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Using the harmonic vibroseismic fields for geodynamic research

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The method of active vibroseismic monitoring being developed recently suggests geological medium probing by seismic waves from artificial controlled sources. Here the changes happening in the stress-deformed state of the medium and its rheologic characteristics are determined by changes in the information parameters of seismograms, i.e., by changes in the amplitudes and times of income of various waves, their spectral and polarization characteristics. Facilities available nowadays, such as powerful low-frequency vibrators and digital distributed seismic signal detection systems, make it possible to perform vibroseismic monitoring of large seismic dangerous zones of 10,000 to 40,000 square kilometers [1–3].

In monitoring on large observation grounds of hundreds of kilometers, there are natural limitations of the possibility to detect weak manifestations of medium changes from analysis of time variations in seismogram parameters. They are caused by the fact that for a finite power of vibration sources and a limited bandwidth the SNR and the contrast ratio of vibration seismograms decreases with increasing distance. Consequently, the accuracy of measuring the amplitude, time, and polarization characteristics of some waves is reduced. Nevertheless, potentialities of the vibration seismic sources make it possible to implement not only the traditional approach resulting in obtaining and analysis of vibration seismograms but also the active monitoring method based on the use of narrow-band harmonic signals and the application of seismic interferometry principles to detect slight changes in the characteristics of the medium. This method can be implemented by using seismic wave vibration sources only and employs to a great extent their metrological merits: the possibility to emit a wide class of quasi-harmonic seismic signals with controllable characteristics and the high stability of the emitted signal parameters.

By the seismic interferometry method it is possible to determine the time variations of stationary wave fields induced in the medium by long-term emission of harmonic signals by the vibrator and recorded by seismic sensors on the surface of the Earth. Here the task of measurements is to determine the amplitudes and phases of the harmonic signals in all channels of the three component area detecting system (seismic antenna). The signal detected at each point is a result of interference (superposition) of all waves arriving in different ways at the point of recording with allowance for their amplitudes and phases. As a result, an interference "moire" pattern is formed on the area system. It is characterized by a very complex and variable spatial distribution of the amplitudes and phases as well as the corresponding polarization characteristics. The characteristic property of this pattern is its high variability with slight variations of the probe signal frequencies.

The main metrological merit of using the narrow-band harmonic signals in active monitoring tasks is the extremely high spectral resolution and the high accuracy of phase and amplitude measurement on the area system of seismic detectors. This property can be efficiently used to detect slight variations in the characteristics of the medium in investigation of geodynamic processes.

The sensitivities of the vibration seismogram method and the seismic interferometry method can be compared using a model of seismic geological medium whose pulse response represents a superposition of delta functions delayed for the transit times of waves corresponding to different rays. For a linear seismic antenna with N sensors it has the form

$$H_n(t) = \sum_i h_i \delta(t - \tau_i - n l_0 / c_i), \tag{1}$$

where h_i is the relative intensity of waves; τ_i, c_i are the transit time and the apparent (phase) velocity of the i-th wave; n is the sensor number; and l_0 is the spacing between sensors in the antenna.

The signals at the antenna sensors $y_n(t)$ represent the superposition of the delayed probe signals of different amplitudes and microseismic noise:

$$y_n(t) = \sum_i S_i b_0(t - \tau_i - n l_0 / c_i) + n(t), \tag{2}$$

where $y_n(t)$ is the signal at the n sensor of the antenna, $b_0(t)$ is the normalized probe signal, S_i is the i signal amplitude, and $n(t)$ is the microseismic noise.

The vibration seismograms $u_n(\tau)$ are obtained by convolution of the detected signal y_n and the probe signal $b_0(t)$ [4]:

$$\begin{aligned} n(\tau) &= \int_0^T y_n(t) b_0(t - \tau)\, dt \\ &= \sum_i S_i \int_0^T b_0\Big(t - \tau_i - \frac{n l_0}{c_i}\Big) b_0(t - \tau)\, dt + \int_0^T n(t) b_0(t - \tau)\, dt. \end{aligned} \tag{3}$$

For the probe sweep signal of unit amplitude $b_0(t) = \cos(\alpha t + \beta t^2/2)$, the autocorrelation function has the form

$$\Psi(\tau) = \int_0^T b_0(t) b_0(t-\tau)\, dt = \frac{T}{2} \frac{\sin \pi \Delta f \tau}{\pi \Delta f \tau} \cos 2\pi f_0 \tau = \frac{T}{2} \Psi_0(\tau), \tag{4}$$

where Δf is the frequency bandwidth, f_0 is the average frequency, and T is the sweep signal duration.

Here, vibration seismograms from different sensors of the seismic antenna represent a superposition of the autocorrelation functions of the sweep signal with different time shifts and amplitudes:

$$u_n(\tau) = \sum_i a_i \Psi_0\Big(\tau - \tau_i - \frac{nl_0}{c_i}\Big) + z_1(\tau), \quad z_1(\tau) = \int_0^T n(t) b_0(t-\tau)\, dt,$$

where $a_i = S_i T/2$ are the amplitudes of the waves on the vibration seismogram, $\Psi_0(\tau)$ is the sweep-signal autocorrelation function normalized to unit amplitude, $z_1(\tau)$ is the noise component of the vibration seismogram.

In the interferometry method the probe signal $b_0(t)$ is harmonic. Here the signals at the antenna sensors after establishment of the stationary wave field are also harmonic:

$$b_0(t) = \cos(\omega_k t), \qquad y_{nk}(t) = \sum_i S_i \cdot \cos\Big(\omega_k\Big(t - \tau_i - \frac{nl_0}{c_i}\Big)\Big) + n(t),$$

where $y_{nk}(t)$ are the signals at the antenna sensors, and ω_k is the harmonic signal frequency.

To preserve the generality of estimates, we apply a procedure similar to (3) to the harmonic signal $y_{nk}(t)$. Hence,

$$\begin{aligned} u_{nk}(\tau) &= \int_0^T y_{nk}(t) b_0(t-\tau)\, dt = \int_0^T \sum_i S_i \cdot \cos\Big(\omega_k\Big(t - \tau_i - \frac{nl_0}{c_i}\Big)\Big) \times \\ &\quad \cos(\omega_k(t-\tau))\, dt + \int_0^T n(t) \cos \omega_k (t-\tau)\, dt \\ &= A_{nk} \cos(\omega_k \tau - \varphi_{nk}) + z_2(\tau), \end{aligned} \tag{5}$$

where

$$z_2(\tau) = \int_0^T n(t) \cos(\omega_k(t-\tau))\, dt.$$

The convolution results in a harmonic signal with the amplitude A_{nk} and the phase φ_{nk} with the noise component adding $z_2(\tau)$. Here

$$A_{nk} \exp(j\varphi_{nk}) = \sum_i a_i \cdot \exp(-j\omega_k(\tau_i + nl_0/c_i)), \tag{6}$$

where $a_i = S_i T/2$, as in the case of the vibration seismogram.

The obtained values A_{nk} and φ_{nk} are essentially the amplitude and phase of the complex spectrum of the harmonic signal y because the performed procedure of convolution is in fact identical to the operation of calculation of the spectral line.

It follows from (5), (6) that the harmonic signal at each antenna sensor is characterized by a complex vector being the sum of the complex vectors with the amplitude a_i of all waves constituting the seismogram whose arguments represent phase incursions connected with the time of arrival of each wave at a concrete sensor and the harmonic signal frequency ω_k.

The determinism of statement of the given problem does not allow us to obtain rigorous statistical estimates. However, we can assume that with a sufficiently large number of waves, irregularity of the distribution of their amplitudes and times of arrival the set of vectors of signal at the antenna sensors will have a quasi-random character. By analogy with the known statistic problem on addition of vectors with a random uniform phase distribution, we can estimate the average value of the amplitudes of harmonic signals at the sensors (nonstrict analogy of mathematical expectation). It will exceed the amplitudes of separate waves and have the form:

$$\bar{A}_{nk} \approx \left(\sum_i a_i^2 \right)^{1/2}. \tag{7}$$

Amplitude variations of signals at different sensors (analogy of variance) will be commensurable with the mean and can be estimated as

$$\Delta A_{nk} \approx \bar{A}_{nk}, \qquad 0 < A_{nk} < \sum_i |a_i|. \tag{8}$$

The obtained estimate (7) has a rather simple physical sense. It is based on the fact that with equal energy of the probe sweep and monochrome signals (as was assumed above) the seismic energy detected by the antenna remains equal in both cases. In the case of seismogram it is composed from individual waves coming sequentially in time, and in the case of harmonic signal it is determined from the interference pattern on the seismic antenna of a sufficient aperture. The estimate of signal amplitude variations also characterizes directly the main property of interference effects.

If we consider the noise components of the correlation convolutions, each being a convolution of microseismic noise and the probe signal, then their autocorrelation functions R_{z_1}, R_{z_2} have the form according to [4]:

$$R_{z_1}(\lambda) = \frac{NT}{2}\Psi_0(\lambda), \qquad R_{z_2}(\lambda) = \frac{NT}{2}\cos\omega_k\lambda,$$

where N is the spectral power density of white noise and T is the session duration.

The values of these functions for $\lambda = 0$ coincide whence it follows the coincidence of the energies of the noise components on the vibration seismogram and on the harmonic signal convolution $u_n(\tau)$, $u_{nk}(\tau)$, which determine the variances of the estimates of the wave amplitudes and the times of arrival a_i, τ_i, as well as those of the amplitudes and phases of the harmonic signal A_{nk}, φ_{nk}. SNR for the harmonic signal is much higher than that on the seismogram primarily because of the great amplitude of the former in accordance with (7), (8). It is well-known from results of experiments with powerful vibrators that the amplitudes of harmonic signals determined by the method of synchronous accumulation during a session exceed many times the amplitudes of individual waves on vibration seismograms [4].

The above estimates of noise components in vibroseismic correlograms are valid if the microseismic noise is white. In fact, the microseismic noise has a variable spectral power density (SPD), its spectrum contains harmonic components from working mechanisms. It is unstationary and contains time intervals with a sharply increased variance caused by earthquake events, traffic, and local industry activities.

The influence of the above-mentioned special features of real microseismic noise on the accuracy of measurement of the vibroseismic signal parameters is different for monochrome and sweep signals. To obtain vibration correlograms it is necessary to perform wide-bandwidth detection of seismic signals. That is why the correlogram noise is determined by the total energy of the noise in the frequency range of a sweep signal, including the regions with the increased SPD and the harmonic components. When using the harmonic signals in the vibroseismic interferometry method, one chooses spectrum regions with the minimal noise SPD and without harmonic components. Moreover, the accuracy of measuring the harmonic vibrosignal parameters is improved by using the weighting algorithms that allow for the nonstationary character of microseismic noise.

Let us consider special features of the geodynamic processes, i.e., time variations of the medium in the two methods described. Assume that some changes (e.g., changes in the stress-deformed state) have occurred in the medium and they resulted in slight irregular changes in the amplitudes and the times of wave arrival:

$$a_i \to a_i + \delta a_i, \qquad \tau_i \to \tau_i + \delta\tau_i.$$

Here the vibration seismogram has the form

$$u_n(\tau) = \sum_i (a_i + \delta a_i)\Psi_0\Big(\tau - \tau_i - \delta\tau_i - \frac{nl_0}{c_i}\Big) + z_1(\tau). \tag{9}$$

The form of harmonic signal convolution also changes:

$$u_{nk}(\tau) = (A_{nk} + \delta A_{nk})\cos(\omega_k\tau - \varphi_{nk} - \delta\varphi_{nk}) + z_2(\tau), \tag{10}$$

where

$$(A_{nk} + \delta A_{nk}) \exp(j(\varphi_{nk} + \delta\varphi_{nk})) \approx \sum_i (a_i + \delta a_i - j a_i \omega_k \delta\tau_i) \exp(-j\omega_k(\tau - \tau_i - nl_0/c_i)). \quad (11)$$

From the expressions above we can clearly see the difference in the manifestations of variations of the wave field in the two methods. Variations in the parameters of individual waves result in changes of the seismogram at points related with the time of their arrival. The changes caused by different waves are independent. In the case of the interference pattern, however, the variations of the amplitudes and the times of arrival result in changes in all vectors of harmonic signals $A_{nk}\exp(j\varphi_{nk})$ at all antenna sensors in accordance with (11). Again, assuming the quasi-random behavior of this set, we obtain that the amplitude variations of signals at the sensors are also quasi-random but, what is very important, they exceed the amplitude variations of individual waves and have the estimate:

$$\overline{\delta A}_{nk} \approx \left(\sum_i (\delta a_i)^2 + (a_i\omega_k\delta\tau_i)^2\right)^{1/2},$$

$$0 < \delta A_{nk} < \sum_i \sqrt{(\delta a_i)^2 + (a_i\omega_k\delta\tau_i)^2}. \quad (12)$$

The property of the interferogram to sum up in the vector form the variations of individual waves provides its higher sensitivity to weak geodynamic processes. The greater number of waves has changed as a result of changes in the characteristics of the medium, the greater the wave field changes on the interferogram. That is why even for the level of variations of the amplitudes of waves lying on the level of vibration seismogram noise, on the interferogram pattern, one will observe the amplitude and phase variations of signals that exceed the noise level. This allows one to obtain a reliable estimate of the fact of the presence of the wave field variations.

In the vibroseismic interferometry method, the parameters of the seismic antenna determine the completeness of information on the interference pattern that can be defined as a 2D array of data received from the antenna sensors with a discrete frequency set. The number of independent values of the stationary wave field parameters A_{nk}, φ_{nk} is equal to $n \cdot k$ and is really limited by the technical possibilities of creating the antenna and by the time of experiment. The spatial amplitude distribution of harmonic signals in the antenna length represents a superposition of sinusoids whose number is equal to the number of waves, and the spatial frequencies are determined by the signal frequency and the virtual (phase) wave velocities. That is why the spacing between the sensors in the antenna must be less than one-half the minimal length of the spatial wave. Similarly, for the set of frequencies ω_k of harmonic frequencies the frequency step $\Delta\omega_k$ is inversely proportional to the maximal time of waves arrival

$$l_0 \approx \min_i \frac{c_i}{\omega_k}, \qquad \Delta\omega_k \approx 1/\max_i \tau_i.$$

To estimate the sensitivity of the vibroseismic interferometry method, we have performed mathematical simulation. The simulation considered a linear seismic antenna with N_d sensors, which was situated distance L_0 from the vibration source. The wave field on the antenna is formed by k_w plane waves coming at times τ_i with the amplitudes a_i and the phase velocities c_i. The model parameters are: the distance to the source $L_0 = 50$ km, the number of sensors $N_d = 15$ spaced 100 m apart, the number of waves 20, the time of arrival from the range 8–50 s, the waves amplitudes a_i from the range 5–50 in relative units, the noise level $N = 1$ relative unit.

The amplitude distribution of harmonic signals at the seismic antenna sensors for the chosen model appeared to be close to experimental results. As in the case of experimental data, one can see the quasi-steady behavior of the amplitude distribution and the high irregularity of the interference pattern. Zones of local maxima on the interferogram are related to seismic antenna sensors where the cophased summation of a part of arriving waves occurs at the given frequency.

Simulation has demonstrated a high variance of the maximal amplitude values (interference) and much lower variations of the mean value. These facts confirm the acceptability of estimate (7) and the sufficiency of the chosen antenna aperture for applying the principle of conservation of energy in the interference pattern and on the seismogram.

To determine the sensitivity of the interference parameters of a stationary wave field we introduced to the model the variations of the amplitudes of some waves and the times of their arrival. The variations were random under one restriction: their maximal value was below the noise level on the seismogram, i.e., it was impossible to detect them reliably by usual analysis of seismograms:

$$\delta a_i < \sigma, \qquad a_i \omega_k \delta\tau_i < \sigma,$$

where $\sigma = \sqrt{NT/2}$ is the root-mean-square deviation of microseismic noise, N is the spectral power density, and T is the session duration.

The introduced variations of the wave parameters have led to amplitude and phase variations at each sensor of the antenna. These variations can be characterized by the complex vectors

$$\delta\vec{A}_{nk} = \vec{A}_{nk}(a_i + \delta a_i, \tau_i + \delta\tau_i) - \vec{A}_{nk}(a_i, \tau_i).$$

Simulation has shown that estimate (12) is quite good for the mean value of signal amplitude variations. In the model considered this estimate yields a better SNR on the average of the order of the square root of the number of waves. Whereas the maximal values of signal variations exceed the average several times and thereby improve the reliability of detecting the wave field variations, which grow with increasing $\delta A_{nk}/\sigma$.

The high response of the vibroseismic interferometry method to low variations of the parameters of seismic waves of the stationary wave field shows that in tasks of active investigation of geodynamic processes this method can be the first and most sensitive indicator of changes in the stress-deformed state of the medium. Here, estimates obtained for the wave field variability have an integral character and the interferogram obtained by using harmonic signals contains information about all waves coming to the point of detection. Determination of variations in the parameters of individual seismic waves passing through the medium in order to learn the geodynamic process mechanism will inevitably require detailed investigation of vibration seismograms, where it will be necessary to increase the energy of probe signals in order to improve the accuracy of determining the varying characteristics of individual waves.

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