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Modeling of the hydroacoustic source for Earth's global tomography*

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The paper deals with the modeling of the resonant vibrational source of the acoustic waves of infrasonic frequency range for the Earth's global tomography problems. The model of the hydroacoustic vibrational source includes a gas-filled resonance radiator operated under the free surface of the liquid. It is shown that for the effective radiation of the acoustic waves in the infrasonic frequency range two conditions must be carried out. First of them defines the depth of immersing of the radiator taking into account the influence of reflected wave from free surface on the full wave field. The second condition is connected with the dependence of the resonant frequency of the gas-filled radiator from its size and gas pressure from the depth of immersing. The combination of these conditions allowed us to define the main parameters of a radiators for the superpower vibrators and to show the opportunity of creation of resonant vibrational sources with power 100–500 kWt in the frequency range 1–5 Hz.

1. Introduction

The opportunity to use artificial seismic wave sources for the Earth's global tomography has not been practically considered until recent time. First of all it was stipulated by the required power characteristics of such sources. Estimates have shown that for the allocation of waves at distances about 1000–10000 km it is necessary to radiate a significant wave energy – about 10000 kWh in a low-frequency range 1–5 Hz [1–3]. Such wave energy can be supplied by the vibrating sources with very high force characteristics – about 10–100 thousand tons. The most powerful existed sources have a force amplitude 2–3 orders less (10–100 tons). Therefore, at the existed level of engineering the creation of superpowerful sources for the Earth's global tomography was considered to be almost a technically impracticable problem. Now there is an opportunity to realize the unconventional approach to the decision of the problem of sources for the Earth's global tomography [4–6]. In the paper, we consider a resonance scheme of the infrasonic source for liquid, containing an oscillatory contour and ensuring the direct compensation of an imaginary part of the medium impedance. Using the conditions

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of radiation and mechanical resonance of a source a dependence of the main parameters of a source on frequency: size of gas-filled radiator, submerged depth, radiation pattern and power of the radiation is obtained.

2. Statement of the problem

The operation of vibrational sources of seismic waves in the marine seismic works has some peculiarities. In contrast to the work carried out on the ground, here it is impossible to use low-frequency vibrators operated at the free surface of liquid or in its vicinity. This is connected with the fact that the free surface of liquid reflects an acoustic signal in the antiphase and at the close location of a source the reflected signal summarizing with the radiating one, decreases its amplitude. Therefore, a seismic vibrator operating in liquid must be located at a required depth for conforming the radiating signal with the reflection conditions at the free surface.

Acoustic wave propagation from the resonant source in the liquid can be written down by the wave equation with the boundary conditions on the free surface of the liquid and on the surface of the source

$$\frac{\partial^2 p}{\partial t^2} - c^2 \Delta p = 0, \quad p|_{z=0} \equiv 0, \quad p|_{r \in S} = p_0(t), \qquad \frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{1}$$

where $p(r,t)$ is the pressure field, $u(r,t)$ is the velocity vector, S is the surface of the radiator.

The solution of the acoustic problem for the unbounded liquid and the harmonic spherical source has the following form in the spherical coordinates

$$\begin{aligned} p(r,t) &= \frac{p_0 a}{r}\exp(ikr - i\omega t), \\ u(r,t) &= \frac{1}{i\omega\rho}\frac{\partial p}{\partial r} = \frac{p_0 a}{i\omega\rho}\Big(-\frac{1}{r^2} + \frac{ik}{r}\Big)\exp(ikr - i\omega t), \end{aligned}$$

where p_0 is amplitude of the pressure on the surface of the radiator, a is the radius of the spherical radiator, r is the radius-vector, ρ and c are the density and wave velocity of the liquid, ω and $k = \omega/c$ are the frequency and wave numbers.

The wave field of the vibrator located below the free surface of liquid can be determined by the image method similar to one applied in electrodynamics. The full wave field can be presented as superposition of wave fields of two sources – the real and its image located symmetrically with respect of free surface and oscillating in the antiphase. In this case, the boundary condition from (1) is automatically fulfilled at the free surface: acoustic pressure at it equals zero. For the pressure field at the distance of $r \gg h^2$ it yields the following expression:

$$
\begin{aligned}
p(r,\theta,t) &= \frac{p_0 a}{|\vec{r}-\vec{h}|}\exp(ik|\vec{r}-\vec{h}|-i\omega t) - \frac{p_0 a}{|\vec{r}+\vec{h}|}\exp(ik|\vec{r}+\vec{h}|-i\omega t) \\
&\approx -ip_0\frac{a}{r}2\sin(kh\cos\theta)\exp(ikr-i\omega t), \qquad (2) \\
\vec{u}(r,\theta,t) &\approx -i\frac{p_0 a}{\rho c}\frac{\vec{r}}{r^2}2\sin(kh\cos\theta)\exp(ikr-\omega t),
\end{aligned}
$$

where $\Phi(\theta) = 2\sin(kh\cos\theta)$ is the function of the radiation patter, h is the depth of immersing, θ is the polar angle.

The factor $\Phi(\theta)$ as a part of the expression for the pressure in the acoustic wave defines the radiation pattern of the vibrational source. It shows that it is most appropriate to locate a source at the depth equal one forth of the wave length of the radiated wave. In this case, the radiation pattern has one lobe with the maximal amplitude of the wave field directed vertically downwards. The power of radiation of the source can be determined as the integral of the energy flow throw the closed surface around the source

$$
W = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\int_S p\vec{u}\cdot\vec{n}\,dS\,dt,
$$

where p, $\vec{u}$ are the pressure and velocity vector in the acoustic wave, $dS = 2\pi R^2\sin\theta\,d\theta$ is the differential of the square, $\vec{n} = \vec{r}/r$ is the normal vector, θ is the polar angle.

Taking into the account the solution for the wave field of the immersed source (2), we can obtain the following expression for its radiation power

$$
W = \frac{2\pi p_0^2 a^2}{\rho c}\left(1 - \frac{\sin(2kh)}{2kh}\right), \qquad W_0 = \frac{2\pi p_0^2 a^2}{\rho c}, \tag{3}
$$

where W_0 is the radiation power of the source in the unbounded liquid. Form (3) shows that power of immersed source tends to zero when the source approach to the free surface. When the depth is equal one forth of the wave length, the power of immersed source is equal the power of the source in the unbounded liquid. The further increase of the depth of the source insignificantly increases the power of radiation, the maximum power is $W_{\max} \approx 1.2W_0$ when $h \approx 0.36\lambda$.

Expression (3) for the radiation power can be rewritten using relative change of the volume of the radiator $\Delta V/V$ in the case of adiabatic process of gas oscillation

$$
\begin{aligned}
p_0 &= \gamma(\rho g h + p_a)\frac{\Delta V}{V}, \\
W &= \frac{2\pi a^2}{\rho c}\gamma^2(\rho g h + p_a)^2\left(1 - \frac{\sin 2kh}{2kh}\right)\left(\frac{\Delta V}{V}\right)^2,
\end{aligned}
$$

where γ is adiabatic air constant, g is gravity acceleration, h is the depth of immersing, p_a is atmospheric pressure.

The size of really conceivable vibrators must be about several meters, therefore when operating at low frequencies, the vibrators are considerably smaller than the wave length of radiated signal, that is, $ka \ll 1$, where k is the wave number, a is the size of a source. The work of sources in such conditions has been theoretically studied and gives the following results referred to the source. For the sphere oscillating in the unbounded liquid, the force and velocity at the surface of the sphere are phase shifted and for small oscillations they are connected by the relation:

$$F = IZ, \qquad F = F_0 e^{i\omega t}, \qquad I = I_0 e^{i(\omega t + \alpha)}, \tag{4}$$

where Z is the resistance of the medium to radiation, including active and reactive components

$$Z = R + iX, \qquad R = \rho c S (ka)^2, \qquad X = \rho c S (ka), \tag{5}$$

where ρ is the density of the liquid, c is the sound velocity, $S = 4\pi a^2$ is the surface of the sphere of the radius a, k is the wave number.

The active component R of the resistance is connected with the energy, radiated by the vibrators in the form of seismic waves. The reactive component X describes the motion of the mass of the liquid surrounding the vibrator and provides energy exchange between the vibrator and medium. The power of radiation (active) and oscillation of liquid (reactive) are respectively equal to:

$$W_R = \frac{I^2 R}{2} = \frac{F_0^2}{2\rho c S}, \qquad W_X = \frac{I^2 X}{2} = \frac{F_0^2}{2cS(ka)}, \tag{6}$$

where W_R is the active power and W_X is the reactive power of the source.

It follows from these expressions that if $ka \ll 1$, the reactive power of the source, is much greater than active. In this case, the active power is constantly used for radiation of acoustic waves and must be replenished by force systems of the vibrator at the account of energy supply. The reactive power can be preserved and would not need to be replenished if the vibrator makes with the medium a mechanical oscillatory system. To do so, the vibrator must contain an elastic volume, and when operated it must produce recuperation of kinetic energy of the moving (oscillating) liquid to the potential energy of the elastic volume compression and back. With such construction of a vibrator, technical advantages of creation of sources of seismic signals acting in a continuous medium are clearly defined. As compared to vibrators operated at the land surface, the necessity of using a special many-ton inertial mass here falls away. Its role being played by the liquid surrounded the source. In this case, the vibrator includes only the elastic volume and the force system for its oscillating. In addition, the total seismic power of the vibrator operating in liquid is embedded in the radiated P-wave, while seismic vibrators operating on land transfer to P-wave an insignificant portion of seismic power, since its major portion is expended in radiation of the Rayleigh surface waves. The oscillating contour "vibrator–medium" can be formed by creating a gas volume in the liquid. The use of the air under pressure equal to the hydrostatic pressure of liquid at the depth of vibrator location $p = \rho g h + p_a$ is technically best suited, since in this case requirements for seals between liquid and gas decrease. In some case this seals need not to use at all. Consider oscillation of a gas volume of the spherical shape of the radius a, under the pressure p in unbounded liquid. The coefficient of rigidity of this volume, that is the ratio between the increment of the force at the surface dF and the change of the radius dr, in the case of adiabatic compression equals

$$n = \frac{dF}{dr} = S^2 \frac{dp}{dV} = -\gamma S^2 \frac{p}{V} = -12\pi a \gamma (\rho g h + p_a).$$

From (4) and (5) for a sphere oscillating in liquid we have

$$\frac{dF}{dr} = i\omega \frac{dF}{dI} \approx -\omega \rho c S(ka). \tag{7}$$

Equating these expressions we obtain the frequency of resonance oscillations of the contour "gas–liquid":

$$\omega^2 = \frac{3\gamma}{\rho a^2}(\rho g h + p_a).$$

For the vibrator located at the most usable (according to radiation) conditions, the depth $h = \lambda/4$, the expression for the frequency is of the form

$$\omega^3 = \frac{3\pi}{2} \frac{\gamma g c}{a^2}\left(1 + \frac{2 p_a \omega}{\pi \rho g c}\right).$$

Numerical values for the frequency $f = \omega/(2\pi)$, the radius a of the gas volume and the operating depth $h = \lambda/4$ are presented in the table.

f, Hz	a, m	$h = \lambda/4$, m	$W_R/(\Delta V/V)^2$, kWt
2	7.16	187.5	$1.6 \cdot 10^6$
4	2.59	93.7	$5.6 \cdot 10^4$
6	1.45	62.5	$9.7 \cdot 10^3$
10	0.70	37.5	$8.6 \cdot 10^2$
20	0.27	18.75	$4.8 \cdot 10^1$

One of the main characteristics of the seismic vibrator is its active power. To evaluate its value for an individual source containing a gas volume, let us set a relative change of this volume $\Delta V/V$. In this case, the amplitude of the force at the surface of the source according to (6), equals

$$F = -4\pi a^2 \gamma(\rho g h + p_a)\frac{\Delta V}{V}.$$

Substituting this expression for the amplitude of the force into (5) and taking into account the dependence of the gas volume radius on the frequency (7), we obtain for the source, operating in the best suited conditions of radiation at $h = \lambda/4$, the following expression for the active of radiation:

$$W_R = \frac{(S\gamma(\rho g h + p_a))^2}{2\rho c S}\left(\frac{\Delta V}{V}\right)^2 = \frac{3\pi^4}{4}\frac{\rho\gamma^3 g^3 c^2}{\omega^5}\left(1 + \frac{2p_a\omega}{\pi\rho g c}\right)^3\left(\frac{\Delta V}{V}\right)^2.$$

The marine vibrator with constant gas volume V can change resonant frequency when change immersing the depth h. The expression for the main characteristics of the source as the function of the depth is as follows:

$$\omega = \omega_0\sqrt{\frac{\rho g h + p_a}{\rho g h_0 + p_a}},$$

$$p = \gamma(\rho g h + p_a)\frac{a}{R}\frac{\Delta V}{V}2\sin\left(\frac{\pi}{2}\frac{h}{h_0}\sqrt{\frac{\rho g h + p_a}{\rho g h_0 + p_a}}\cos\theta\right)e^{ikR - i\omega t},$$

$$W = \frac{2\pi a^2}{\rho c}\gamma^2(\rho g h + p_a)^2\left(\frac{\Delta V}{V}\right)^2\left[1 - \frac{\sin(\pi h/h_0)\sqrt{\dfrac{\rho g h + p_a}{\rho g h_0 + p_a}}}{\pi h/h_0\sqrt{\dfrac{\rho g h + p_a}{\rho g h_0 + p_a}}}\right],$$

where ω_0 and h_0 are the optimal resonant frequency and the depth of immersing for the source with the gas-filled radiator of the volume V.

3. Conclusion

From the above expression it is seen that seismic power to be obtained from the vibrator, strongly depends on its frequency. The rapid growth of it value with the decrease of frequency is due to the simultaneous effect of some factors: the increase of the depth of the location of the vibrator, the increase of the pressure of the air in gas volume and its amplitude of oscillations, the increase of the surface area of the source. The numeical values of seismic power of the source per relative change of its volume are shown in the Table. The conductive calculations show that the seismic vibrator which operates in liquid and contains the oscillating contour vibrator–medium could be effectively used for solving problems of vibrosounding of deep layers of the crust as well as in seismic prospecting. It is shown that for efficient sensing of the Earth at the depth more than 100 km, a vibrational source of seismic power about 100 kWt operating within the frequency range 2–5 Hz is needed. For evaluation, let us present characteristics of the considered vibrator operating with the frequency 4 Hz (see the table). This vibrator must contain the gas volume of the radius 2.59 m and be located at the depth 94 m below the liquid surface. With a relative change of gas volume in the course of the work $dV/V = 0.1$ the pressure oscillation amplitude being $dp = 1.45$ atm which corresponds to the amplitude of the force on the surface $F = 1200$ tons. In this case, the seismic power of the source equals 560 kWt. Thus, it is possible to attain the needed seismic power for deep sounding of the Earth with use of only one vibrator. To conclude let us note that when creating vibrators operating in the liquid, there is no need to realize a three-dimensional oscillating motion of the sphere along the radius. With the source of dimension essentially less than a wavelength $k \ll 1$, the sonic field of the source weakly depends on the geometry of a radiator. It is determined by the characteristic size of the radiator and relative change of its volume in the course of oscillations. Therefore, instead of a three-dimensional volume movement it is possible to use a one-dimensional one, that is to change the size of a vibrator in one direction, which is easier to realize in its construction.

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