Bull. Nov. Comp. Center, Math. Model. in Geoph., 24 (2022), 1–6
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Two-phase flows in an inclined channel*

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Abstract. The application of a hydrodynamic model of the two-velocity media flow of to non-stationary problems of the heterophase flows motion in channels is considered. Various flow regimes of such media are studied for given penetration rates for initially inhomogeneous flow at various values of channel inclination.

The presented mathematical model can be used for describing various types of geological fluid, fluid-magmatic, hydrothermal systems, as well as for describing convective heat and mass transfer in ore-magmatic systems.

Keywords: heterophase medium, channel flow, fluids, mathematical modeling

1. Mathematical model

Let us consider a system of equations for a pressure-equilibrium two-velocity model of a compressible two-phase medium that takes into account energy dissipation through phase viscosity, thermal conductivity, and interfacial friction. The heterophase medium flow is determined by the equations describing the conservation laws for mass, momentum, and entropy [1]:

$$\frac{\partial \rho_s}{\partial t} + \operatorname{div}(\rho_s \boldsymbol{u}) = 0, \quad \frac{\partial \rho_l}{\partial t} + \operatorname{div}(\rho_l \boldsymbol{v}) = 0,$$

$$\rho_s\Big(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}, \nabla)\boldsymbol{u}\Big) + \frac{\rho_s}{\rho}\nabla P + \frac{\rho_s\rho_l}{\rho}\nabla Q = -b\rho_l(\boldsymbol{u}-\boldsymbol{v}) + \eta_s\Delta\boldsymbol{u} + \rho_s\boldsymbol{g},$$

$$\rho_l\Big(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}, \nabla)\boldsymbol{v}\Big) + \frac{\rho_l}{\rho}\nabla P - \frac{\rho_s\rho_l}{\rho}\nabla Q = b\rho_l(\boldsymbol{u}-\boldsymbol{v}) + \eta_l\Delta\boldsymbol{v} + \rho_l\boldsymbol{g},$$

$$\frac{\partial(\rho s)}{\partial t} + \operatorname{div}\Big((\rho_s\boldsymbol{u} + \rho_l\boldsymbol{v})s - \aleph\frac{1}{T}\nabla T\Big) = b\frac{1}{T}\rho_l(\boldsymbol{u}-\boldsymbol{v})^2.$$

This system is closed by the equations of state

$$\frac{1}{\rho_0}\delta\rho = \alpha_T\delta P - \beta_P\delta T,$$

$$\delta s = \frac{c_p}{T_0}\delta T - \frac{1}{\rho_0}\beta_P\delta P,$$

$$\frac{1}{\rho_{0s}}\delta\rho_s = \alpha_T\delta P + \rho_{0s}\alpha_Q\delta Q - \beta_P\delta T,$$

*The work was carried out according to the state order for the Institute of Geology and Mineralogy of the Siberian Branch of the Russian Academy of Sciences.

where

$$c_p = c_{ps}^{\mathrm{ph}}(1-\phi) + c_{pl}^{\mathrm{ph}}\phi, \qquad \alpha_T = \alpha_{Ts}^{\mathrm{ph}}(1-\phi) + \alpha_{Tl}^{\mathrm{ph}}\phi,$$
$$\alpha_Q = \alpha_{Qs}^{\mathrm{ph}}(1-\phi), \qquad \beta_P = \beta_{Ps}^{\mathrm{ph}}(1-\phi) + \beta_{Pl}^{\mathrm{ph}}\phi.$$

Here u_x, u_y, v_x, v_y, ρ, ρ_l, ρ_s, s, P, Q, T are unknowns for the models, $\alpha_{Qs}^{\mathrm{ph}}$, $\alpha_{Ts}^{\mathrm{ph}}$, $\alpha_{Tl}^{\mathrm{ph}}$, β_{Ps}^{ph}, β_{Pl}^{ph}, c_{ps}^{ph}, c_{pl}^{ph} are thermodynamic parameters of phases, ϕ is the volumetric content of dispersed phase, η_s, η_l are phase viscosities, $\aleph$ is the coefficient of thermal conductivity, $b = \dfrac{\eta_l}{\rho k}$ is the coefficient of interfacial friction, and k is the permeability coefficient.

2. Problem statement

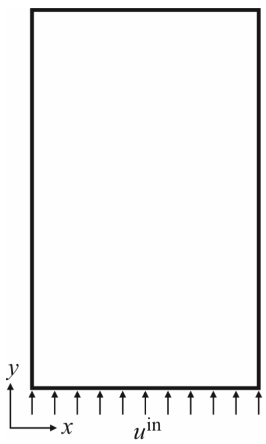

Figure 1

The paper considers the pressure-driven flow of a viscous two-phase medium in a flat channel in a gravity field. As the calculation area, a rectangular area was chosen with the side walls parallel to the coordinate axes x, y (Figure 1).

On the lateral boundary $x = 0$, the no-flow and no-slip boundary conditions are set:

$$u_x|_{x=0} = u_y|_{x=0} = 0,$$

Similar statements are set for the components of velocity vector of the second phase and on the lateral boundary $x = L_x$ of the computational domain.

At the "inlet" boundary $y = 0$, the velocity vector components for the dispersed phase are specified:

$$u_x|_{y=0} = u_x^{\mathrm{in}}, \quad u_y|_{y=0} = u_y^{\mathrm{in}},$$

and similar relations can be written for the components of the carrier phase velocity vector.

The following conditions are imposed on the "output" boundary for the components of the velocity vector of the dispersed phase:

$$\left.\frac{\partial u_x}{\partial y}\right|_{y=L_y} = 0, \quad \left.\frac{\partial u_y}{\partial y}\right|_{y=L_y} = 0,$$

and similarly for the components of the carrier phase velocity vector.

3. Numerical results

Numerical calculations were carried out and results were obtained for the models with the following physical parameters of a two-phase medium: physical phase densities $\rho_s = 8.8 \cdot 10^2$ kg/m^3, $\rho_l = 9.9 \cdot 10^2$ kg/m^3, phase viscosities of the carrier phase $\eta_l = 0.001$ N $\cdot$ s/m^2, and dispersed phase viscosity $\eta_s = 0.1$ N $\cdot$ s/m^2. The rest of the parameters correspond to the normal conditions. The value of the velocity vector component of the dispersed phase $u_{sy} = 0.1$ m/s and, similarly, of the carrier phase $u_{sy} = 0.1$ m/s.

The calculations were carried out in a channel with the inclination angle of 0, 5, and 25°.

3.1. Calculations for a horizontally oriented channel. The distribution of the volumetric content of the dispersed phase and temperature in a channel with the inclination angle of 0° for high and low values of the Reynolds number are shown in Figures 2–5.

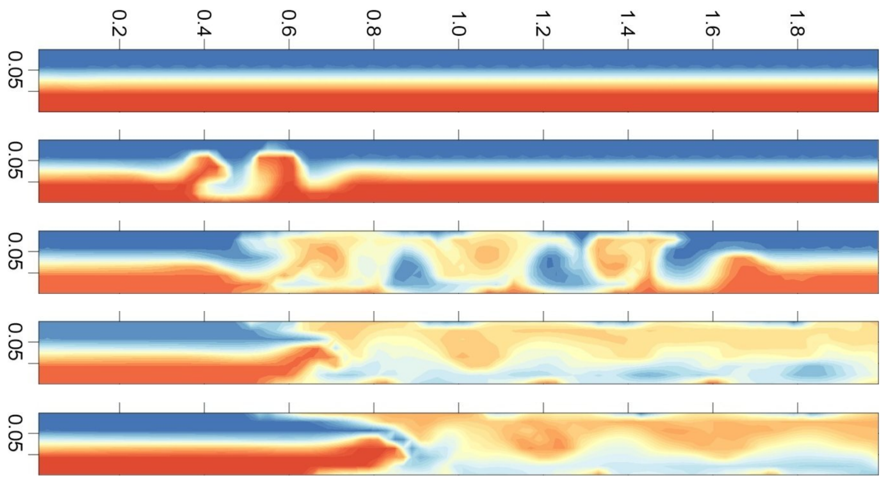

Figure 2. Volumetric dispersed phase content at the time points of 1,000, 5,000, 10,000, 20,000, 40,000 (Re = 1875, inclination angle 0°)

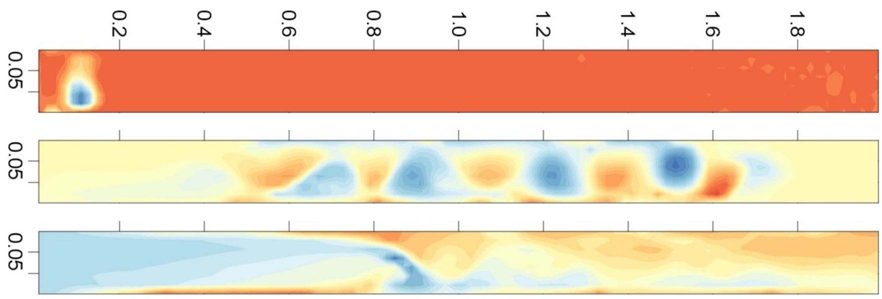

Figure 3. The two-phase medium temperature at the time points of 1,000, 10,000, 40,000 (Re = 1875, inclination angle 0°)

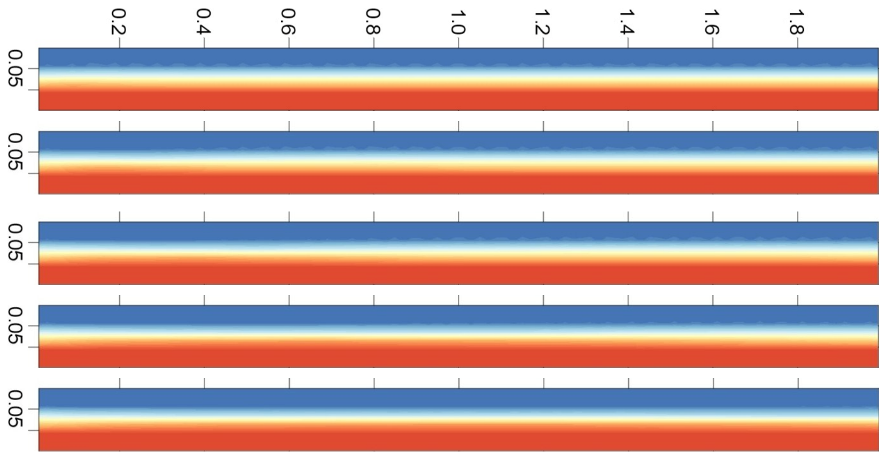

Figure 4. Dispersed phase volumetric content at at the time points of 1,000, 5,000, 10,000, 20,000, 40,000 (Re = 188, inclination angle 0°)

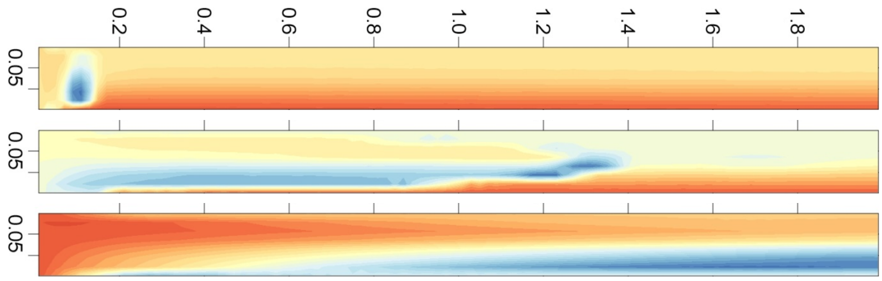

Figure 5. Two-phase medium temperature at the time points of 1,000, 10,000, 40,000 (Re = 188, inclination angle 0°)

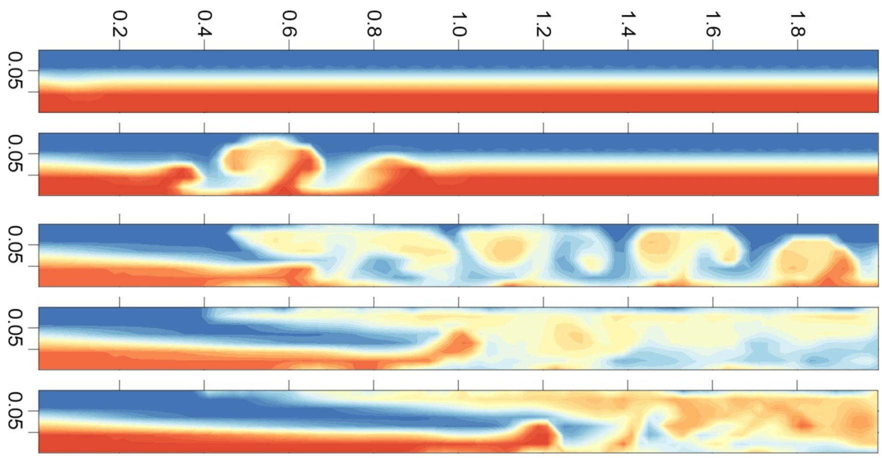

Figure 6. Volumetric content of the dispersed phase at the time points of 1,000, 5,000, 10,000, 20,000, 40,000 (Re = 1875, inclination angle 5°)

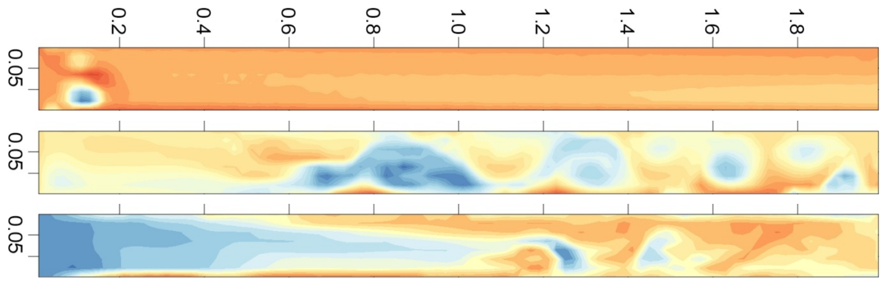

Figure 7. Two-phase medium temperature at the time points of 1,000, 10,000, 40,000 (Re = 1875, inclination angle 5°)

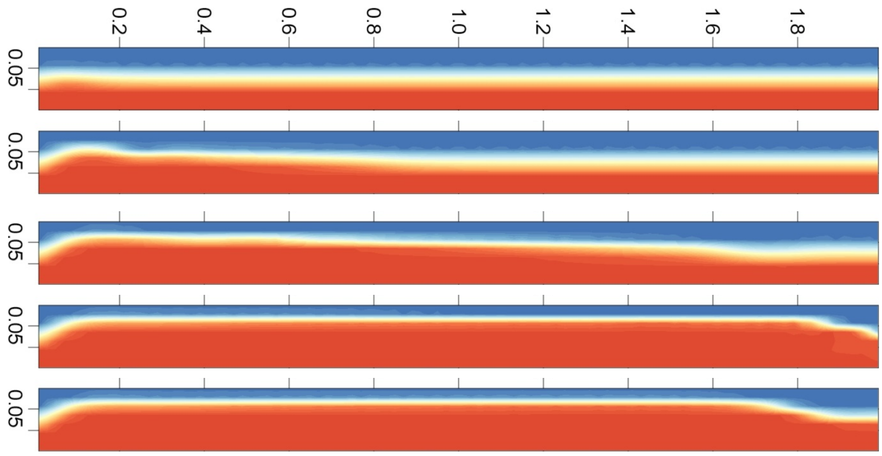

Figure 8. Volumetric content of the dispersed at the time points of 1,000, 5,000, 10,000, 20,000, 40,000 (Re = 188, inclination angle 5°)

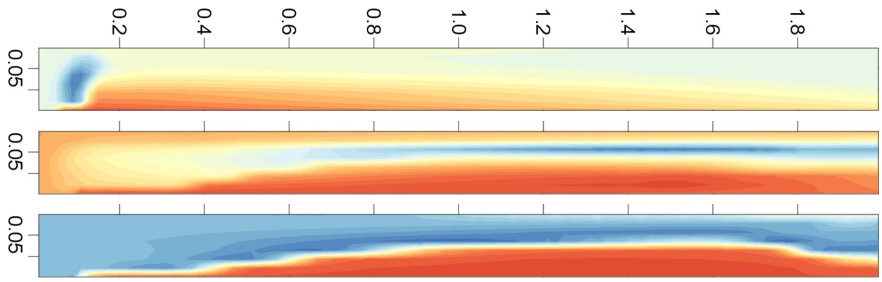

Figure 9. Two-phase medium temperature at the time points of 1,000, 10,000, 40,000 (Re = 188, inclination angle 5°)

3.2. Calculations for a channel with a small inclination angle. The volumetric content distribution for dispersed phase and the temperature distribution in a channel with the inclination angle of 5° for high and low values of the Reynolds number are shown in Figures 6–9.

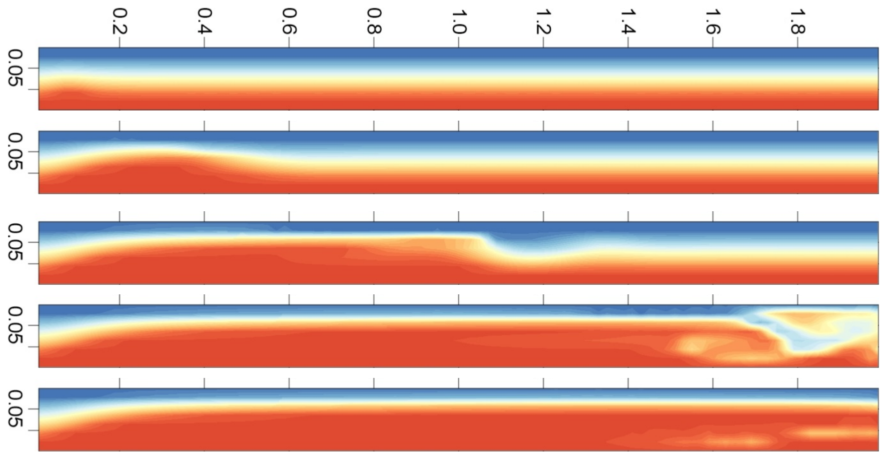

Figure 10. Volumetric content of the dispersed phase at the time points of 1,000, 5,000, 10,000, 20,000, 40,000 (Re = 188, inclination angle 25°)

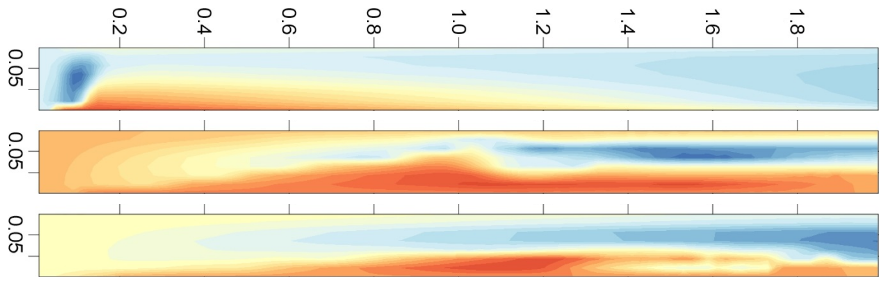

Figure 11. Two-phase medium temperature at the time points of 1,000, 10,000, 40,000 (Re = 188, inclination angle 25°)

3.3. Calculations for a channel with a big inclination angle. The volumetric fraction for the dispersed phase and the temperature for different times for the case of two-phase pressure flown a channel with the inclination angle of 25° for low value of the Reynolds number are shown in Figures 10, 11.

Conclusion

Setting different values for the phase velocities at the inlet boundary does not lead to a change in the qualitative flow pattern. For a difference in the boundary values of the velocities for both phases (even by a factor of three), we observe only a minor quantitative difference in the nature of flow.

References

[1] Perepechko Y.V., Sorokin K.E. Two-velocity dynamics of compressible heterophase media // J. Eng. Thermophys. — 2013. — Vol. 22, No. 3. — P. 241–246.