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Numerical solution to the inverse problem for a system of elasticity for vertically inhomogeneous medium*

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In the paper, the results of numerical solution to the inverse problem for a system of elasticity for vertically inhomogeneous medium are presented. The center of compression excites the elastic oscillations in medium. This mathematical model of source does not give a decomposition of the elastic system and we need to solve numerically the inverse problem for all system. It is necessary to find the longitudinal velocity v_p and the transverse velocity v_s in each layer if longitudinal and transverse displacements are known on the surface. The results of reconstructions of velocities for medium typical for Western Siberia are presented. It is shown that satisfactory reconstruction is possible if the inverse problem data with error in 5–40%.

1. Introduction

At present, there are a lot of works devoted to one-dimensional and multidimensional inverse problems for system of elasticity. We must note the basic works of A.S. Alekseev [1], A.S. Blagovestchenskii [2], V.G. Romanov [3], and V.G. Yakhno [4]. For one-dimensional inverse problem for the Lame system of elasticity there are a lot of investigations (see, for example, references in the works noted above). A characteristic feature of all those works is the use of combined sources such as combinations of vertical impact with instantaneous center of rotation [1], or sloping impacts [2, 3, 4]. Such a choice of sources gives a possibility to decompose systems of elasticity and hence original inverse problem can be reduced to a sequence of more simple inverse problems for scalar hyperbolic equations.

Applied geophysics takes an interest numerically to solve the one-dimensional inverse problem of elasticity with a source which can be described as

$$\pi\nabla_{x,y,z}\delta(x,y,z-z_*)f(t), \qquad f(t)=\rho A\,\mathrm{e}^{-bt}\cos(\omega_0 t+\psi). \tag{1}$$

This source is interpreted as a center of compression which is a model of explosion. Here ρ is the density of medium in which the explosion takes place, A, b, ω_0, ψ are the values characterizing the explosion, z_* is the coordinate (depth) of the explosion ($z_* \neq 0$). This mathematical model of

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source does not give a decomposition of the elastic system, and we need to investigate and to solve numerically the inverse problem for all system.

2. Statement of inverse problem

Let us have the medium as n-layered structure with points of the boundaries z_k, $k = \overline{0,n}$, $z_0 = 0$; the layer with the number of m is found in $z_{m-1} < z < z_m$, the last (underlying) layer with the number $n+1$ in $z_n < z < \infty$. We assume that the density ρ is the known constant. Each layer is characterized by the longitudinal velocity v_p and the transverse velocity v_s, i.e., $v_p(z)$ and $v_s(z)$ are step functions of the variable z, $0 < z < \infty$. Source (1) excites the elastic oscillations in medium. The source is found in one of layers, i.e, $z_* \neq z_k$, $k = \overline{1,n}$. Applying to the system of elasticity in cylindrical system of coordinates the Fourier–Bessel transform with respect to r and the Laplace transform with respect to t we obtain the following system:

$$\frac{\partial}{\partial z}\left(M\frac{\partial}{\partial z}U + \nu NU\right) - \nu N^T\frac{\partial}{\partial z}U - KU = F, \tag{2}$$

$$U = \begin{bmatrix} u \\ v \end{bmatrix}, \qquad F = \begin{bmatrix} \nu A\delta(z-z_*) \\ -A\delta'(z-z_*) \end{bmatrix}, \qquad M = \begin{bmatrix} v_s^2 & 0 \\ 0 & v_p^2 \end{bmatrix},$$

$$N = \begin{bmatrix} 0 & -v_s^2 \\ v_p^2 - 2v_s^2 & 0 \end{bmatrix}, \qquad K = \begin{bmatrix} p^2 + \nu^2 v_p^2 & 0 \\ 0 & p^2 + \nu^2 v_s^2 \end{bmatrix}.$$

Here ν is a parameter of the Fourier–Bessel transform, $p = -\alpha + i\omega$ is a parameter of the Laplace transform, the superscript T denotes that matrix is transposed. The boundary conditions are as follows:

$$\left(M\frac{\partial}{\partial z}U + \nu NU\right)\bigg|_{z=0} = \mathbf{0}, \qquad U \to \mathbf{0} \quad (z \to \infty). \tag{3}$$

The conditions on the gaps are as follows:

$$\left[M\frac{\partial}{\partial z}U + \nu NU\right]_{z_k} = \mathbf{0}, \quad [U]_{z_k} = \mathbf{0}, \qquad k = \overline{1,n}. \tag{4}$$

Here we use the notation $[f]_x = f(x+0) - f(x-0)$ for a value of gap of a function f in the point x.

We assume that the longitudinal velocity and the transverse velocity are known in the first and the last layers. We assume also that coordinates of the gaps z_k, $k = \overline{0,n}$, are known too.

Inverse problem. *Find the longitudinal velocity v_p and the transverse velocity v_s in each layer if we know the following additional information on solution of the direct problem* (2)–(4)

$$U(0,\nu,p) = U_0(\nu,p), \qquad U_0 = \begin{bmatrix} u_0 \\ w_0 \end{bmatrix}. \tag{5}$$

where the $U_0(\nu,p)$ is the given vector-function.

3. Numerical solution to inverse problem

To solve the direct problem (2)–(4) we use the ideas from works [5–10]. In [9, 10], A.G. Fat'yanov developed the basic principles for solution of elastic system.

To numerically solve the inverse problems of seismic a minimization of a cost functional is widely used in practice. We can note works [11–22], and it is by no means full list. Excepting [12], in most cases the inverse problems were considered for one equation.

To solve inverse problem (2)–(5) we minimize the cost functional

$$J[v_s,v_p] = \sum_{\nu}\sum_{\omega} h_\nu h_\omega \Big(|u(0,\nu,p)-u_0(\nu,p)|^2 + |w(0,\nu,p)-w_0(\nu,p)|^2\Big). \tag{6}$$

Here the summation is conducted with respect to ν and ω which belong to certain discrete sets, the values h_ν and h_ω are normalization constants depending on the number of the frequencies ν, ω and its frequencies intervals.

It is necessary to note that inverse problem (2)–(5) is nonlinear. This course affects on the behavior of functional (6). The values of α and ν, frequencies intervals, numbers of time and space frequencies N_ω and N_ν, the discreteness of sets of frequencies, the combination of these parameters affect on the behavior of the cost functional too.

We made series of calculations for the medium typical for Western Siberia.

Velocities (km/s) and gaps coordinates (m)

n	Velocity		z_k	n	Velocity		z_k	n	Velocity		z_k
	v_p	v_s			v_p	v_s			v_p	v_s	
1	2.3	1.5	1500	8	4.2	2.3	1555	15	3.3	2.0	1607
2	3.2	1.6	1507	9	4.3	2.4	1562	16	3.6	1.9	1615
3	3.4	1.7	1514	10	4.1	2.5	1570	17	3.5	1.8	1622
4	3.3	1.8	1525	11	4.0	2.4	1577	18	3.4	2.0	1630
5	3.1	1.6	1533	12	3.9	2.3	1586	19	3.5	2.1	1640
6	3.2	1.7	1540	13	4.0	2.3	1593	20	2.9	2.1	1658
7	3.1	1.8	1548	14	4.1	2.2	1600	21	4.0	2.5	

Parameters of source: $z_* = 20$ m, $A = 10^9$ is a normalization constant, $b = 100$, $\omega_0 = 2\pi \cdot 30$ Hz, $\psi = 0$.

We made several steps to minimize the cost functional (6):

- at the beginning we put $\nu = 0$;
  - to reconstruct the longitudinal velocity we choose initial approach $v_p = \text{const}$ in all layers;
  - we choose the interval of time frequencies 10–25 Hz;
  - we minimize the functional, if the value of functional or the norm of functional gradient is less than ε_1, then the minimization process is stopped;
  - we choose the interval of time frequencies 10–85 Hz;
  - using the obtained approach as initial one we minimize the functional again, if the value of functional or the norm of functional gradient is less than ε_2, then the minimization process is stopped;
- second, we put $\nu = 10^{-2}$;
  - we choose the interval of time frequencies 5–65 Hz;
  - to reconstruct the transverse velocity we choose initial approach $v_s = c_1 v_p + c_2$ (c_1 and c_2 are correlative constant typical for this region), here we use the longitudinal velocity reconstructed;
  - we minimize the functional, if the value of functional or the norm of functional gradient is less than ε_3, then the minimization process is stopped.

To solve the inverse problem the additional information (5) was computed with the help of the solution to direct problem, and after that the random error was introduced in this data. In Figures 1–5, we show the results of reconstruction of the longitudinal velocity v_p and the transverse velocity v_s introducing errors in 5, 10, 20, 30, and 40% respectively. Solid line denotes the exact solution, and dashed line denotes the reconstructed one.

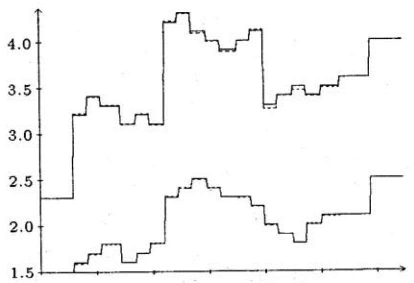

Figure 1

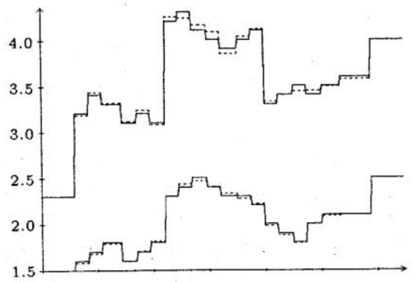

Figure 2

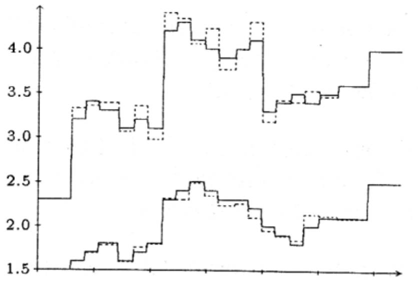

Figure 3

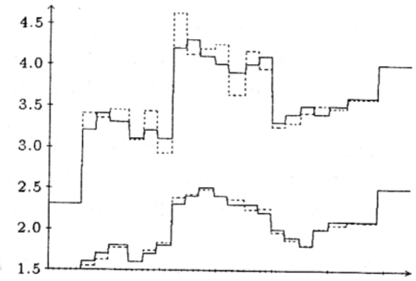

Figure 4

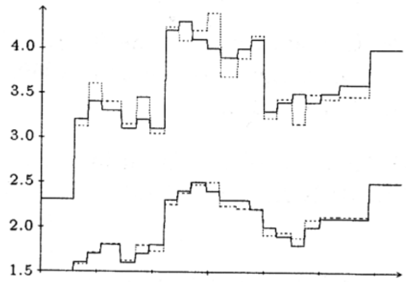

Figure 5

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