Bull. Nov. Comp. Center, Math. Model. in Geoph., 27 (2025), 1–5

Traveling SH waves in a fluid-saturated porous medium*

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Abstract. New solutions to wave equations with variable coefficients describing traveling SH waves in a porous medium, taking energy dissipation into account, are obtained. These solutions are obtained using transformation methods, where equations with variable coefficients can be reduced to equations with constant coefficients.

Introduction

In oil field development, the question of reservoir parameter changes during well operation is of particular interest. For this purpose, reservoir filtration parameters (RPP) studies were conducted at the Romashkinskoye field site in September 1980 and July 1981 using the automated reservoir production monitoring system. An experiment to determine the dependence of RP on reservoir pressure was also conducted at this site. The RPP studies, using the filtration pressure wave (FPW) method at a reservoir pressure of 155 ppm atm, revealed slight RPP heterogeneity across various wellbore intervals. Harmonic Fourier analysis of the experimental results revealed significant phase shift changes in two of the three studied directions. The increase in piezoconductivity at the interwell interval 4379a–9793 was 45 %, and at the interval of wells 4379a–9794 — 26 % [1]. As noted in [2], a feature of fluid motion in fractured-porous media is the exchange of fluid between blocks and fractures. Non-stationary processes in fractured-porous media after a certain time, which is defined as the delay time, proceed in the same way as in a homogeneous porous medium. With regard to the FVD method, it can be concluded that the greater the period of oscillations created in the formation compared to the delay time, the more accurately the fractured-porous formation is described by the model of a homogeneous porous formation.

In the book [3] an important experimental fact is noted that if a sound vibration receiver is lowered into the borehole of a well filled with water and the energy spectrum of the noise is measured, then a resonant frequency can be identified at the level of the fluid-saturated formation.

Non-stationary processes are described by the equations of filtration theory in fractured-porous media. As noted in [4,5] and other works, due to the relatively high permeability of the interblock space, Darcy's approximations

*Supported by Project FWNM-2025-0004 of the SB RAS state assignment.

for describing non-stationary processes may, in principle, be invalid. The law of conservation of mass and the equations of fluid motion in the case where the velocity of an elastic porous body $\boldsymbol{u}f = 0$ and the stress tensor $h_{ik} = 0$ are described by the following system of differential equations [6]

$$\frac{\partial \rho_l}{\partial t} = \operatorname{div}(\rho_l \boldsymbol{v}) = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \chi \rho_l \boldsymbol{v} = \frac{1}{\rho} \nabla p. \tag{2}$$

In formulas (1) and (2), $\boldsymbol{v} = \boldsymbol{v}(t, \boldsymbol{x})$ is the filtration rate of liquid in an elastic porous medium with partial density $\rho_l = \rho_l(t, \boldsymbol{x})$, $\rho = \rho_l(t, \boldsymbol{x}) + \rho_s(t, \boldsymbol{x})$, $\rho_s(t, \boldsymbol{x})$ and $p = p(t, \boldsymbol{x})$ are the partial density of the matrix and pressure, respectively, $\chi = \chi(t, \boldsymbol{x})$ is the coefficient of friction, t is time, $\boldsymbol{x} = (x, y, z)$ is the point from R^3.

As noted in [6], when considering problems of isothermal non-stationary rectilinear-parallel filtration of a homogeneous liquid in an isotropic porous medium, taking into account the finite speed of propagation of disturbances, the filtration law is usually used

$$w + \tau \frac{\partial w}{\partial t} = -\frac{k}{\mu_l} \frac{\partial p}{\partial x}. \tag{3}$$

Here τ is the time constant, k is the permeability coefficient, and μ_l is the viscosity of the liquid.

Comparing the first equation (2) with equation (3), we obtain an expression for determining the constant τ through the friction coefficient χ_0 and the partial density $\rho_{0,l}$ of the liquid in the Dorovsky model

$$\tau = \frac{1}{\chi_0 \rho_{0,l}}.$$

The paper studies the one-dimensional direct problem of seismic wave propagation in a porous medium.

1. Statement of the problem

Let us consider the propagation of seismic waves in a heterogeneous porous medium, taking into account the energy loss due to the friction coefficient $\chi(x)$:

$$\rho_s(x) \frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}\Big(\mu(x) \frac{\partial u}{\partial x}\Big) - \rho_l^2(x)\, \chi(x) \Big(\frac{\partial u}{\partial t} - \frac{\partial v}{\partial t}\Big) = 0, \tag{4}$$

$$\frac{\partial v}{\partial t} + \rho_l(x) \chi(x)(u - v) = 0, \tag{5}$$

where $\mu(x)$ is the shear modulus.

To find the solution to this system in the form of a traveling wave, we will use a transformation (mapping) technique that reduces the system of equations (4), (5) with variable coefficients to a system of equations with constant coefficients [7–9]. To do this, we make the following substitution into the system of equations (4), (5):

$$u(t,x) = A(x)U(t,\xi(x)), \quad v(t,x) = A(x)V(t,\xi(x)),$$

where $A(x)$, $U(t,\xi(x))$, $V(t,\xi(x))$, and $\xi(x)$ are four unknown functions to be determined. Then this system of equations (4), (5) is transformed into a system of equations with variable coefficients

$$\rho_s(x)\left[\frac{\partial^2 U}{\partial t^2} - \mu(x)\Big(\frac{d\xi(x)}{dx}\Big)^2\frac{\partial^2 U}{\partial x^2}\right] +$$

$$(\mu(x)A'(x))' - A(x)\rho_l^2(x)\chi(x)\Big(\frac{\partial U}{\partial t} - \frac{\partial V}{\partial t}\Big) = 0, \tag{6}$$

$$A(x)\frac{\partial V}{\partial t} + A(x)\rho_l(x)\chi(x)(U - V) = 0, \tag{7}$$

Since system (6), (7) contains three unknown functions, three conditions can be imposed to determine them uniquely. In [10], the following choice of these conditions in the form of three equations was proposed:

$$c_t^2(x)\Big(\frac{d\xi(x)}{dx}\Big)^2 = 1, \tag{8}$$

$$\frac{dc_t(x)}{dx} + 2\frac{c_t}{A}\frac{dA(x)}{dx} + \frac{c_t}{\rho_s}\frac{d\rho_s}{dx} = 0, \tag{9}$$

$$\frac{1}{\rho_s A}\frac{d}{dx}\Big(\rho_s c_t^2(x)\frac{dA(x)}{dx}\Big) = P, \tag{10}$$

where P is an arbitrary constant, $c_t(x)$ is the shear wave velocity. When conditions (8)–(10) are satisfied, the system of equations (6), (7) is reduced to a system of equations of the form with constant coefficients at the highest derivatives

$$\frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 U}{\partial x^2} - PU - \omega(x)\varepsilon(x)\Big(\frac{\partial U}{\partial t} - \frac{\partial V}{\partial t}\Big) = 0, \tag{11}$$

$$\frac{\partial V}{\partial t} + \omega(x)(U - V) = 0, \tag{12}$$

where $\omega(x) = \rho_l(x)\chi(x)$, $\varepsilon(x) = \frac{\rho_l(x)}{\rho_s(x)}$.

The system of equations (11), (12) implies the existence of evanescent traveling waves (in particular, monochromatic traveling waves). If the wave is not monochromatic, we can use the Fourier or the Laplace transform to analyze the waveform. However, waves traveling in opposite directions do not interact with each other.

Equation (8) is called the eikonal equation [11, 12] and defines the transition to the phase of the wave (for definiteness, a wave propagating to the right is taken) or the time of passage

$$\xi = \int_{x_0}^{x} \frac{dy}{c_t(y)}.$$

Equation (9) defines the relationship between the wave amplitude and the acoustic stiffness of the porous medium ($\rho_s(x)c_t(x)$):

$$A(x) = \frac{\text{const}}{\sqrt{\rho_s(x)c_t(x)}}$$

and this formula coincides with the famous Green's law (for liquid media in the case of constant density), known from the law of conservation of energy flow in media with slowly changing parameters [13, 14]. Note that here we do not impose a condition on the slow change of the medium with depth.

Conclusion

The problem of the existence of traveling waves in inhomogeneous porous media is important for explaining wave propagation over long distances. New solutions to wave equations with variable coefficients describing traveling SH waves in a porous medium, taking energy dissipation into account, are considered. These solutions are obtained using transformation methods, where equations with variable coefficients can be reduced to an equation with constant coefficients.

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