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Simulation of a superpower shaft hydroresonance vibrational source

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1. Introduction

The paper presents the results of mathematical modeling of superpower vibrational sources for the global tomography of the Earth. A hydroresonance scheme of a seismovibrator is considered, in which an oscillating in the vertical shaft liquid column with the mass of several tens thousands tons serves a seismic waves source. A mathematical model of the shaft source has been constructed. This model reflects the most significant features for the considered process of seismic waves radiation. Being combined, the model includes an elastic half-space with a vertical cylindric cavity, a column of a compressible fluid, and the ideal gas volume at the bottom of the shaft. The problem has been formulated in general in terms of mathematics, thus reducing to a combination of three systems of equations – dynamic elasticity theory, compressible fluid dynamics, and ideal gas dynamics. The boundary conditions are the known stresses and velocities relations at the interfaces between different media. For low frequencies when the wavelength essentially exceeds the shaft diameter, it appeared possible to separate the problem the determination of pressure distribution in the fluid from the 1D problem of finite oscillations of the compressible fluid column on the gas volume with allowance for quasi-static elastic deformations of the shaft walls, and, further, the solution of the dynamic problem for the elastic half-space with determined stresses on the cylindrical cavity boundary as boundary conditions. The results of numerical calculations of the full wave field of the shaft source for various model of media are presented.

2. Superpowerful vibrational seismic sources

Vibrational sources of seismic waves are currently used in active seismology for the research into the structure of the Earth's crust and upper mantle, monitoring of temporal variations of the stress-deformed state of the medium as well as in industrial applications [1, 2]. Several projects on superpower seismic vibrators with radiated power of several hundreds kilowatt in a low-frequency band [3, 4] have been developed for the Earth's global tomography.

These sources are based on the hydroresonant scheme, and their construction contains a mechanical oscillatory circuit with an oscillating inertial mass and elastic elements. The main peculiarity of the hydroresonant scheme of vibrators is the possibility of scaling, that is, increase in the size of the source and, hence, of the power and force parameters. At the same time, the developed structural scheme and technical decisions turned out to be equally acceptable for the creation of sources of various power, including superpowerful vibrators.

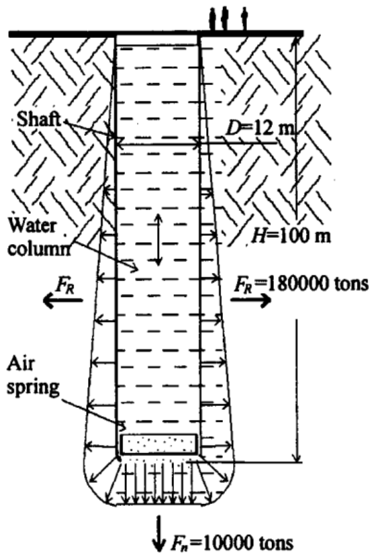

Figure 1. Mine hydroresonant source with an action of 10000 tons

In superpowerful seismic vibrators, the source of seismic waves is a liquid column of several tens of thousands of tons that is resonantly oscillating in a vertical mine. To create a vibrator with a force of 10000 tons, it is necessary to have a mine 12 m in diameter and 100 m deep which is filled with water (Figure 1). The oscillatory circuit is formed in such a source by locating a compressed air volume near the mine bottom. Vertical oscillations of the liquid column at the elastic volume lead to periodic pressure variations in the compressed air volume at the mine bottom and in the entire liquid column. There appear periodic vertical forces applied to the mine bottom and radial forces applied to the mine walls. This causes radiation of seismic waves. The frequency range of the mine vibrator for global seismology constitutes 0.5–5 Hz. The lower frequency of the range is determined by the magnitude of the inertial liquid mass and the maximal volume of the pneumatic spring. The upper operating frequency is determined by the elasticity of the liquid column and mine walls. It is limited by the value when the wavelength in the liquid column is comparable to the size of the liquid column itself in the mine (see the table).

Frequency, Hz	0.5	1	2	3	4	5
Air volume, m^3	150	37	9	4	2.5	1.5
Amplitude, mm	1000	250	60	25	15	10
Energy, J	10^8	2.5×10^7	6×10^6	3×10^6	1.5×10^6	10^6

The scheme of mine vibrational source under consideration is a rather complex oscillation system, in which resonant frequencies depend on elastic characteristics of the air volume, mass, and elasticity of the liquid column and the ground surrounding the mine. The distribution of stresses over the mine surface, as well as the geometrical dimensions determine the radiated seismic power, the distribution of wave types, and the directional pattern of the source. The main force, frequency, and power characteristics of the mine seismic vibrator were determined by mathematical modeling.

2.1. The mathematical model

The mathematical model of mine hydroresonant source, which reflects the properties that are most significant for the radiation process of seismic waves under consideration, is a combined one.

It includes an elastic half-space with a vertical cylindrical cavity, a column of compressible liquid, and a volume of ideal gas at the mine bottom (Figure 2). The general mathematical statement of the problem is reduced to a combination of the following three systems of equations: the dynamic theory of elasticity, the dynamics of compressible liquid, and ideal gas.

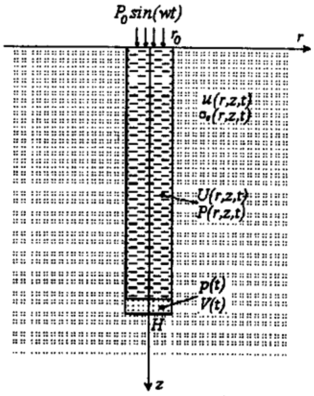

Figure 2. The model of mine hydroresonant source

2.2. System of equations

1. For an elastic half-space, the equations of the dynamic theory of elasticity have the following form in cylindrical coordinates (r, z, θ), taking into account the axial symmetry:

$$
\begin{gathered}
(\lambda+\mu)\operatorname{grad}\operatorname{div}\vec{u}+\mu\Delta\vec{u}-\rho\ddot{\vec{u}}=0,\\
\sigma_{rr}=\lambda\Theta+2\mu\frac{\partial u_r}{\partial r},\quad \sigma_{zz}=\lambda\Theta+2\mu\frac{\partial u_z}{\partial z},\quad \sigma_{rz}=\mu\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right),\\
\Theta=\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}+\frac{u_r}{r},\\
r>r_0 \quad \text{at} \quad 0<z<H, \qquad r>0 \quad \text{at} \quad z>H,
\end{gathered} \tag{1}
$$

where λ and μ are the elastic Lame constants, ρ is the density, $u(r,z,t)$ are the displacements, $\sigma_{rr}(r,z,t)$, $\sigma_z(r,z,t)$, $\sigma_{rz}(r,z,t)$ are stresses, Θ is the volumetric expansion, r_0 is the mine radius, and H is the mine depth.

2. For the column of compressible liquid, the equations of dynamics in an acoustic approximation are as follows:

$$
K\Delta\vec{U}-\rho_0\ddot{\vec{U}}=0, \quad P=-K\operatorname{grad}\vec{U}, \quad r<r_0 \text{ at } 0<z<H, \tag{2}
$$

where $U(r, z, t)$ are the displacements, $P(r, z, t)$ is the pressure, $K = \rho_0 c_0^2$ is the coefficient of liquid compressibility, c_0 is the sound speed in the liquid, and ρ_0 is the density of the liquid.

3. For the compressed gas volume, the relation between the pressure and volume is determined by the adiabatic equation

$$p = p_0 \left(\frac{V_0}{V}\right)^{\gamma}, \qquad p_0 = \rho_0 g H, \tag{3}$$

where $p(t)$ is the pressure, $V(t)$ is the gas volume, V_0 is the initial volume at the pressure equal to the static pressure p_0 in the liquid at the mine bottom at the depth H.

2.3. Boundary conditions

The boundary conditions of the problem are the well-known relation between the stresses and velocities at the common boundaries between different media.

1. At the free surface of the half-space, there are no normal stresses:

$$\sigma_{zz}(r, 0, t) = 0 \qquad \text{at} \quad r > r_0, \;\; z = 0. \tag{4}$$

The periodic pressure is given at the upper boundary of the liquid column as an excitation source of oscillations:

$$P(r, 0, t) = P_0 \sin \omega t \qquad \text{at} \quad r < r_0, \;\; z = 0. \tag{5}$$

2. At the surface of the cylindrical cavity, at the *liquid-elastic half-space* boundary, we have

$$\begin{gathered} \sigma_{rr}(r_0, z, t) = P(r_0, z, t), \quad \sigma_{zz}(r_0, z, t) = \sigma_{rz}(r_0, z, t) = 0, \quad u_r = U_r \\ \text{at} \quad 0 < z < H, \end{gathered} \tag{6}$$

where $\sigma_{rr}(r, z, t)$, $\sigma_{zz}(r, z, t)$, and $\sigma_{rz}(r, z, t)$ are the stresses, $u(r, z, t)$ are the displacements from (1), $P(r, z, t)$ is the pressure, and $U(r, z, t)$ are the displacements from (2).

3. At the *liquid-gas* boundary, at the bottom of the cylindrical cavity,

$$P(r, H, t) = p(t), \quad V(t) = V_0 - \int_S U_z(r, H, t)\, dS \quad \text{at} \quad r < r_0, \;\; z = H, \tag{7}$$

where $P(r, z, t)$ is the pressure, $U(r, z, t)$ are the displacements from (2), $p(t)$ is the pressure, $V(t)$ is the gas volume from (3), and $S = \pi r r_0^2$ is the cross-section area.

4. At the *gas-elastic half-space* boundary, at the bottom of the cylindrical cavity,

$$\sigma_{zz}(r, H, t) = p(t) \quad \text{at} \quad r < r_0, \ z = H, \tag{8}$$

where $\sigma_{zz}(r, z, t)$ are the stresses from (1), $U(r, z, t)$ are the displacements from (2), $p(t)$ is the pressure, and $V(t)$ is the gas volume from (3).

Direct solving of the formulated problem is associated with considerable difficulties, not only due to the complexity of the combined model, but also because the finite amplitude of oscillations of the liquid column on the gas volume must be taken into account, which leads to a nonlinear statement.

The problem of modeling is considerably simplified by considering the radiation process of seismic waves at low frequencies, when the characteristic wavelengths both in the elastic half-space and in the liquid are much greater than the mine diameter. In this case, the problem can be divided into two independent problems. The first one is the determination of the pressure distribution in the liquid from a 1D problem of finite oscillations of the column of compressible liquid on a gas volume with allowance for the influence of the elasticity of the surrounding half-space. The other one is solving the dynamic problem for the elastic half-space with the found stresses at the boundary of the cylindrical cavity as boundary conditions.

2.4. Resonant oscillations of the liquid column

The wave propagation problem in elastic half-space with a cylindrical cavity of infinite length filled with a liquid was considered earlier both in the full statement [9], and in a long-wave approximation (the classical theory of water hammer) [10]. The solution shows that in the case of low frequencies when the wavelength is much greater than the diameter of the cylindrical cavity, the 1D acoustic approximation can be used. In this case, the velocity of elastic waves propagating along the liquid column has a smaller value than the sound speed in an unbounded liquid. This is due to the fact that the effective coefficient of liquid compressibility decreases owing to elastic deformations of the surrounding space. The value of velocity of elastic waves for low frequencies is given by the following expression [9]:

$$c^2 = \frac{c_0^2 \mu}{\rho_0 c_0^2 + \mu}, \tag{9}$$

where ρ_0 and c_0 are the density and sound speed in the liquid from (2), $\mu = \rho V_S^2$ is the shear modulus for the elastic medium, and ρ and V_S are the density and velocity of shear waves.

The resonant characteristics of the column of compressible liquid at an adiabatic pneumatic spring are determined from the 1D equation of acoustics with the following two boundary conditions: the equality of the pressure to zero at the upper boundary of the liquid at $z = 0$ and the equality of the pressure in the liquid and gas at $z = H$:

$$\frac{\partial^2 U(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 U(z,t)}{\partial t^2} = 0, \quad P(z,t) = -\rho_0 c^2 \frac{\partial U(z,t)}{\partial z},$$
$$\rho_0 c^2 \frac{\partial U(z,t)}{\partial z}\bigg|_{z=H} = \rho_0 g H \left(\frac{V_0}{V_0 - U(z,t)S}\right)^{\gamma}\bigg|_{z=H}, \tag{10}$$
$$\frac{\partial U(z,t)}{\partial z}\bigg|_{z=H} = 0 \quad \text{at} \quad 0 < z < H,$$

where $U(z,t)$ are the displacements, $P(z,t)$ is the pressure, c is the velocity of elastic waves in the liquid in accordance with (9), ρ_0 is the density of the liquid, V_0 is the initial gas volume, $S = \pi r_0^2$ is the cross-section area of the cavity in accordance with (3), (7).

For small oscillations, the second boundary condition admits the linearization:

$$\rho_0 c^2 \frac{\partial U(z,t)}{\partial z}\bigg|_{z=H} = -\frac{N}{S}U(z,t)\bigg|_{z=H}, \quad N = \gamma \rho_0 g H \frac{S^2}{V_0}, \tag{11}$$

where N is the elasticity coefficient of the gas volume at small 1D deformations.

The solution of 1D equation of acoustics (10) with boundary condition (11) is a superposition of the eigen forms of oscillations (of standing waves) whose frequencies are determined from the following characteristic equation:

$$U(z,t) = \sum_i A_i \sin(\omega_i t)\cos(k_i z),$$
$$P(z,t) = \rho_0 c^2 \sum_i A_i k_i \sin(\omega_i t)\sin(k_i z), \tag{12}$$
$$k_i = \frac{\omega_i}{c} = \frac{2\pi}{\lambda_i}, \quad \omega_i \operatorname{tg}\left(\frac{\omega_i H}{c}\right) = \frac{N}{\rho_0 c S},$$

where ω_i, k_i, λ_i are the frequency, wave number, and wavelength of the i-th eigen form of oscillations, and A_i is its amplitude.

The form of the solution for the displacements and pressure, as well as characteristic equation for frequencies (12) show that for any value of elasticity of the gas volume there exists an infinite number of eigen forms, with arbitrarily high frequency values. We are interested in the first form of oscillations with the least frequency value and in the dependence of this frequency on the parameters of the gas volume. The least values of oscillation frequency admitting of the analytical estimate (because $\operatorname{tg} x \approx x$) correspond to small values of N (i.e., large gas volumes). At maximally high values of N (i.e., when the mine has no bottom), the frequency is determined from the condition that the argument of the tangent tends to $\pi/2$:

$$
\begin{aligned}
N \to 0: \quad & U(z,t) \approx U_{\max} \sin(\omega t) \cos(\omega z/c), \\
& P(z,t) \approx P_{\max} \sin(\omega t) \frac{z}{H}, \quad \omega^2 \approx \frac{N}{\rho S H} = \frac{N}{M}; \\
N \to \infty: \quad & U(z,t) \approx U_{\max} \sin(\omega t) \cos\left(2\pi \frac{z}{4H}\right), \\
& P(z,t) \approx P_{\max} \sin(\omega t) \sin\left(2\pi \frac{z}{4H}\right), \quad \frac{\omega H}{c} \approx \frac{\pi}{2}; \quad \lambda \approx 4H,
\end{aligned}
\tag{13}
$$

where M is the total mass of the liquid column.

Thus, in the case of small rigidity of the elastic gas volume, the liquid column oscillates as one mass and has a linear distribution of pressure with height. Here the influence of the liquid compressibility is insignificant. In the second limiting case, in the absence of gas in the pneumatic spring, an elastic standing wave is excited. The length of this wave is by a factor of 4 greater than the mine depth, and the pressure distribution is described by one fourth of the period of sinusoid with the maximal value at the mine bottom.

The solution of the equation of acoustics with nonlinear boundary condition (10) for finite oscillations of the liquid leads to the appearance of periodic solution. The solution contains harmonics with frequencies that are multiples of the fundamental frequency, and their own forms of natural oscillations correspond to them. A simple representation of the solution in the form of series of multiple frequencies similarly to (12) does not give a direct result, because the problem is reduced to nonlinear equations for coefficients of the infinite series. Therefore, the solution is found numerically with the use of a finite difference approximation of equation (10). The result of the calculation is the sought-for pressure distribution $P(z,t)$, which is in qualitative agreement with (12), i.e., it is a superposition of eigen forms of oscillations, from the first one and higher. This pressure distribution is boundary condition (6) for the dynamic problem of elasticity theory (1) and determines all characteristics of the mine vibrator as a source of seismic waves.

3. The dynamic problem for elastic half-space

After the determination of the pressure distribution at the walls of the cylindrical cavity, the problem of modeling of the full wave field of the mine source is reduced to the dynamic problem of elasticity theory for a half-space with complex boundary (1) and specified stresses at it as boundary conditions (4) and (6). In the case of low frequencies under consideration, when the lengths of elastic waves in the half-space and the mine depth are much greater than its diameter, the stresses at the boundary of the cylindrical cavity can be approximated by the stresses from a system of point sources of the "pressure center" and "vertical force" types, linearly distributed along the z- axis from the surface to the depth H in a homogeneous elastic half-space.

If the source is considered in the form of a sphere with a uniform radial pressure on the surface (a "pressure center" of finite size), in a cylindrical system of coordinates, it can be represented as a superposition of sources of two types: a source of circular horizontal force F_r and a source of vertical force F_z. The values of these forces are determined by the integration of the corresponding projections of forces over the sphere surface.

$$\begin{aligned} F_r &= \int_S p\cos\vartheta\, dS = p\int_{-\pi/2}^{\pi/2}\cos^2\vartheta\, d\vartheta\int_0^{2\pi} d\varphi = p\pi^2 r_0^2, \\ F_z &= \pm\int_S p\sin\vartheta\, dS = \pm p\int_0^{\pi/2}\sin\vartheta\cos\vartheta\, d\vartheta\int_0^{2\pi} d\varphi = \pm p\pi r_0^2, \end{aligned} \tag{14}$$

where p is the pressure at the sphere surface, (r,ϑ,φ) are the spherical coordinates, $dS = r^2\cos\vartheta\, d\vartheta\, d\varphi$ is the differential of area on the sphere.

The source of circular horizontal force approximates an element of the cylindrical cavity with normal stresses. Two sources of vertical source, from the upper and lower hemisphere, respectively, determine the forces along the z-axis. At a linear location of sources of the "pressure center" type with different intensities along the cavity axis, for the compensation of the vertical component of force, it is necessary to introduce a linear system of sources of the "vertical force" type, with the intensity of each of them equal to the difference of vertical components of the forces from the neighboring sources of the "pressure center" type. Therefore, the system of point sources approximating the cylindrical cavity with the distribution of normal stresses $P(z,t)$ has the following form:

$$f_{1n}(t) = P(z,t)\big|_{z=nh}, \quad f_{2n}(t) = \frac{\partial P(z,t)}{\partial z}h\bigg|_{z=nh}, \quad n = 1,\dots,N, \tag{15}$$

where $f_{1n}(t)$ is the intensity of the n-th source of the "pressure center" type, $f_{2n}(t)$ is the intensity of the n-th source of the "vertical force" type, h is the distance between the sources, and $N = H/h$ is the number of sources.

For a system of distributed point sources, the dynamic problem of elasticity theory for elastic half-space has the following form:

$$\begin{aligned} \rho\ddot{\vec{u}} &= (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\vec{u} + \mu\Delta\vec{u} + \vec{F}, \\ \vec{F}(z,r,t) &= F_r\vec{e}_r + F_z\vec{e}. \end{aligned} \tag{16}$$

The right-hand part of equation (16) describes the action of a system of sources localized in time and space.

For a system of sources of the "pressure center" type,

$$\vec{F} = \vec{F}_1(z, r, t,) = \sum_{n=1}^{N} \mathrm{grad}\left[\frac{\delta(r)}{2\pi r}\delta(z - nh)\right] f_{1n}(t). \tag{17}$$

For a system of sources of the "vertical force" type,

$$\vec{F} = \vec{F}_2(z, r, t,) = \sum_{n=1}^{N} \frac{\delta(r)}{2\pi r}\delta(z - nh)\, f_{2n}(t)\, \vec{e}_z. \tag{18}$$

The initial and boundary conditions of the problem are as follows:

$$\vec{u}|_{t=0} = \dot{\vec{u}}|_{t=0} = \sigma_{zz}|_{z=0} = \sigma_{rz}|_{z=0} = 0. \tag{19}$$

Let us use the integral Hunkel transform with respect to the variable r [11]:

$$\begin{aligned} R(z, k_i, t) &= \int_0^a r u_z(z, r, t) J_0(k_i r)\, dr, \\ S(z, k_i, t) &= \int_0^a r u_r(z, r, t) J_1(k_i r)\, dr, \end{aligned} \tag{20}$$

with the following inverse formulas:

$$\begin{aligned} u_z(z, r, t) &= \frac{2}{a^2} \sum_{i=1}^{\infty} R(z, k_i, t) \frac{J_0(k_i r)}{[J_1(k_i a)]^2}, \\ u_r(z, r, t) &= \frac{2}{a^2} \sum_{i=1}^{\infty} S(z, k_i, t) \frac{J_1(k_i r)}{[J_0(k_i a)]^2}, \end{aligned} \tag{21}$$

where k_i are the roots of the Bessel equation $J_1(k_i a) = 0$. The system of the 1D equations obtained after the transformations contains terms $u_z|_{r=a}$, $u_z|_{r=a}$. Let us introduce new boundary conditions at the right-hand side:

$$u_z(z, r, t)|_{r=a} = u_z(z, r, t)|_{r=a} = 0 \tag{22}$$

and consider the wave field to the time $t < T$, where T is the minimal time of propagation of the wave leading edge to the reflecting surface $r = a$. We can do this by virtue of the hyperbolicity of the problem.

The new boundary value problem of smaller dimensionality obtained as a result of using the finite integral Hunkel transform with respect to r has the following form:

$$
\begin{aligned}
\rho\frac{\partial^2 S}{\partial t^2} &= \mu\frac{\partial^2 S}{\partial z^2} - k_i\lambda\frac{\partial R}{\partial z} - k_i\mu\frac{\partial R}{\partial z} - k_i^2(\lambda+2\mu)S + \bar{F}_r, \\
\rho\frac{\partial^2 R}{\partial t^2} &= (\lambda+2\mu)\frac{\partial^2 R}{\partial z^2} + k_i\lambda\frac{\partial S}{\partial z} + k_i\mu\frac{\partial S}{\partial z} - k_i^2\mu R + \bar{F}_z.
\end{aligned} \tag{23}
$$

The initial conditions are

$$
S(z,k_i,t)\Big|_{t=0} = R(z,k_i,t)\Big|_{t=0} = 0, \qquad \frac{\partial S}{\partial t}\Big|_{t=0} = \frac{\partial R}{\partial t}\Big|_{t=0} = 0. \tag{24}
$$

The boundary conditions are

$$
\lambda k_i S + (\lambda+2\mu)\frac{\partial R}{\partial z}\Big|_{z=0} = 0, \qquad \mu\Big(\frac{\partial S}{\partial z} - k_i R\Big)\Big|_{z=0} = 0. \tag{25}
$$

The right-hand part of the equations has the following components:

a) in the case of sources of the "pressure center" type

$$
\bar{F}_r = -\sum_{n=1}^{N}\frac{k_i}{2\pi}\delta(z-nh)f_{1n}(t), \quad \bar{F}_z = \sum_{n=1}^{N}\frac{1}{2\pi}\frac{d}{dz}(\delta(z-nh))f_{1n}(t); \tag{26}
$$

b) in the case of sources of the "vertical force" type

$$
\bar{F}_r = 0; \quad \bar{F}_z = -\sum_{n=1}^{N}\frac{k_i}{2\pi}\delta(z-nh)f_{2n}(x). \tag{27}
$$

To solve the reduced problem, one can use the finite difference approximation over time and the space coordinate z. At this approach, each time when the number of sources, their location, or intensity is changed, calculations have to be carried out again. At a series of computational experiments on the modeling of wave fields generated by a set of sources, this approach is not economical.

A combination of the method of straight lines and the method of matrix decomposition is more effective [12]. This numerical-analytical approach for specific models of media makes it possible to model wave fields at any time of interest for an arbitrary combination of sources of various types, various intensity, and localization without great computational expenditures.

To solve problem (23)–(27), let us use the method of straight lines, which in contrast to the method of grids, approximates the operation of differentiation only with respect to the space variable z. For this purpose, we introduce a uniform grid on the interval $z \in (0,b)$ as follows:

$$
\omega = \{z_j = jh;\ j = 1,\dots,M;\ h(M+1) = b\}. \tag{28}
$$

The derivation of differential-difference equations to find the functions $S(z_j,k_i,t)$ and $R(z_j,k_i,t)$ at the straight lines $z = z_j$ was performed in accordance with the scheme proposed in [13]. The differential-difference problem obtained can be written in the vector form:

$$\frac{d^2\vec{X}}{dt^2} + A\vec{X} = f(t)\vec{\varphi}, \quad \vec{X}|_{t=0} = \left.\frac{d\vec{X}}{dt}\right|_{t=0} = 0, \tag{29}$$

where $\vec{X}(t) = (S(z_1, k_i, t), R(z_1, k_i, t), \dots, S(z_M, k_i, t), R(z_M, k_i, t))^T$, A is the symmetric positive-definite band matrix, the width of which depends on the order of approximation over the variable z; $\vec{\varphi}$ is the vector of the right-hand side approximating (26) or (27) on the grid ω, $f(t)$ is assumed to be equal to $f_1(t)$ or $f_2(t)$, depending on the type of sources considered.

With the help of orthonormal expansion [14]:

$$A = Q\,\mathrm{diag}\{\lambda_1, \dots, \lambda_M\}Q^{-1} \tag{30}$$

and change of the variables $\vec{Y}(t) = Q^{-1}\vec{X}(t)$, system (29) breaks down into N independent Cauchy problems for each Y_i component of the vector $\vec{Y}(t)$:

$$\frac{d^2Y_i}{dt^2} + \lambda_i Y_i = \Psi_i, \quad \Psi_i = (Q\vec{\varphi})_i f(t), \quad Y_i|_{t=0} = \left.\frac{dY_i}{dt}\right|_{t=0} = 0. \tag{31}$$

The solution to this problem is written in the following form:

$$Y_i = \int_0^t \Psi_i(\tau)\frac{1}{\sqrt{\lambda_i}}\sin\left(\sqrt{\lambda_i}(t-\tau)\right)d\tau. \tag{32}$$

Depending on the type of function of the signal $f(t)$, the values of $Y_i(t)$ can be obtained either in the analytical form or numerically by computing the integral in the right-hand part of (32) with the use of an approximate method. For the vibrational source radiating a harmonic signal, the function $f(t)$ has the following form:

$$f(t) = \sin\omega_0 t, \tag{33}$$

where ω_0 is the radiation frequency. Then formula (32) admits of the following analytical representation:

$$\begin{aligned} Y_i &= (Q\vec{\varphi})_i \frac{1}{\sqrt{\lambda_i}(\omega_0^2 - \lambda_i)}\left(\omega_0 \sin\sqrt{\lambda_i}t - \sqrt{\lambda_i}\sin\omega_0 t\right) \quad \text{at } \omega_0^2 \neq \lambda_i, \\ Y_i &= (Q\vec{\varphi})_i \frac{1}{2\omega_0^2}(\sin\omega_0 t - \omega_0 t\cos\omega_0 t) \qquad \text{at } \omega_0^2 = \lambda_i. \end{aligned} \tag{34}$$

After the vector $\vec{Y}(t)$ is determined, it is sufficient to return to the variable $\vec{X}(t) = Q\vec{Y}(t)$ and then find the solution $\vec{u}(z_i, r, t)$ to the initial problem (16)–(19) by formulas (21).

Notice that the main computational time of the algorithm is devoted to the construction of orthonormal expansion (30) for matrix A, the coefficients

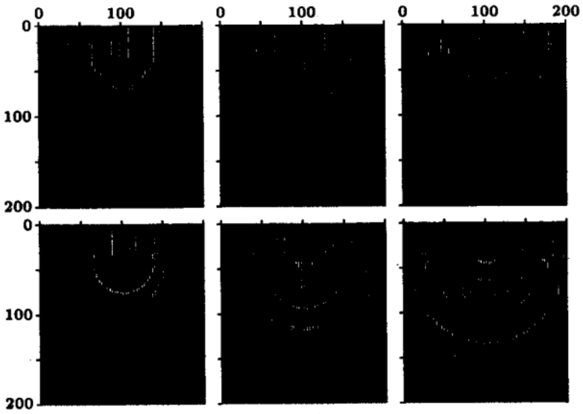

Figure 3. The wave field of the mine source (top) and the low-velocity layer (bottom) in a homogeneous half-space

of which depend only on the structure of the medium and do not depend on the parameters and location of the sources. After the expansion (30) for a specific model of the medium is obtained, the modeling of the wave field for an arbitrary combination of sources-receivers is realized only in accordance with formulas (32) and (35), which are analytical formulas for certain types of sources, as it was shown.

The algorithm developed was used for mathematical modeling of a full wave field of the mine vibrator for various models of the medium. The wave field of a mine source in a homogeneous elastic half-space for successive times is presented in Figure 3. The cylindrical longitudinal wave from the mine walls and a spherical wave from its bottom are clearly seen. Figure 3 also represents the wave field of the source for the mine location in a layer with velocities of elastic waves that are lower than those in the underlying half-space. The leading propagation of the longitudinal wave in the high-velocity part of the cross-section and the penetration of the conical wave into the low-velocity layer is clearly seen.

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