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Estimation of sensitivity of the active monitoring method by harmonic signals

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Abstract. The paper considers the estimation of sensitivity of the method of active monitoring of changes in the elastic characteristics in the Earth's interior crust zone. The model of the Earth's crust-mantle system in the form of a layer in a half-space with different velocity values of elastic waves is presented. The mathematical statement of the problem is done in the approximation of the wave equation. It is assumed that a vibrational source is a point and a harmonic one with a constant oscillation frequency and the zone of changes of the characteristics in the medium is spherical. The wave field in the medium is calculated in the ray approximation. The wave field variations in the medium and at the free surface are determined for the case of insignificant velocity changes in the spherical region by calculating a beam pattern of a fictitious 3D source in diffraction approach. As a result of the modeling, the sensitivity of the active monitoring method with harmonic vibrational signals is estimated. The relation between the quantitative changes in the amplitudes and phases of the oscillations recorded at the surface, the geometry and location of the zone of changes in the medium, and the magnitude of changes in the elastic characteristics are determined.

Introduction

To determine the capability of the active vibroseismic monitoring method in investigations of geodynamic processes in seismic prone-zones, in problems of detecting internal zones of media with changes or redistribution of tectonic stresses, one should consider the problem of interrelation between the variations of parameters of the vibroseismic wave field and the variations of medium parameters in internal zones, which affect the characteristics of waves passing through these zones.

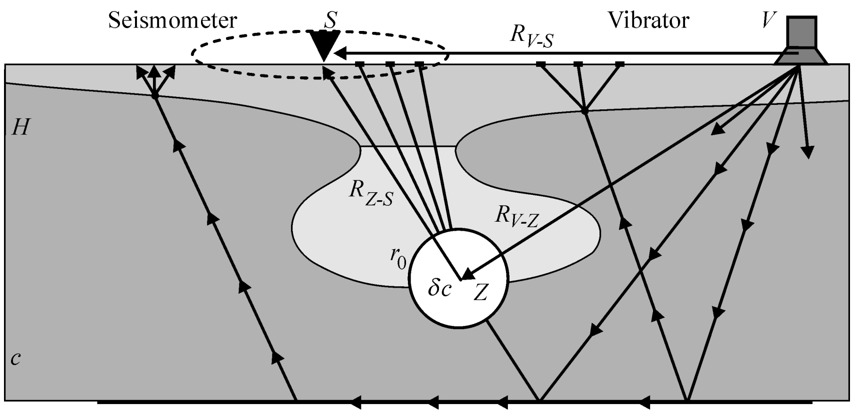
Below, some results of mathematical modeling of vibroseismic monitoring with the use of a stationary wave field formed in the medium at long radiation of harmonic signals by a vibrator with constant characteristics, such as frequency, amplitude, and phase, are presented.

It is possible to distinguish two main problems (direct and inverse), of modeling the monitoring with the use of stationary harmonic wave fields. The direct problem is in the determination of changes in the characteristics of a stationary wave field recorded at the surface with appearance of changes in the density and velocity of seismic waves in some internal zone of the medium. Also, it is the determination of the relation between the quantitative changes in the amplitudes and phases of oscillations recorded at the surface and the geometry and location of a zone of changes in the medium and the magnitude of changes in the elastic characteristics. For the vibroseismic monitoring method and organization of an observation system, it is important to determine the geometry of a zone of maximum changes in the amplitude-phase characteristics of the surface field depending on the mutual location of the vibrational source and the zone of changes in the medium. The inverse problem is associated with determination of the geometry of the zone of changes inside the medium and the quantitative changes in the elastic characteristics based on changes in the amplitudes and phases of the stationary harmonic field recorded at the surface.

1. A model and a system of equations

The method to monitor dilatant zones presented in [1, 2] is shown schematically in the figure. In the general case, to determine the relation between variations of a field and those of parameters of dilatant zone, it is necessary to calculate the full wave field. This can be done only numerically even for relatively simple models of the medium and the geometry of the zone of parameter changes.

To obtain analytical estimates of the method sensitivity, let us consider the direct problem of vibroseismic monitoring of changes of elastic characteristics in the Earth's crust interior zone in approximation of the wave equation and a model of the "Earth's crust-mantle system" in the form of an elastic layer in an elastic half-space with different velocities of elastic waves. We assume the vibrational source to be a point and to operate in the harmonic mode with a constant oscillation frequency. The zone of changes of characteristics in the medium is taken to be spherical with a radius from several fractions to several wavelengths (see the figure).



Calculation scheme: V — vibrator, S — seismometer at a recording point, Z — a zone of the medium parameters changes

The system of equations with boundary conditions has the following form:

$$\begin{cases} \left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \Delta\right) u(\vec{r},t) = F_0 \delta(\vec{r}) \exp(-i\omega t) \quad \text{for } 0 \le z \le H, \\ \left(\dfrac{1}{c_1^2}\dfrac{\partial^2}{\partial t^2} - \Delta\right) u_1(\vec{r},t) = 0 \quad \text{for } H \le z, \\ \dfrac{\partial u(\vec{r},t)}{\partial z}\Big|_{z=0} = 0, \\ u(\vec{r},t)\big|_{z=H} = u_1(\vec{r},t)\big|_{z=H}, \quad \dfrac{\partial u(\vec{r},t)}{\partial z}\Big|_{z=H} = \dfrac{\partial u_1(\vec{r},t)}{\partial z}\Big|_{z=H}, \end{cases} \tag{1}$$

where $u(\vec{r},t)$, $u_1(\vec{r},t)$ are the displacement functions in the layer and half-space, c, c_1 are the velocities of waves in the layer and half-space, F_0, ω are the intensity and frequency of the point harmonic source, and H is the layer thickness.

The wave equations and boundary conditions for the model "Earth's crust–mantle" take into account the presence of a harmonic source in the layer, the absence of stresses on the free surface, and the equality of stresses and velocities at the layer–half-space interface.

For the wave field changes caused by small changes of the velocities of seismic waves in some area V, the wave equation and boundary conditions will have the following form, accurate to the second order terms [3]:

$$\begin{cases} \left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \Delta\right) \delta u(\vec{r},t) = -\dfrac{2\delta c(\vec{r})}{c^3}\dfrac{\partial^2 u(\vec{r},t)}{\partial t^2} \quad \text{for } 0 \le z \le H, \\ \delta c(\vec{r}) \ne 0 \qquad \text{for } \vec{r} \in V, \\ \left(\dfrac{1}{c_1^2}\dfrac{\partial^2}{\partial t^2} - \Delta\right) \delta u_1(\vec{r},t) = 0 \quad \text{for } H \le z, \\ \dfrac{\partial \delta u(\vec{r},t)}{\partial z}\Big|_{z=0} = 0, \\ \delta u(\vec{r},t)\big|_{z=H} = \delta u_1(\vec{r},t)\big|_{z=H}, \quad \dfrac{\partial \delta u(\vec{r},t)}{\partial z}\Big|_{z=H} = \dfrac{\partial \delta u_1(\vec{r},t)}{\partial z}\Big|_{z=H}, \end{cases} \tag{2}$$

where $\delta u(\vec{r},t)$, $\delta u_1(\vec{r},t)$ are the variations of solutions for displacements in the layer and half-space, δc is the variation of wave velocity in the area V located in the layer. In the case of a spherical area, it is characterized by the coordinates of the center R_0 and a radius r_0.

Thus, distortions of the wave field that occur with appearance of an area of small changes of seismic waves velocity are described by the wave equations for the initial medium with a volume source in the area V. The function in the right-hand side of (2) describes the density of the 3D volume source. The density is proportional to the product of the velocity change into the initial solution of the field obtained for the vibrational source $F_0\delta(\vec{r}) \cdot e^{-i\omega t}$. It is not equal to zero only in the area V.

2. Solution for the initial field in the ray approximation

The initial solution for the displacement field of the point vibrational source in a layer in the half-space for an unpertubed medium can be represented as ray approximation. It is a superposition of spherical waves that had multiple reflections from the free surface and the layer–half-space interface

$$\begin{cases} u(\vec{r},t) = \sum\limits_{m=1}^{\infty} \dfrac{1}{R_1} \exp(i\vec{k}_1\vec{R}_1 - i\omega t + im\varphi_1)\alpha_1^m \beta^{m-1} + \\ \qquad\qquad \sum\limits_{m=0}^{\infty} \dfrac{1}{R_2} \exp(i\vec{k}_2\vec{R}_2 - i\omega t + im\varphi_2)\alpha_2^m \beta^{m}, \\ \vec{R}_1 = \vec{r} + \vec{z}_m, \qquad \vec{k}_1 = (\omega/c)\vec{R}_1/R_1, \\ \vec{R}_2 = \vec{r} - \vec{z}_m, \qquad \vec{k}_2 = (\omega/c)\vec{R}_2/R_2, \\ \vec{z}_m = (0,0,m\cdot 2H), \end{cases} \tag{3}$$

where $\vec{k}_1$, $\vec{k}_2$ are the vectors of the waves multiply reflected from the free surface and a half-space, m is multiplicity of reflections from the half-space, β, α_1, α_2 are factors of reflection from the free surface and interface, φ_1, φ_2 are the phase shifts at supercritical reflection from the interface. For the free surface, the reflection factor for displacements is $\beta = -1$.

The reflection and phase shift factors are given by the following Fresnel formulas [4]:

$$\begin{cases} \alpha_i = \dfrac{\cos\theta_i - \sqrt{n^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n^2 - \sin^2\theta_i}}, \ \ \varphi_i = 0 \ \text{ for } \theta_i < \arcsin\dfrac{c}{c_1}, \\ \alpha_i = 1, \quad \varphi_i = -2\,\mathrm{arctg}\,\dfrac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i} \quad \text{for } \ \arcsin\dfrac{c}{c_1} < \theta_i < \dfrac{\pi}{2}, \\ \theta_i = \mathrm{arctg}\,\dfrac{R_{ix}}{R_{iy}}, \end{cases} \tag{4}$$

here $i = 1, 2$ is the subscript for different waves in (4.31).

3. Solution for wave field variations

It is necessary to know the solution in the area V to solve problem (2). If a radius of the spherical area of the parameter changes is much less than the distance to the vibrational source, $r_0 \leq R_0$, the spherical waves from (3) in this area can be approximately considered as locally plane ones. For this, in (3), in the denominators describing a spherical divergence of waves and in the formulas determining wave vectors, the radius-vector R_0 of the sphere center is used as radius-vector r. Thus, the solution $u(\vec{r}, t)$ to problem (3) is as follows:

$$u(\vec{r}, t) = \sum_{j=1}^{\infty} A_j \exp(i\vec{k}_j\vec{r} - i\omega t + i\psi_j) \quad \text{for} \quad \vec{r} \in V, \tag{5}$$

where A_i, $\vec{k}_1$, $\vec{k}_2$, ψ_i are the amplitudes of waves, wave vectors, and phases of the plane waves determined from (3).

Let us consider the solution of the first wave equation from (2) for the case when the displacement field in the right-hand side is a single plane wave propagating along the axis x. This solution is not equal to zero only in the spherical area V. Sources of such a type are called the traveling-wave antennas in the theory of antennas, and are used in hydro-acoustics and a radio-location. In our case, the area of parameter changes forms variations of the wave field as a 3D volume traveling-wave antenna.

$$\begin{cases} \left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \Delta\right)\delta u(\vec{r}, t) = -\dfrac{2\delta c(\vec{r})}{c^3}\dfrac{\partial^2 u(\vec{r}, t)}{\partial t^2}; \\ u(\vec{r}, t) = A_0 \exp(ikx - i\omega t); \\ \delta c(\vec{r}) \neq 0 \quad \text{for } \tilde{\mathrm{r}} \in \mathrm{V}. \end{cases} \tag{6}$$

The field in the direction under the angle θ with the axis X (a directional pattern) is determined as the integral of the elementary point sources field in the area V, with allowance for their amplitudes and phases at each point of this area. The field from an elementary volume $dx\,dy\,dz$ is given by the relation

$$\delta u = dx\,dy\,dz \frac{2\delta c\omega^2}{c^3} A_0 \exp\big(ikx - i\omega t - ik(x\cos\theta + z\sin\theta)\big), \tag{7}$$

where the term $k(x\cos\theta + z\sin\theta)$ takes into account the phase shift relative to the centre of the sphere.

In the case when the area V is a sphere $x^2 + y^2 + z^2 \leq r_0$, the integral determining the directional pattern $u(\theta)$ is as follows:

$$u(\vartheta) = \int_{-r_0}^{r_0} dx \int_{-\sqrt{r_0^2 - z^2}}^{\sqrt{r_0^2 - z^2}} dz \int_{-\sqrt{r_0^2 - x^2 - z^2}}^{\sqrt{r_0^2 - x^2 - z^2}} dy \exp\big(ikx - ik(x\cos\theta + z\sin\theta)\big)$$

$$= \int_{-r_0}^{r_0} dx \exp\big(ikx(1 - \cos\vartheta)\big) \int_{-\sqrt{a^2 - z^2}}^{\sqrt{a^2 - z^2}} 2\sqrt{a^2 - x^2 - z^2} \exp(ikz\sin\vartheta)\, dz$$

$$= 2 \int_{-r_0}^{r_0} dx \exp\big(ikx(1-\cos\vartheta)\big) \frac{\sqrt{r_0^2 - x^2}}{k \sin\vartheta} \pi J_1\Big(k\sqrt{r_0^2 - x^2} \sin\vartheta\Big)$$

$$= \frac{4\pi r_0^3}{\left(2kr_0 \sin\frac{\vartheta}{2}\right)^2} \left(\frac{\sin\left(2kr_0 \sin\frac{\vartheta}{2}\right)}{2kr_0 \sin\frac{\vartheta}{2}} - \cos\left(2kr_0 \sin\frac{\vartheta}{2}\right) \right). \tag{8}$$

Thus, the wave field variations formed by the spherical area in the case of a harmonic plane wave can be approximated at distances greater than the area size by the following expression:

$$\delta u(\vec{r}, t) = \frac{2\delta c\omega^2 A_0}{r \cdot c^3} \frac{4\pi r_0^3}{(2kr_0 \sin\frac{\vartheta}{2})^2} \left(\frac{\sin\left(2kr_0 \sin\frac{\vartheta}{2}\right)}{2kr_0 \sin\frac{\vartheta}{2}} - \cos\left(2kr_0 \sin\frac{\vartheta}{2}\right) \right) \times \exp(ikr - i\omega t), \tag{9}$$

where $\vec{r}$ is the coordinate of the point relative to the area center and θ is the angle between the vector $\vec{r}$ and the axis x.

In expression (9), the angle-dependent function determines a beam pattern of a virtual source determining the wave field variations. For the ratios $r/\lambda = 0.1$, a beam pattern is close to a circular one, which corresponds to the case of wave diffraction on a small inclusion. As the ratio r/λ increases, the property of directivity of a maximum of field variations along the passing wave ray manifests itself, which corresponds to the ray model.

It is of interest to consider some limiting cases of the solution to (9).

1. If the size of the area V is much less than a wavelength, the directional pattern will not depend on the angle, i.e., a small area of the parameter changes produces field variations as a point source. This is a well-known effect of diffraction with small inclusions:

$$\begin{cases} r_0 \to 0 \quad \text{or} \quad \theta \to 0, \\ \delta u(\vec{r}, t) \to \dfrac{2\delta c\omega^2 A_0}{rc^3} \dfrac{4\pi r_0^3}{3} \exp(ikr - i\omega t). \end{cases} \tag{10}$$

2. For an area of any finite size, the wave field variations along the axis x have a maximum value, which is given by the same relation (10). That is, along the axis x, the traveling-wave source radiates as a single point source with an intensity proportional to the volume. This is the basic property of the traveling-wave antennas.

3. In the case when the area size increases in comparison with a wavelength, the directional pattern is sharply directed. A maximum in the direction of the axis x remains unchanged. The radiation in all other directions decreases because of a large value of the denominator in formula (9).

As shown above, the wave field in the area of parameter changes can be considered as superposition of locally plane waves (3) and (5). For each plane wave, the field perturbation by a spherical area with a velocity change is determined by (9). Therefore, the total variations of the wave field will be produced by the area as sum of sources with their own amplitudes and directional patterns:

$$\delta u(\vec{r},t) = \sum_{j=1}^{\infty} A_j \frac{2\delta c\omega^2}{|\vec{r}-\vec{R}_0| r \cdot c^3} \frac{4\pi r_0^3}{\left(2k_j r_0 \sin\frac{\vartheta_j}{2}\right)^2} \times \left(\frac{\sin\left(2k_j r_0 \sin\frac{\vartheta_j}{2}\right)}{2k_j r_0 \sin\frac{\vartheta_j}{2}} - \cos\left(2k_j r_0 \sin\frac{\vartheta_j}{2}\right)\right) \times \exp\left(ik_j|\vec{r}-\vec{R}_0| - i\omega t + i\psi_j\right); \tag{11}$$

where A_j are the amplitudes of waves from (5), θ_j is the angle between the wave vector $\vec{k}_j$ and the vector of direction to the recording point from the sphere center $(\vec{r}-\vec{R}_0)$, and ψ_j are the initial phases of constituent waves from (5).

Solution (11) can be considered as first approximation of determination of variations of the stationary wave field caused by the presence of an area with small velocity changes in the layer. It takes into account the influence of all the waves passing from a vibrational source through the area of parameter changes. The field on the surface is determined by the choice of the vector $\vec{r}$ with the zero component z. The following approximation is made taking into account the presence of the free surface and the layer–half-space interface. This can be done in the above-mentioned ray approximation to calculate an unperturbed wave field.

The geometry change of the zone of wave field maximum variations at the surface depending on the location of the zone of change in the medium parameters, its dimensions, and wavelength of the harmonic signal was investigated in a series of calculations and analysis of solution (11). Numerical solutions show that one can easily trace the character of displacement of the zone of amplitudes variation maximum described above. This result is important when planning experiments on the monitoring of seismic-prone zones.

4. Analytical estimates of the monitoring method sensitivity with stationary wave fields

The above problem of determining variations in the parameters of the stationary wave field with changing characteristics in the medium internal zone makes it possible to obtain analytical estimates of sensitivity of the active monitoring method in the model considered. Such estimates can be obtained when the main factors determining the problem solution are taken into account. These include geometrical parameters of location of the source and receiver in the zone of changes and the allowance for the main waves with the greatest contribution to the wave field. The figure at page 106 shows the calculation scheme.

In experiments on the monitoring method, the wave field parameters at the surface at a distance $R_{V–S}$ from the source and their variations caused by changes in the medium are recorded. The greatest contribution to the recorded wave field in the model taken is from the direct wave, whose amplitude with allowance for the spherical divergence law can be estimated as

$$u = \frac{A_0}{R_{V–S}}, \tag{12}$$

where A_0 is a characteristic amplitude at the unit distance from the source and $R_{V–S}$ is a vibrator–seismometer distance.

Variations of the amplitudes of the recorded field at the surface are determined from (11). For a direct wave passing to the parameter changes zone in the direction of maximum variations, they can be estimated as

$$\delta u = \frac{4}{3}(2\pi)^3 \alpha \frac{A_0}{R_{V–Z}} \frac{r_0}{R_{Z–S}} \frac{\delta c}{c} \left(\frac{r_0}{\lambda}\right)^2, \tag{13}$$

where A_0 is a characteristic amplitude at a unit distance from the source, R_{V-Z} is a distance between the vibrator and the parameter variation zone, R_{Z-S} is a distance from the parameter variation zone to the recording point (seismometer), r_0 is a radius of the parameter variation zone, λ is a wavelength of the sounding signal, $\delta c/c$ are relative variations of wave velocities in the parameter variation zone, and α is a reflection factor lying within 0.15–1.0 for the model taken and the wave velocities in the core and the mantle.

The relation between relative variations of the velocity in the parameter variation zone and those of the amplitudes of the recorded signal is as follows:

$$\frac{\delta c}{c} = 3 \cdot 10^{-3} \alpha \frac{\delta u}{u} \left(\frac{R_{V–Z} R_{Z–S}}{R_{V–S} \cdot r_0}\right) \left(\frac{\lambda}{r_0}\right)^2. \tag{14}$$

One can see from (14) that the relative variations of the velocity in the parameter variation zone that can be determined by the active monitoring method with the use of harmonic signals are proportional to those of the recorded signal amplitudes. They are also proportional to the coefficient for the relation between the typical (source—receiver, source—zone of parameter variation, and the zone of parameter variation—receiver) distances, and the size of the parameter variation zone. In addition, they are proportional to the square of the relation between a sounding signal wavelength and a radius of the parameter variation zone and the reflection factor at the core-mantle boundary.

The estimate obtained (14) is the one of sensitivity of the active monitoring method with the use of harmonic signals and measurement of variations of the stationary wave field. It makes it possible to obtain numerical estimates for possible values of velocity variations in the zone of parameter variation, which can be determined from variations of the recorded amplitudes of oscillations.

Experience shows that variations in the amplitudes of harmonic signals at distances of 100–400 km from the vibrator at the existing microseismic noise level can be determined with an accuracy of 10^{-2}. Therefore, monitoring at the frequency $f = 6$ Hz (wavelength $\lambda = 1$ km) and typical source-recorder and source-zone of variation distances of 50–100 km, and for the zone of parameter variation with a radius of 1–10 km, gives the following estimates of possible determination of the relative variations in seismic wave velocities:

$$\begin{aligned} r_0 &= 1 \text{ km}, \quad \delta c/c = 10^{-2}\text{–}10^{-3}, \\ r_0 &= 10 \text{ km}, \quad \delta c/c = 10^{-5}\text{–}10^{-6}. \end{aligned} \tag{15}$$

The obtained estimates (15) show that the sensitivity of the active monitoring method is very high for seismologic methods. This proves its key role in methods for tracing changes in the stressed-deformed state of the medium in the earthquake preparation zone.

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