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Modeling of the acoustoseismic induction process

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The paper is devoted to modeling of the acoustoseismic induction process caused by the interaction of a harmonic wave with elastic half-space. This effect was first detected in experiments with powerful vibrational sources of seismic waves, carried out at the vibroseismic test site of Siberian Branch of RAS. The 2D problem of the elastic surface wave propagation with good allowance for the influence of an acoustic wave as boundary conditions is considered. The amplitude and the polarization characteristics of the induced surface wave and their dependence on elastic constants of the half-space are defined.

The surface seismic waves, induced by the acoustic radiation from a vibrator, were detected at distances of 20 and 50 km in experiments with the low-frequency vibrational sources, carried out in the ICMMG SB RAS [1, 2].

The effect of excitation of the surface elastic waves by an acoustic impulse from surface explosions are well-known [3], and the results of the mathematical modeling of this process are presented in [4].

The effect of the acoustoseismic induction at the distance of several tens kilometers from powerful seismic vibrators was observed for the first time, therefore the mathematical modeling of the process of excitation of the surface waves by the harmonic acoustic wave, propagating along the free surface, is necessary to obtain the quantitative characteristics of the induction process [5].

A 2D problem for the homogeneous ideally elastic half-space $z > 0$, with the Lame constants λ, μ and the density ρ is considered.

The acoustic wave, propagating along the boundary on the axis x, is taken into account in the form of boundary conditions for normal stresses on the free surface of the elastic half-space ($z = 0$).

A harmonic acoustic wave has a constant velocity v_0 equal to the velocity of sound in the air, the amplitude of pressure p and the frequency ω.

The Lame equations with the boundary conditions are solved:

$$(\lambda + \mu)\,\mathrm{grad\,div}, \boldsymbol{u} + \mu \Delta \boldsymbol{u} - \rho \ddot{\boldsymbol{u}} = 0, \tag{1}$$

$$t_{xz}|_{z=0} = 0, \qquad t_{zz}|_{z=0} = p \exp i(\omega t - kx), \tag{2}$$

where $k = \omega / v_0$ is the wave number of the acoustic wave.

Introducing the potentials $\varphi(x, z, t)$ and $\psi(x, z, t)$, connected with the field of displacements by the formulas

$$u_x = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z}, \qquad u_z = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x},$$

and assuming their dependence on x, z, t as

$$\varphi = A\exp i(\omega t - kx - k_{\varphi z}z), \qquad \psi = B\exp i(\omega t - kx - k_{\psi z}z), \tag{3}$$

after substitution in (1) we define the wave numbers

$$k_{\varphi z} = k\left(\frac{v_0^2}{v_p^2} - 1\right)^{1/2}, \qquad k_{\psi z} = k\left(\frac{v_0^2}{v_s^2} - 1\right)^{1/2}.$$

Substituting φ and ψ from (3) in boundary conditions (2), we obtain an inhomogeneous set of equations for the factors A and B:

$$\begin{aligned} t_{xz}|_{z=0} &= \mu\left[2\frac{\partial^2\varphi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right]_{z=0} = 0, \\ t_{zz}|_{z=0} &= \left[\lambda\frac{\partial^2\varphi}{\partial x^2} + (\lambda+2\mu)\frac{\partial^2\varphi}{\partial z^2} + 2\mu\frac{\partial^2\psi}{\partial x\partial z}\right]_{z=0} = p\exp i(\omega t - kx); \end{aligned} \tag{4}$$

$$\begin{aligned} 2i\left(1 - \frac{v_0^2}{v_p^2}\right)^{1/2} A - \left(2 - \frac{v_0^2}{v_s^2}\right) B = 0, \\ \left(2 - \frac{v_0^2}{v_s^2}\right) A + 2i\left(1 - \frac{v_0^2}{v_s^2}\right)^{1/2} B = \frac{p}{k^2\rho v_s^2}. \end{aligned} \tag{5}$$

The determinant of system (5) is the Rayleigh function [6]

$$R = \left(2 - \frac{v_0^2}{v_s^2}\right)^2 - 4\left[\left(1 - \frac{v_0^2}{v_p^2}\right)\left(1 - \frac{v_0^2}{v_s^2}\right)\right]^{1/2}.$$

The values of the factors A, B and C are the following:

$$A = C\left(2 - \frac{v_0^2}{v_s^2}\right), \qquad B = 2iC\left(1 - \frac{v_0^2}{v_p^2}\right)^{1/2}, \qquad C = \frac{p}{k^2\rho v_s^2 R}.$$

For components of the field of displacements u_x and u_z, introducing the notation $\gamma = v_s/v_p$, $\theta = v_0/v_s$, we arrive at:

$$\begin{aligned} u_x &= -ikC\left[(2-\theta^2)e^{-kz\sqrt{1-\gamma^2\theta^2}} - 2\sqrt{(1-\theta^2)(1-\gamma^2\theta^2)}e^{-kz\sqrt{1-\theta^2}}\right]e^{i(\omega t - kx)}, \\ u_z &= kC\sqrt{1-\gamma^2\theta^2}\left[(\theta^2-2)e^{-kz\sqrt{1-\gamma^2\theta^2}} + 2e^{-kz\sqrt{1-\theta^2}}\right]e^{i(\omega t - kx)}, \end{aligned} \tag{6}$$

where

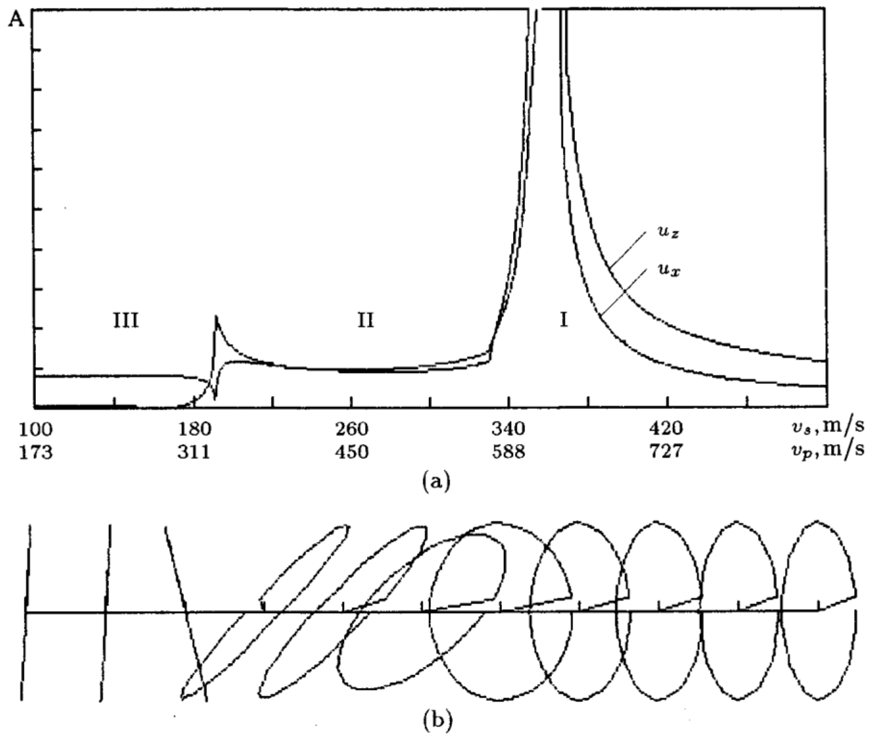
$$C = \frac{p}{k^2\rho v_s^2 R(\theta)}, \qquad R(\theta) = (2-\theta^2) - 4\sqrt{(1-\theta^2)(1-\gamma^2\theta^2)}.$$

The field of displacements is determined by the real parts of formulas (6). Their concrete form depends on the signs of radicands, included in these formulas.

It is possible to distinguish three areas of values of parameters of the elastic medium, that define various forms of solution (6) and, correspondingly, various character of the process of interaction of the acoustic wave with the elastic half-space. Graphs of the amplitudes of the field of displacements on the free surface of the half-space and the respective polarization curves are shown in the figure.

Area I. $0 < v_0 < v_s$, $\theta < 1$. The acoustic wave propagation above the elastic half-space with the velocities of longitudinal and transverse waves, that are higher than the velocity of sound in the air.

Solution (6) shows that in this case the surface wave, propagating with the velocity of acoustic wave, is induced in the elastic half-space. Amplitudes of the field of displacements exponentially decrease at $z > 0$, and there is no energy flux in the direction of the axis $z > 0$. The induced surface wave has an ellipsoidal polarization, the relation of the axes of the ellipse depending on values of the parameters v_s and v_p.



Amplitudes of the displacemrnt on the free surface (a) and the polarization (b)

On the free surface of the half-space ($z = 0$) the displacements have the form

$$u_x|_{z=0} = Ck\Big[2 - \theta^2 - 2\sqrt{(1-\gamma^2\theta^2)(1-\theta^2)}\Big]\sin(\omega t - kx),$$
$$u_z|_{z=0} = Ck\theta^2\sqrt{1-\gamma_2\theta^2}\cos(\omega t - kx).$$

In this area of parameters there are such values v_p and v_s at which the velocity of sound coincides with the velocity of the surface Rayleigh wave.

Solution (6) has singularity at this point, because the Rayleigh function, included in the denominator of the factor C, becomes equal to zero.

In this case there occurs an unlimited magnification of the amplitude of the field of displacements. In terms of Physics it corresponds to the resonance excitation of the surface elastic wave with the continuous energy input from the acoustic wave.

Area II. $v_s < v_0 < v_p$, $1 < \theta$, $\gamma\theta < 1$. Here the wave number $k_{\psi z}$ is real, and solution (6) represents superposition of the two wave processes: the surface wave, propagating with the velocity v_0 along the axis x and having the exponential decay of the amplitude with depth, and the wave with a constant amplitude and the wave vector $(k, k_{\psi z})$, transporting the energy from the free surface of the elastic half-space in the direction of the wave vector.

Polarization of the field of displacements on the free surface of the half-space remains ellipsoidal with variable inclination of the ellipse, and the ratio of semi-axes in terms of the parameters v_s, v_p;

$$u_x|_{z=0} = Ck\Big[(2-\theta^2)\sin(\omega t - kx) - 2\sqrt{(\theta^2-1)(1-\gamma^2\theta^2)}\cos(\omega t - kx)\Big],$$
$$u_z|_{z=0} = Ck\sqrt{1-\gamma^2\theta^2}\theta^2\cos(\omega t - kx).$$

Area III. $v_p < v_0$, $1 < \theta$, $1 < \gamma\theta$. The wave numbers $k_{\varphi z}$ and $k_{\psi z}$ are real, and solution (6) represents a superposition of two waves with constant amplitude and the wave vectors $(k,\ k_{\varphi z})$ and $(k,\ k_{\psi z})$, propagating under various angles to the free surface and transporting the energy in the direction of wave vectors.

Polarization of the field of displacements on the free surface of the elastic half-space represents degenerate ellipses with variable inclination:

$$u_x|_{z=0} = Ck\Big[(2-\theta^2) + 2\sqrt{(\theta^2-1)(\gamma_2\theta^2-1}\Big]\sin(\omega t - kx),$$
$$u_z|_{z=0} = -Ck\sqrt{\gamma^2\theta^2-1}\theta^2\sin(\omega t - kx).$$

It is possible to make a conclusion, that in the case of propagation of the acoustic wave above the "rigid" half-space (with high values of velocities of longitudinal and transverse waves with respect to the velocity v_0) the surface elastic wave is induced and has the velocity of sound in the air.

For an elastic half-space with the velocity of the Rayleigh wave, equal to the velocity of sound in the air, there occurs a resonance absorption of energy of the acoustic wave and resonance increase of amplitude of the surface wave, propagating along the axis x.

In case of the "soft" half-space (with the lower values v_p, v_s as compared to v_0) two waves are induced: the surface wave, and the wave, propagating under an angle to the free surface and transporting the energy from the acoustic wave to the elastic half-space. The relation of the parameters v_p, v_s, v_0 uniquely defines the form of ellipsoidal polarization of the field of displacements on the free surface of the half-space.

The detection of the surface wave, induced by the acoustic radiation of a vibrator [1], was apparently connected with the origin of a nearsurface sound channel, stipulated by the temperature inversion due to the cooling of the Earth at the night time.

In this case the amplitude of the pressure p in the acoustic wave at the distance R from a source can be evaluated as

$$p = \left(\frac{W\rho c}{\pi RH}\right)^{1/2},$$

where W is the acoustic power of the source; ρ, c are density of the air and the velocity of sound; H is thickness of the inversion layer in the nearsurface sound channel.

For the power $W = 0.1$–1.0 kWt and the thickness $H = 150$ m, this expression gives the evaluation of the amplitude of pressure at the distance of 20 km, $p = 0.07$–0.2 Pa.

From solution (6) and the numerical calculation shown in the figure, it follows that the characteristic velocities of displacements have the magnitude of order

$$v_x, v_z \approx \frac{p}{\rho v_s}.$$

In the region of experiments, the surface ground layer (a zone of small velocities) has $v_s = 200$–400 m/s, that gives the velocity of oscillations of the ground in the induced wave $v_x, v_z = 0.1$–0.3 mkm/s. These values are of the same order as the magnitude corresponding to the level of the recorded signal.

The proximity of the value of the velocity of the surface Rayleigh wave in the region of the experiment to the velocity of sound in the air, is an important circumstance, that makes the resonance excitation of the induced surface wave possible.

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