let $\Omega = Q \cap \{z \geq 0\} \neq \emptyset$, $P(z,\partial) = n_t^2(z)\partial_t^2 - \partial_z^2$ be D'Alembert's operator, $\partial_t = \partial/\partial t$.

Lemma 1. *Let, for all* $(t,z) \in \Omega$,

$$|\varphi_z| \geq 1, \qquad \varphi_{zz} \geq \delta > 0, \tag{11}$$

$$n_t(z) > 0, \qquad -\frac{\partial n_t(z)}{\partial z} > 0. \tag{12}$$

Then at $n_t \in C^1(\overline{\Omega})$ *there exist numbers* c_0 *and* $\tau_0 > 0$ *such that for all* $u \in C_0^\infty(\Omega)$, $\tau \geq \tau_0$ *the estimate*

$$\tau\Big(\|u\|_\tau^2 + \|\partial_t u\|_\tau^2 + \|\partial_z u\|_\tau^2\Big) \leq c_0\|Pu\|_\tau^2 \tag{13}$$

holds.

Proof. In accordance with Hörmander's theorem [9], it is sufficient to check the positiveness of the function

$$H(x,\xi) = \langle\varphi_{xx}\nabla_\xi P, \nabla_\xi P\rangle + \langle\nabla_\xi P, (\nabla_x P)'_\xi \nabla_x\varphi\rangle - \langle\nabla_x P, (\nabla_\xi P)'_\xi \nabla_x\varphi\rangle, \tag{14}$$

$$(x,\xi) \in \mathcal{A} = \{(x,\xi) \mid x \in \bar{\Omega},\ \xi \in R^2,\ |\xi| \neq 0,\ P(x,\xi) = 0,\ \langle\nabla_\xi P, \nabla_x\varphi\rangle = 0\}.$$

Here the matrices $(\nabla_x P)'_\xi$ and $(\nabla_\xi P)'_\xi$ are the derivatives of the vectors $\nabla_x P$ and $\nabla_\xi P$ with respect to the variables ξ, $\langle\cdot,\cdot\rangle$ is the scalar product in R_2.

Since $P(x,\partial)$ is a differential second order operator with a real coefficient, then, in accordance with [9, Theorem 8.6.3], $\varphi(x) = \exp(\lambda\psi(x))$ can be taken as a weight function, where λ is a sufficiently large positive number, $x = (z,t)$. The symbol of the operator $P(x,\partial)$ has the form $P(x,\xi) = n_t^2(z)\xi_2^2 - \xi_1^2$. We choose $\psi(x) = -z$. Note that $\mathcal{A} = \emptyset$. Then

$$\nabla_\xi P = (-2\xi_1, 2n_t^2(z)\xi_2), \quad \nabla_\xi P = \left(2n_t(z)\frac{\partial n_t(z)}{\partial z}\xi_2^2, 0\right), \quad \nabla_x\varphi = -(\lambda\varphi, 0),$$

$$\varphi_{zz} > 0, \quad \varphi_{tz} = \varphi_{zt} = \varphi_{tt} = 0, \quad P_{z\xi_2} = 4n_t(z)\xi_2\frac{\partial n_t(z)}{\partial z},$$

$$P_{\xi_1\xi_1} = -2, \quad P_{\xi_2\xi_2} = 2n_t^2(z), \quad P_{\xi_1\xi_2} = P_{t\xi_1} = P_{t\xi_2} = P_{z\xi_1} = 0.$$

Substituting these expressions into (14), after easy transformations and taking into account the estimate (12), we obtain

$$H(x,\xi) = 4\lambda\varphi n_t(z)\xi_2^2\left(\lambda - \frac{\partial n_t(z)}{\partial z}\right) > 0$$

as was to be shown. □

Assume that $\Omega_\varepsilon = \Omega \cap \{\varphi > \varepsilon\}$ and $\eta \in C^\infty(\Omega_\varepsilon)$, so that $0 \le \eta \le 1$ in Ω, with $\eta = 1$ at $(t,z) \in \Omega_\varepsilon$ and $\eta = 0$ at $(t,z) \in \Omega \setminus \Omega_{\varepsilon/2}$.

Let us use the notation $\mathcal{M}_1 = \{u \mid u \in C^2,\ \|u\|_{C^2(\Omega_0)} \le M_1\}$, where M_1 is a positive number.

Theorem 1. *Assume that $c_t(z), \rho_{0,s}(z), \rho_{0,l}(z) \in C^1(\overline{\Omega})$ and $c_t(z)$ satisfies conditions $c_t(z) > 0$, $\frac{\partial c_t(z)}{\partial z} > 0$.*

Let $(u,v) \in \mathcal{M}_1 \times C^2(\Omega)$ be the solutions to problem (7), (8), (9), and (10) and $U_2(z) \ne 0$. Then the following estimate of conditional stability is valid:

$$\|v\|_{(0)}(\Omega_\varepsilon) + \|v_t\|_{(0)}(\Omega_\varepsilon) + \|u\|_{(1)}(\Omega_\varepsilon) \le C\|u_0\|^k_{(3/2)}(\{z=0\})\|u\|^{1-k}_{(2)}. \qquad (15)$$

Here C does not depend on u and v, $k \in (0,1)$.

Proof. Solving the Cauchy problem (8), (10) for v_t and estimating it in the weight L_2 norm, we obtain

$$\int_{\Omega_\varepsilon} e^{2\tau\varphi(z)}|v_t|^2 dz\,dt \le Ce^{2TM_1}\int_{\Omega_\varepsilon} e^{2\tau\varphi(z)}|u_t|^2 dz\,dt, \qquad (16)$$

Here we used the meanvalue theorem and the boundedness of the operator of integration J, $(Ju(t) = \int_0^t u(\tau)\,d\tau)$ in the space $L_2(-T,T)$.

Below $C(\varepsilon)$ stands for the positive constant. To avoid the numbering of the constants $C(\varepsilon)$, each constant is considered to be larger than the preceding one.

In accordance with the continuation theorem [9, 10], there exists such a function $\tilde{u}$ that

$$\|\tilde{u}\|^2_{(2)}(\Omega_0) \le C\|u_0\|^2_{(3/2)}(\{z=0\}), \qquad (17)$$

$$\tilde{u}(t,0) = u_0(t), \quad \tilde{u}_z(t,0) = 0, \quad t \in [-T,T],$$

we determine $w = u - \tilde{u}$.

Note that

$$|P(\eta w)|^2 \le C\Big(|u|^2 + |u_t|^2 + |u_z|^2 + |Pu|^2 + |P\tilde{u}|^2\Big). \qquad (18)$$

Therefore, from estimate (13) we obtain

$$\begin{aligned}
\tau\int_{\Omega_\varepsilon} & e^{2\tau\varphi(z)}\Big(|u|^2 + |u_t|^2 + |u_z|^2\Big)dz\,dt \\
&\le \tau\int_{\Omega_0} e^{2\tau\varphi(z)}\Big(|\eta u|^2 + |\eta u_t|^2 + |\eta u_z|^2\Big)dz\,dt \\
&\le c_0\int_{\Omega_0} e^{2\tau\varphi(z)}|P(\eta u)|^2 dz\,dt
\end{aligned}$$

$$\leq C \int_{\Omega_0} e^{2\tau\varphi(z)} \Big(|u|^2 + |u_t|^2 + |u_z|^2 + |v_t|^2 + |P\tilde{u}|^2\Big) dz\, dt$$
$$\leq C \int_{\Omega_0} e^{2\tau\varphi(z)} \Big(|u|^2 + |u_t|^2 + |u_z|^2 + |P\tilde{u}|^2\Big) dz\, dt. \tag{19}$$

Here we used inequality (16) at $\varepsilon = 0$.

Choosing $\tau > 2C$, from (19) we obtain

$$\tau \int_{\Omega_\varepsilon} e^{2\tau\varphi(z)} \Big(|u|^2 + |u_t|^2 + |u_z|^2\Big) dz\, dt$$
$$\leq 2C \Bigg(\int_{\Omega_0} e^{2\tau\varphi(z)} |P\tilde{u}|^2 dz\, dt + \int_{\Omega_0 \setminus \Omega_{\varepsilon/2}} e^{2\tau\varphi(z)} \Big(|u|^2 + |u_t|^2 + |u_z|^2\Big) dz\, dt \Bigg). \tag{20}$$

Factoring $\min \exp(2\tau\varphi) = \exp(2\tau\varepsilon)$ outside the integral over Ω_ε, and factoring the weight maxima $\max \exp(2\tau\varphi) = \exp(\tau\varepsilon)$ and $\max \exp(2\tau\varphi) < \exp(2\tau)$ outside the integral over $\Omega_0 \setminus \Omega_{\varepsilon/2}$, after dividing by factor $\exp(2\tau\varepsilon)$, we obtain

$$\tau \int_{\Omega_\varepsilon} \Big(|u|^2 + |u_t|^2 + |u_z|^2\Big) dz\, dt \leq C\big(A_1 e^{2\tau(1-\varepsilon)} + A_2 e^{-\tau\varepsilon}\big). \tag{21}$$

Here $A_1 = \|\tilde{u}\|^2_{(2)}(\Omega_0)$, $A_2 = \|u\|^2_{(1)}(\Omega_0)$. We choose

$$\tau = (2-\varepsilon)^{-1} \ln(A_2/A_1) + C(\varepsilon).$$

Then the first term in the right-hand side of (21) is not larger than the second term. Since $\tilde{u} = u - w$, at the given τ estimate (21) is valid with u instead of $\tilde{u}$. Taking into account the choice of τ and estimate (17), we derive from (21) that

$$\|u\|^2_{(1)}(\Omega_\varepsilon) \leq C A_1^{\varepsilon(2-\varepsilon)^{-1}} A_2^{1-\varepsilon(2-\varepsilon)^{-1}}. \tag{22}$$

As $v(t,z) = (Jv_t)(t,z)$, and since the operator of integration J in the space $L_2(-T,T)$ is bounded, the following estimate holds:

$$\|v\|^2_{(0)}(\Omega_\varepsilon) \leq C\|v_t\|^2_{(0)}(\Omega_\varepsilon). \tag{23}$$

Assuming that $\tau = 0$ in estimate (17), we have

$$\|v_t\|^2_{(0)}(\Omega_\varepsilon) \leq C\|u_t\|^2_{(0)}(\Omega_{-\varepsilon}). \tag{24}$$

Hence, from estimates (23) and (24) we obtain

$$\|v\|^2_{(0)}(\Omega_\varepsilon) + \|v_t\|^2_{(0)}(\Omega_\varepsilon) \leq C\|u_t\|^2_{(0)}(\Omega_\varepsilon) \leq C\|u\|^2_{(1)}(\Omega_\varepsilon). \tag{25}$$

Therefore, taking into account estimate (22), we get

$$\|v\|^2_{(0)}(\Omega_\varepsilon) + \|v_t\|^2_{(0)}(\Omega_\varepsilon) \le C A_1^{\varepsilon(2-\varepsilon)^{-1}} A_2^{1-\varepsilon(2-\varepsilon)^{-1}}.$$

Combining this estimate with inequality (22), we obtain estimate (14). □

Corollary 1. *Let $c_t(z)$, $\rho_{0,s}(z)$, $\rho_{0,l}(z)$ satisfy the conditions of Theorem 1. Then the solution u, v, χ to Problem 1 is unique.*

The proof of Corollary 1 is performed in the same way as in [11].

3. One-dimensional inverse problem for longitudinal waves

Let us consider in half-space R_2^+ the following initial boundary value problem for the equations of porous media [8, 12]:

$$u_{tt} - L_1(u,v) + \chi(z)\frac{\rho_{0,l}^2(z)}{\rho_{0,s}(z)}(u_t - v_t) = 0, \quad (t,z) \in R_2^+, \tag{26}$$

$$v_{tt} - L_2(u,v) - \chi(z)\rho_{0,l}(z)(u_t - v_t) = 0, \quad (t,z) \in R_2^+, \tag{27}$$

$$u|_{t=0} = U_1(z), \quad u_t|_{t=0} = U_2(z), \quad v|_{t=0} = v_t|_{t=0} = 0, \quad z \in R_1^+, \tag{28}$$

$$\hat{h}_{33} + p|_{z=0} = 0, \quad \left.\frac{\rho_{0,l}}{\rho_0}p\right|_{z=0} = 0, \quad t \in R_1. \tag{29}$$

Here $u(t,z)$ and $v(t,z)$ are the normal components of the velocity displacement vector of the elastic porous body and liquid, $L_k(u,v)$, $k=1,2$, are the second-order differential operators with respect to z (they depend on the parameters of the medium $\rho_{0,l}(z)$, $\rho_{0,s}(z)$, $\lambda(z)$, $\mu(z)$, and $\alpha(z)$) determined in [12].

Problem 2. *Determine the functions $\chi(z)$ (the other functions $\rho_{0,l}(z)$, $\rho_{0,s}(z)$, $\lambda(z)$, $\mu(z)$, and $\alpha(z)$ are given) from relations (26)–(29) and*

$$u|_{z=0} = u_0(t), \quad t \in [-T,T]. \tag{30}$$

We consider that $\chi(z) \in C(R_1^+)$, $\rho_{0,l}(z)$, $\rho_{0,s}(z)$, $\lambda(z)$, $\mu(z)$, $\alpha(z) \in C^2(R_1^+)$ and the velocities of the first longitudinal wave $c_{l_1}(z)$ are as follows:

$$c_{l_1}(z) > 0, \quad \frac{\partial c_{l_1}(z)}{\partial z} > 0, \quad z \in R_1^+. \tag{31}$$

Assuming that $t=0$ in (26), we obtain the following formula for $\chi(z)$:

$$\chi(z) = \frac{\rho_{0,s}(z)}{\rho_{0,l}^2(z)} \frac{L_1(U_1, 0) - u_{tt}(0, z)}{U_2(z)}. \tag{32}$$

Let us exclude the friction coefficient $\chi(z)$ from equations (26) and (27) by using (32). We obtain differential second-order equations with a nonlocal operator for u, v:

$$u_{tt} - L_1(u, v) + \frac{L_1(U_1, 0) - u_{tt}(0, z)}{U_2(z)}(u_t - v_t) = 0, \quad (t, z) \in R_2^+, \tag{33}$$

$$v_{tt} - L_2(u, v) - \frac{\rho_{0,s}(z)}{\rho_{0,l}(z)} \frac{L_1(U_1, 0) - u_{tt}(0, z)}{U_2(z)}(u_t - v_t) = 0, \quad (t, z) \in R_2^+. \tag{34}$$

Let us use the following notation:

$$\mathcal{M}_2 = \{u, v \mid u, v \in C^2, \ \|u\|_{C^2(\Omega_0)} + \|v\|_{C^2(\Omega_0)} \le M_2\},$$

where M_2 is a positive number.

Theorem 2. *Assume that the velocity $c_{l_1}(z)$ satisfies condition* (31).

Let $(u, v) \in \mathcal{M}_2$ be the solutions to problems (28)–(30), (33), *and* (34) *and $U_2(z) \neq 0$. Then the following estimate of conditional stability is valid:*

$$\begin{aligned} &\|v\|_{(0)}(\Omega_\varepsilon) + \|v_t\|_{(0)}(\Omega_\varepsilon) + \|u\|_{(1)}(\Omega_\varepsilon) \\ &\qquad \le C\|u_0\|_{(3/2)}^k(\{z = 0\})(\|u\|_{(2)} + \|v\|_{(2)})^{1-k}. \end{aligned}$$

Here C does not depend on u and v, $k \in (0, 1)$.

The proof of this theorem is similar to that of Theorem 1.

Corollary 2. *Let c_{l_1} satisfy the conditions of Theorem 2. Then the solution u, v, χ to Problem 2 is unique.*

The proof of Corollary 2 is performed in the same way as in [11].

4. Multidimensional inverse problem for equations of porous media

Let us use the following notation: $\boldsymbol{u}(\tilde{x}) = (u_1(\tilde{x}), u_2(\tilde{x}), u_3(\tilde{x}))$ and $\boldsymbol{v}(\tilde{x}) = (v_1(\tilde{x}), v_2(\tilde{x}), v_3(\tilde{x}))$ are the velocity vectors of the elastic porous body and liquid, respectively,

$$\mathcal{M}_3 = \{\boldsymbol{u}, \boldsymbol{v} \mid \boldsymbol{u}, \boldsymbol{v} \in W_2^4, \|\boldsymbol{u}\|_{(4)}(Q_0) + \|\boldsymbol{v}\|_{(4)}(Q_0) \le M_3\},$$

M_3 is a positive number,

$$\tilde{x} = (x, t) \in R^4, \quad x = (x_1, x_2, x_3), \quad t = x_4,$$
$$(\boldsymbol{u}, \boldsymbol{v}) = u_1 v_1 + u_2 v_2 + u_3 v_3, \quad |\boldsymbol{u}| = (\boldsymbol{u}, \boldsymbol{v})^{1/2},$$
$$D^\alpha = D_1^{\alpha_1} D_2^{\alpha_2} D_3^{\alpha_3} D_4^{\alpha_4}, \quad D_j = -i\frac{\partial}{\partial x_j}, \quad \alpha = (\alpha_1, \ldots, \alpha_4), \quad |\alpha| = \sum_{j=1}^{4} \alpha_j,$$
$$\Delta = \sum_{j=1}^{3} \left(\frac{\partial}{\partial x_j}\right)^2, \quad \nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right),$$
$$\psi = -x_1^2 - x_2^2 - 2x_3^2 + 4ax_3 - \theta t^2 + r^2, \quad a = \frac{r^2}{4d} - \frac{d}{2}, \quad \theta = \frac{r^2}{T^2},$$
$$Q_\sigma = Q \cap \{\psi > \sigma\}, \quad Q = G \times (-T, T), \quad \Sigma = \Gamma \times (-T, T),$$
$$\Gamma = \partial G \cap \{x_3 = 0\}, \quad G = \left\{x \;\middle|\; -d < x_3 < 0, \ \sqrt{x_1^2 + x_2^2} < r\right\},$$

d, T are some positive numbers.

Assume that the domain $D = \{G \times (0, T)\} \cap \{\psi > 0\}$ is filled with an isotropic-porous medium. The parameters of the medium $\lambda(x)$, $\mu(x)$, $\alpha(x)$, $\rho_{0,s}(x)$, and $\rho_{0,l}(x)$ are elements of the space $C^2(\overline{\Omega}) \cap W_2^4(\Omega)$, $\Omega = \{G \times \{t = 0\}\} \cap \{\psi > 0\}$. In what follows these conditions are considered satisfied.

The wave propagation process in such a medium is described by the following initial boundary value (direct) problem:

$$\boldsymbol{u}_{tt} - L_1(\boldsymbol{u}, \boldsymbol{v}) = \boldsymbol{F}_t, \quad \boldsymbol{v}_{tt} - L_2(\boldsymbol{u}, \boldsymbol{v}) = \boldsymbol{F}_t \quad \text{at} \quad D, \quad \boldsymbol{u}, \boldsymbol{v} \in W_2^4(D), \tag{35}$$
$$\boldsymbol{u} = \boldsymbol{v} = \boldsymbol{u}_t = \boldsymbol{v}_t = 0 \quad \text{at} \quad \{G \times \{0\}\} \cap \{\psi > 0\}, \tag{36}$$
$$h_{i3} + P\delta_{i3} = 0, \quad \frac{\rho_{0,l}}{\rho_0} P = 0 \ \text{at} \ \{G \times (0, T)\} \cap \{\psi > 0\}, \quad i = 1, 2, 3. \tag{37}$$

Here δ_{ij} is the Kronecker symbol, the operators $L_k(\boldsymbol{u}, \boldsymbol{v})$, $k = 1, 2$, are determined in [12].

Problem 3 [3, 4]. *Determine three functions* $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{g})$, $\boldsymbol{F}_t = \mathcal{F}\boldsymbol{g}$, $\mathbf{g}_t = 0$, *from* (35)–(37) *using the additional condition*

$$\boldsymbol{u} = \boldsymbol{u}^0 \quad \text{at} \quad \{\Gamma \times (0, T)\} \cap \{\psi > 0\} \tag{38}$$

at the given $\mathcal{F}(\tilde{x})$, $\lambda(x)$, $\mu(x)$, $\alpha(x)$, $\rho_{0,s}(x)$, $\rho_{0,l}(x)$.

Assuming that $t = 0$ in the first equation of system (35), we exclude the unknown function $\boldsymbol{g}(x)$ from the system of equations (35) and making an even continuation $(\boldsymbol{u}(t, x) = \boldsymbol{u}(-t, x)$, $\boldsymbol{v}(t, x) = \boldsymbol{v}(-t, x))$ of the functions $\boldsymbol{u}$ and $\boldsymbol{v}$ over t at $[-T, 0]$, we obtain

$$\boldsymbol{u}_{tt} - L_1(\boldsymbol{u}, \boldsymbol{v}) = \mathcal{F}\mathcal{F}^{-1}(0, x)\boldsymbol{u}_{tt}(0, x) \quad \text{and}$$
$$\boldsymbol{v}_{tt} - L_2(\boldsymbol{u}, \boldsymbol{v}) = \mathcal{F}\mathcal{F}^{-1}(0, x)\boldsymbol{u}_{tt}(0, x) \quad \text{at} \quad Q_0, \tag{39}$$
$$\boldsymbol{v} = \boldsymbol{v}_t = 0 \quad at \quad \{G \times \{0\}\} \cap \{\psi > 0\}, \tag{40}$$
$$\boldsymbol{u} = \boldsymbol{G}^0, \quad \boldsymbol{u}_{x_3} = \boldsymbol{G}^1 \quad at \quad \Sigma_0. \tag{41}$$

Here

$$\boldsymbol{G}^0 = \boldsymbol{u}_t^0, \quad \boldsymbol{G}^1 = (G_1^1, G_2^1, G_3^1), \quad G_k^1 = -u_{3tx_k}^0, \quad k = 1, 2,$$
$$G_3^1 = \frac{\lambda - (\frac{\rho_{0,s}}{\rho_0} + 1)K + \rho_0\rho_{0,s}\alpha + (\rho_0^2\alpha - K)(\frac{K}{\alpha} - \rho_0\rho_{0,s})/\rho_0^2}{\lambda + 2\mu - (\frac{\rho_{0,s}}{\rho_0} + 1)K + \rho_0\rho_{0,s}\alpha + (\rho_0^2\alpha - K)(\frac{K}{\alpha} - \rho_0\rho_{0,s})/\rho_0^2} \times$$
$$(u_{1tx_1}^0 + u_{2tx_2}^0).$$

Let us introduce the following conditions:

$$0 < n_j^2, \quad 0 < \frac{\partial n_j^2}{\partial x_3} \quad at \quad \overline{\Omega}, \quad j = t, l_1. \tag{42}$$

Theorem 3. *Assume that $n_j(x), j = t, l_1$, satisfy condition (42), the coefficients of the matrices $\mathcal{F}$ belong to $C^2\left(\overline{Q}_0\right)$ and $\det \mathcal{F} \neq 0$ at $Q_0 \cap \{t = 0\}$.*

If a and T are chosen such that

$$4d^2 + 2\sqrt{2}n_j r d < r^2, \quad j = t, l_1,$$
$$3n_j^2 + \frac{1}{2}\left(x_1 n_{jx_1}^2 + x_2 n_{jx_2}^2\right) < a\frac{\partial n_j^2}{\partial x_3} \quad at \quad \overline{\Omega}, \quad j = t, l_1,$$
$$r^2 n_j^2 \leq T^2, \quad r^2(n_j^2/T + |\nabla n_j^2|/n_j^2) < T \quad at \quad \overline{\Omega}, \quad j = t, l_1,$$

then the solutions $\boldsymbol{u}, \boldsymbol{v} \in \mathcal{M}_3$ to problem (39)–(41) satisfy the following estimate of conditional stability:

$$\|\boldsymbol{v}\|_{(1)}^2(Q_\varepsilon) + \|\boldsymbol{u}\|_{(3)}^2(Q_\varepsilon)$$
$$\leq C(\varepsilon)\|\boldsymbol{u}_t^0\|_{(7/2)}^{2k}(\Sigma_0)\left[\|\boldsymbol{v}\|_{(4)}^2(Q_0) + \|\boldsymbol{u}\|_{(4)}^2(Q_0)\right]^{(1-k)}.$$

Here $C(\varepsilon)$ is a positive constant that does not depend on $\boldsymbol{u}$ and $\boldsymbol{v}$, $k \in (0, 1)$.

The proof of the theorem is similar to that from [4] with the use of the idea from Section 1.

Corollary 3. *Let $\mathcal{F}, \mathcal{F}_t$, $n_j^2, j = t, l_1$, satisfy the conditions of Theorem 3. Then the solution $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{g})$ to Problem 3 is unique.*

The proof of Corollary 3 is performed in the same way as in [13].

5. Combined one-dimensional inverse problem for equations of electrodynamics and porous media

Let us consider the following initial boundary value problems for a parabolic equation in [6, 14] in the half-space R_2^+:

$$E_t - \sigma^{-1}(z)E_{zz} = H\Big(\frac{\sigma_s(z)}{\sigma(z)}u_t + \frac{\sigma_l(z)}{\sigma(z)}v_t\Big), \quad (t,z) \in R_2^+, \tag{43}$$

$$E|_{t=0} = E_1(z), \quad z \in R_1^+, \tag{44}$$

$$\sigma^{-1}E_z|_{z=0} = 0, \quad t \in R_1. \tag{45}$$

Here $E(t,z)$ is the horizontal component of electric field intensity, $\sigma(z) = \sigma_l(z) + \sigma_s(z)$, $\sigma_l(z)$ and $\sigma_s(z)$ are the conductivities of the liquid and elastic porous body, respectively.

Problem 4 (5). *Determine the functions $\sigma_l(z)$, $\sigma_s(z)$, and $\chi(z)$ (the other functions $\rho_{0,l}(z)$, $\rho_{0,s}(z)$, $\mu(z)$ (or $\rho_{0,l}(z)$, $\rho_{0,s}(z)$, $\lambda(z)$, $\mu(z)$, $\alpha(z)$), and the constant H are known) from relations* (1)–(5), (26)–(30), (43)–(45), *and*

$$E|_{z=0} = E_0(t), \quad t \in [-T,T]. \tag{46}$$

Note that the right-hand side of equation (43) has a known quantity. Actually, solving the Cauchy problems (7)–(10), (28)–(30), (33), and (34) for transverse (longitudinal) waves, we determine the functions u and v. Then, using Archie's law [7] and the formula $\sigma_s/\sigma = 1 - \rho_{0,l}^m$, we determine the right-hand side of (43).

Lemma 2. *Let $\sigma_l(z), \sigma_s(z) \in C^2(R_1^+)$, $\sigma_l(z) > 0$, $\sigma_s(z) > 0$, and $E(t,z)$ be the solution to the initial boundary value problem* (43)–(45) *such that $E(t,z) \in \mathcal{M}_1$. Besides, $\partial^2 E_1(z)/\partial z^2 \neq 0$, $z \in R_1^+$, $u(t,z), v(t,z) \in C^1(R_2^+)$. Then the function $E_0(t)$, $t \in (-T,T)$, uniquely determines the function $\sigma(z)$.*

The proof of the lemma follows from [15, Theorem 1, p. 95].

Theorem 4 (5). *Let $u(t,z)$, $v(t,z)$, $E(t,z)$ be the solution to the initial boundary value problem* (1)–(5), (26)–(30), (43)–(45) *such that $E(t,z) \in \mathcal{M}_1$, $(u(t,z), v(t,z)) \in \mathcal{M}_1 \times C^2(\Omega)$ $(u(t,z), v(t,z) \in \mathcal{M}_2)$. Besides, $U_2(z) \neq 0$, $\partial^2 E_1(z)/\partial z^2 \neq 0$, $z \in R_1^+$, $\chi(z) \in C[0,\infty)$, $\sigma_l(z), \sigma_s(z) \in C^2[0,\infty)$ and conditions* (12) (*or* (31)) *are satisfied. Then the functions $u_0(t)$, $E_0(t)$, $t \in (-T,T)$, uniquely determine the functions $\chi(z)$, $\sigma_l(z)$, $\sigma_s(z)$.*

The proof of the theorem follows from Theorem 1 (2) and Lemma 2 with the use of Archie's law (as in [6, Section 1]).

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