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Solutions to the inverse coefficient problem for the system of poroelasticity equations based on neural networks*

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Abstract. This paper examines the use of neural networks to solve an inverse coefficient problem for a system of poroelastic equations. The problem consists of finding the desired functions of the system along with an unknown coefficient of the equation under an additional boundary condition. In this paper, the unknowns are the shear modulus coefficient and the interfacial friction coefficient. This approach assumes the possibility of setting problems for various coefficients and parameters of the environment involved in the system of equations under consideration or their combination.

1. Statement of direct and inverse problems

A direct initial boundary value problem for a system of poroelasticity equations with homogeneous boundary conditions is formulated as follows [1–3]:

$$\rho_s u_{tt} = (\mu(u_x)u_x)_x - \rho_l^2((u-v)\chi(u-v))_t, \quad x \in (0,L), \ t \in (0,T), \tag{1}$$

$$\rho_l v_t = \rho_l^2 (u-v)\chi(u-v), \quad x \in (0,L), \ t \in (0,T), \tag{2}$$

$$u|_{t=0} = \phi_0(x), \quad u_t|_{t=0} = \phi_1(x), \quad x \in (0,L), \tag{3}$$

$$v|_{t=0} = 0, \quad x \in (0,L), \tag{4}$$

$$u|_{x=0} = 0, \quad t \in (0,T). \tag{5}$$

The problem of finding the functions u and v from the system of equations (1)–(5) will be called the direct problem.

Let the coefficient $\mu(u_x)$ be unknown and additional information on the boundary $\tilde{L}$ be given:

$$u(\tilde{L}, t) = \tilde{u}(t). \tag{6}$$

Then the problem of finding the coefficient $\mu(u_x)$ and the functions u and v from the system of equations (1)–(5) and the additional condition (6) is called the inverse problem.

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2. Description of the method for solving the inverse coefficient problem of proroelasticity

To solve the inverse problem (1)–(5) with additional condition (6), it is proposed to use a neural network [4] trained on a sufficiently large data set consisting of a set of approximations of the problem coefficients and additional boundary conditions.

This article considers inverse problems separately for cases of constant coefficients and coefficients depending on their arguments.

We will consider two approaches to the formation of input and search data for the algorithm.

The first approach concerns the case of constant coefficients $\mu(u_x) =$ const; $\chi(u - v) =$ const. In this case, the training data is a set of coefficients $\{\mu_i\}$, $\{\chi_i\}$ from the ranges $[\mu_{\min}, \mu_{\max}]$, $[\chi_{\min}, \chi_{\max}]$ and a set of corresponding additional boundary conditions $u_i(\tilde{L}, t)$, obtained by solving the direct problem with coefficients μ_i, χ_i. In a similar manner, test data sets are formed from the same ranges (for example, μ_j, χ_j are randomly selected from $[\mu_{\min}, \mu_{\max}]$, $[\chi_{\min}, \chi_{\max}]$, and additional conditions $u_j(\tilde{L}, t)$ are calculated for them.

In the second approach, the coefficients μ, χ have a given dependency on their arguments with some constant values μ_c, χ_c. The choice of training and test data in this case is similar to the first approach, only instead of the coefficients μ_i, χ_i, we choose their constant components μ_{c_i}, χ_{c_i}.

For training and test data, it is necessary to enter sets of coefficients $\{\mu_i\}$, $\{\chi_i\}$, where each element μ_i and χ_i are vectors of dimension $N \times M$ or less, if we restrict ourselves to the data in the set of control points.

3. Solution to the inverse problem for constant coefficients

Let us consider the inverse problem for the system of equations (1)–(6) in the case of constant coefficients. It is necessary to find the coefficients μ and χ using the additional condition (6).

To solve this problem, a neural network with an LSTM (Long Short-Term Memory) architecture was chosen. This neural network uses additional hidden layers [5]. The use of a neural network to solve this problem consists of two parts: training the neural network and applying the obtained network coefficients to the input parameters of the problem being solved. The input parameters in this case are the sets of coefficients μ, χ and the additional conditions $\tilde{u}(t)$. First, we will collect a training database for our neural network. To do this, we form P sets of coefficients μ, χ in a uniform grid with a step of $0.1 \cdot 10^6$ from the ranges $[2 \cdot 10^9, 3 \cdot 10^9]$ for μ and $[10^{-2}, 2 \cdot 10^{-2}]$ for χ. We calculate additional conditions on the boundary $\tilde{L}$ by solving the direct problem (1)–(5) for each set of coefficients.

As test data, we use a random set of coefficients μ and χ from the same ranges. We also calculate additional conditions on the boundary $\tilde{L}$ for the selected coefficients.

The result of the neural network operation is demonstrated in Figure 1. The figure shows a comparison of the test coefficients μ and χ with the coefficients reconstructed using additional conditions from the test data.

We can conclude that the neural network successfully reconstructed the coefficients using additional boundary conditions for constant coefficients. The accuracy of this reconstruction can be increased or decreased by changing the calculation parameters and the neural network used.

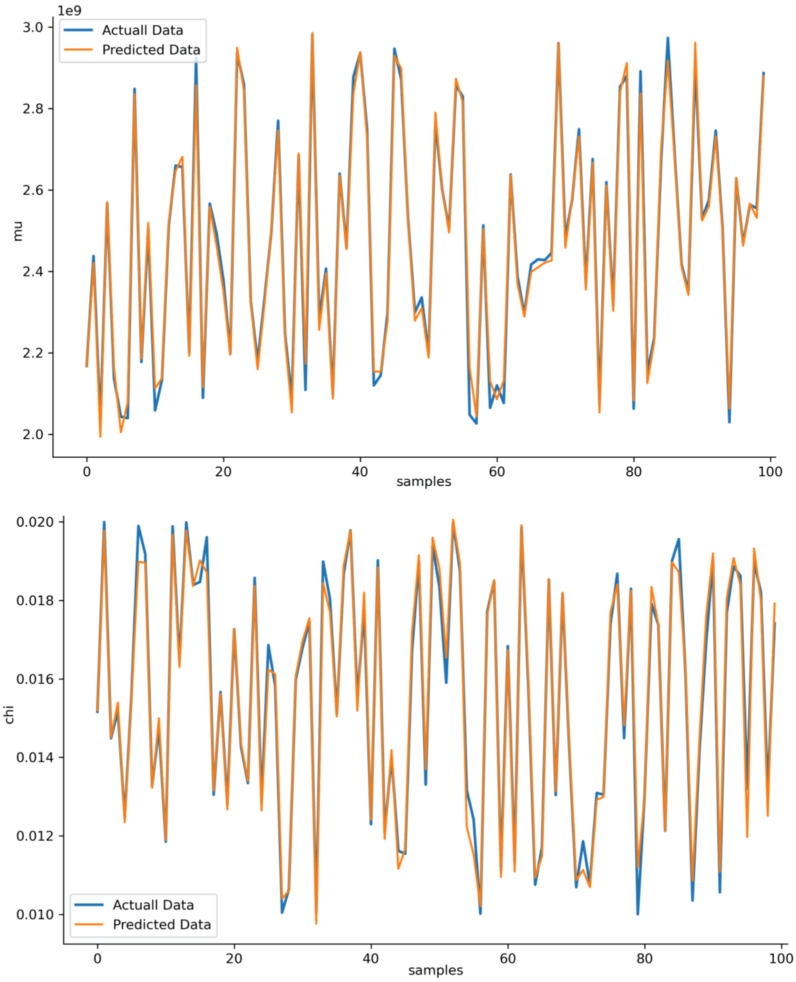

Figure 1. Comparison of test and reconstructed coefficients for a problem with constant coefficients

4. Solution to the inverse problem for variable coefficients with a given dependence

In this case we will consider problem (1)–(5) with coefficients μ, χ having the following dependency on their arguments

$$\mu(u_x) = \mu_c\big(1 + e^{-(u_x)^2}\big), \quad \chi(u - v) = \chi_c\big(1 + e^{-(u-v)^2}\big),$$

where $\mu_c = \text{const}$, $\chi_c = \text{const}$.

We use the same approach as for the case with constant coefficients, but as training and test data we will take sets of coefficients μ_c, χ_c from the same ranges and the additional conditions on the boundary $\tilde{L}$.

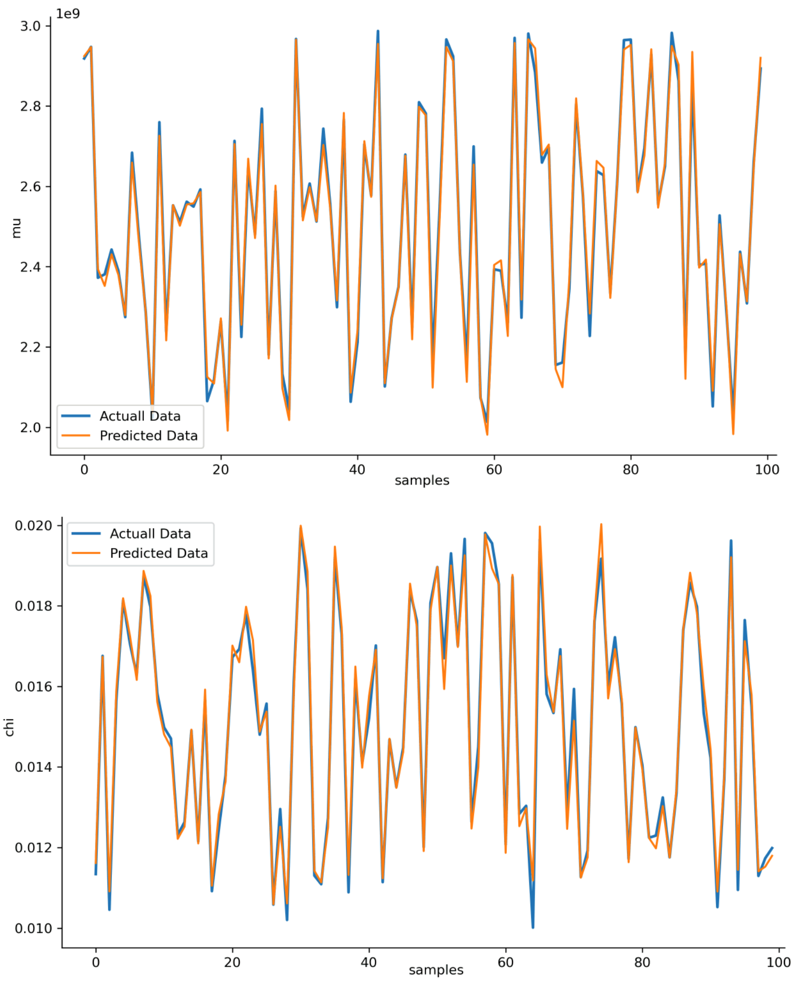

Figure 2. Comparison of test and reconstructed coefficients for a problem with a known dependency of the coefficients

The result of the restoration the coefficients μ_c , χ_c according to additional conditions from the test data set is shown in Figure 2.

Based on the results obtained, we can conclude that, using the described technology, it is possible to reconstruct the solution to problem (1)–(5) under the additional condition (6) within the ranges for the coefficients μ_c, χ_c in which the training data were specified. The accuracy of the reconstructed solution, with this approach, will depend only on the detail and quality of the training data.

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