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Numerical modeling of the filtration theory problems for saturated porous media*

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Abstract. In the given paper, a mathematical model for layered porous media is constructed. A series of numerical experiments is carried out. There is observed a weak dependence of the filtration velocity on the physical properties of a medium as compared to the boundary pressure value.

Introduction

When developing the oil deposits and in the oil borehole operation, of particular interest are filtration processes in a layer.

The given work is dealt with a numerical solution of the filtration problems theory. Numerically, the equation of filtration is solved for layered porous media with the use of the Buleev method of incomplete factorization.

1. Statement of problem

Let there be a rectangular collector $[0, H_x] \times [0, H_y]$ (m^2) in a petroliferous layer, in which an excess of pressures, under the action of which, starts the process of filtration of a fluid from the collector in the borehole. Thus, the oil collector is considered to consist of several layers.

Stationary processes are described by the equations of the filtration theory in porous media. Let us suppose that the conservation law of mass and the Darcy law are valid in the case of vertical inhomogeneous layered porous media. Pressures and velocity of a fluid in the layer β satisfy the following system of differential equations [1, 2]:

$$\operatorname{div}(\rho_{l_\beta}\boldsymbol{v}) = 0, \qquad \rho_{l_\beta}\boldsymbol{v} = -\frac{1}{\chi_\beta\rho_\beta}\nabla p. \tag{1}$$

Here $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{x})$ is the filtration velocity of a fluid (m/s), $p = p(\boldsymbol{x})$ is pressure (N/m^2), ρ_β is density of a porous medium ($\rho_\beta = \rho_{l_\beta} + \rho_{s_\beta}$), ρ_{l_β} is a partial density of the fluid ($\rho_{l_\beta} = \rho^f_{l_\beta} d_{0_\beta}$), $\rho^f_{l_\beta}$ is the physical density of the fluid (kg/m^3), ρ_{s_β} is a partial density of an elastic porous body

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$(\rho_{s_\beta} = \rho^f_{s_\beta}(1 - d_{0_\beta}))$, $\rho^f_{s_\beta}$ is the physical density of the elastic porous body (kg/m^3), d_{0_β} is porosity, χ_β is an intercomponent friction coefficient (m^3/(s $\cdot$ kg)), $\boldsymbol{x} = (x, y)$ is a point from R^2.

Let us set the boundary conditions for each layer β and the conjugation requirements between the layers. On the left, the condition of constant pressure p_{0_β} is set as follows:

$$p|_{x=0} = p_{0_\beta}, \tag{2}$$

while on the right, the zero normal velocity condition is set:

$$\rho_{l_\beta} \boldsymbol{v}_x|_{x=H_x} = 0. \tag{3}$$

In the upper from above layer and in a sublayer from below, the zero normal velocity conditions are assigned:

$$\rho_{l_0} \boldsymbol{v}_y|_{y=0} = 0, \qquad \rho_{l_N} \boldsymbol{v}_y|_{y=H_y} = 0. \tag{4}$$

Between the layers, the conjugation condition satisfies the following:

$$\rho_{l_\beta} \boldsymbol{v}|_{y \to H_{y_\beta}+0} = \rho_{l_{\beta+1}} \boldsymbol{v}|_{y \to H_{y_\beta}-0}. \tag{5}$$

2. Numerical method of solution

For solving the system of equations (1)–(5) and constructing a difference scheme, we use the Buleev method of incomplete factorization [3] and the integro-interpolation method [4]:

$$k_{i+1/2,j}(u_{i+1,j} - u_{i,j}) - k_{i-1/2,j}(u_{i,j} - u_{i-1,j}) + \\ k_{i,j+1/2}(u_{i,j+1} - u_{i,j}) - k_{i,j-1/2}(u_{i,j} - u_{i,j-1}) = 0,$$

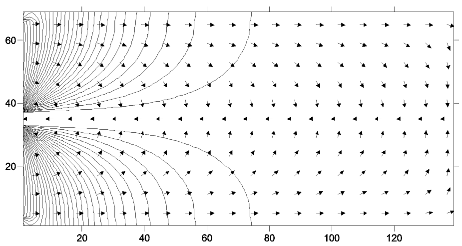
$$k_{i+1/2,j} = -\frac{1}{h_y} \int_{y_{j-1/2}}^{y_{j+1/2}} \left(\frac{1}{h_x} \int_{x_i}^{x_{i+1}} \chi_\beta \rho_\beta \, dx \right)^{-1} dy,$$

$$k_{i,j+1/2} = -\frac{1}{h_x} \int_{x_{i-1/2}}^{x_{i+1/2}} \left(\frac{1}{h_y} \int_{y_j}^{y_{j+1}} \chi_\beta \rho_\beta \, dy \right)^{-1} dx,$$

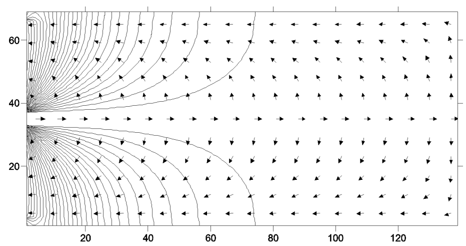
where $i = 1, \ldots, N_x$, $j = 1, \ldots, N_y$, $h_x = \frac{H_x}{N_x}$, $h_y = \frac{H_y}{N_y}$.

3. Numerical results

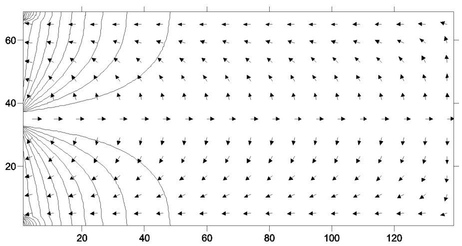
In the given paper, numerical modeling of a solution of the forward problem is carried out for the given parameters of media. The difference domain has the following dimensions: 71×141 space with nodes ($H_x = 200$, $H_y = 100$), divided into five layers by $\{5, 30, 5, 30, 5\}$. In the figure, the distribution of pore pressure and the field of fluid velocities are shown.



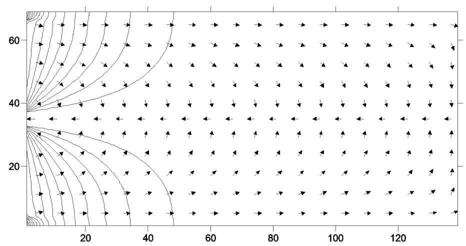
$p|_{x=0} = \{0, 1, -2, 1, 0\}$



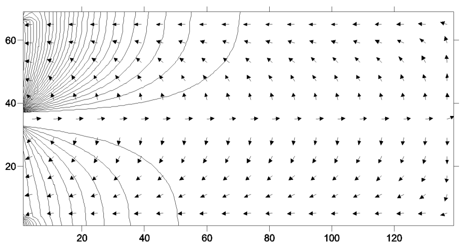
$p|_{x=0} = \{0, -1, 2, -1, 0\}$



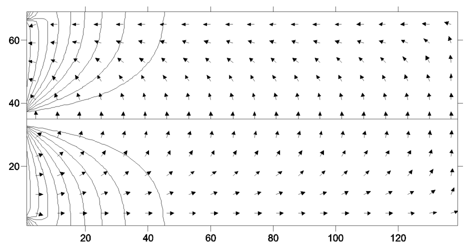
$p|_{x=0} = \{0, 1, 2, 1, 0\}$



$p|_{x=0} = \{0, -1, -2, -1, 0\}$



$p|_{x=0} = \{0, 1, 2, -1, 0\}$



$p|_{x=0} = \{0, 1, 0, -1, 0\}$

For all numerical calculations, the following parameters are taken:

$$\rho_s^f = \{2.7, 2.7, 2.7, 2.7, 2.7\}, \qquad d_0 = \{0.01, 0.1, 0.2, 0.1, 0.01\},$$
$$\rho_l^f = \{0.9, 0.9, 0.9, 0.9, 0.9\}, \qquad \chi = \{100.0, 10.0, 0.01, 10.0, 100.0\}.$$

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