Bull. Nov. Comp. Center, Math. Model. in Geoph., 19 (2016), 11–16

A 2D numerical model of Novosibirsk reservoir flows using a mixed finite element method

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Abstract. The calculation of the Novosibirsk reservoir flows with the help of the scheme with two splitting steps and a finite element method (FEM) has been carried out. At these steps, for constructing the FEM operators different types of finite elements are used. In particular, at the step corresponding to the vorticity advection and diffusion, the non-conforming finite elements are used.

Introduction

The Novosibirsk reservoir is the largest artificial lake in West Siberia. It was created with the primary purpose of power generation but it has played and continues to play an important role in the regional economy in many other ways. Various studies of hydrological, hydrochemical and hydrobiological processes for the reservoir during the period of its formation and later have been conducted [1]. Also, the regular monitoring of the water quality is carried out. Complex ecological monitoring is very important because of a great role of the reservoir in the operational regulation of the region energy supply, in the operational regulation of very uneven throughout the year runoff of the Ob River, in the water supply for population, agriculture and industry of the Novosibirsk Region and Altai Territory. The reservoir is used for the commercial and recreational fishing; on its shores and water, the tourist and recreational activities are carried out.

Today, the development of a mathematical model is usually done for studying a water body, and hydrodynamic models are most important since the success gained in specific applications is directly associated with the quality of the water exchange structure simulation. However, a reliable and verified Novosibirsk reservoir simulation model that would provide the spatial detailing of the flows has not been created yet.

1. Statement of the problem

From the town of Kamen-na-Obi up to the hydroelectric power station (HPS) dam the Novosibirsk reservoir is rather a narrow elongated body of water of about 200 km long. The calculation was carried out for the lower part of the reservoir (from the village of Zavialovo to the HPS dam)

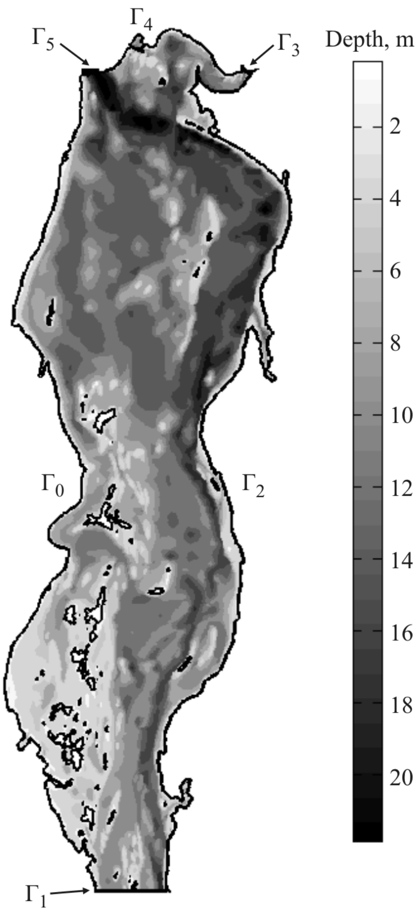


Figure 1. Novosibirsk reservoir bottom topography

which is a lake-like water body strictly stretched from south-west to north-east. There are numerous isles in the left part of the submerged floodplain. The main inflow of water (about 95 %) takes place from the Ob River. The inflow from the banks is less than 5 % of the whole volume of inflows and its greatest part is from the Berd River, passing into the Gulf of Berd.

Figure 1 represents the bottom topography built with a 120 m × 120 m grid on the basis of a large navigable map dated by 1978. The liquid boundaries are the cross-section at the village of Zavialovo (Γ_1), the section by the Berd highway of the Gulf of Berd (Γ_3) and the HPS (Γ_5). In the future, there is an intention to take into consideration the overflow dam with additional water discharges in the course of flooding.

During the winter and spring seasons the prevailing wind direction is south-west with the average speed 5.2 m/s above the reservoir. In May, June and September the prevailing wind direction is south-west, in July and August it is from the north. The monthly average wind speed above the water surface is 4.3–5.3 m/s.

The stormiest month is October. From September to October the wind repeatability with the speed exceeding 8 m/s increases almost by 10 %. There are south-west winds with the speed 25–30 m/s and, sometimes, nearly 40 m/s. Figure 2 represents the wind direction and the wind speed repeatability based on the information about the wind recurrence during the period when the Novosibirsk reservoir is ice-free, i.e. from the 15th of April to the 15th of November, for the years of 1990–2010 (Obskaya Hydrometeostation [1]).

Currently, there are available calculations only for different wind directions with the model front defined by sinus function, but in the near future the simulation with real-time wind changes based on statistics will be developed.

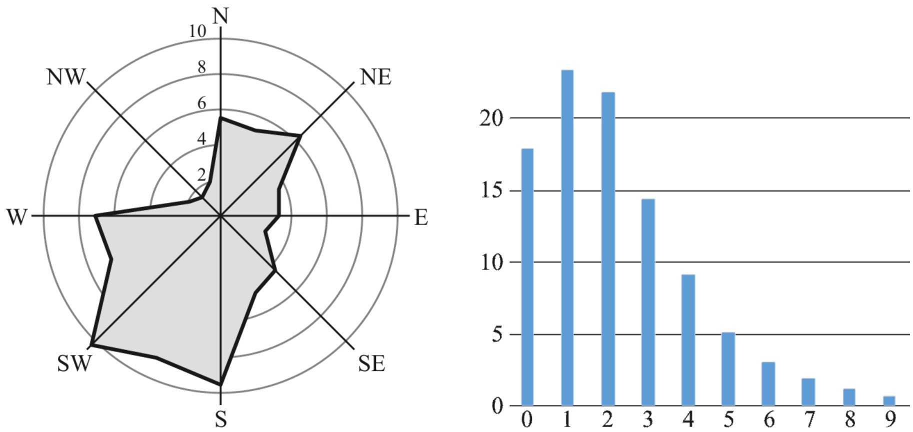


Figure 2. Wind direction and speed repeatability (%) during the ice-free period

In order to determine the velocity field (u, v), a 2D nonlinear vorticity equation is used. The solid boundary conditions are impermeability and slipping; on the liquid boundaries the stream function value is calculated from a predetermined flow rate:

$$u = -\frac{1}{H}\Psi_y, \quad v = \frac{1}{H}\Psi_x.$$

$$\frac{\partial}{\partial t}(\Delta_H \Psi) - \mathrm{rot}\Big(\frac{1}{H}\Delta_H\Psi\,\nabla\Psi\Big) - \mathrm{rot}\Big(\frac{f}{H}\,\nabla\Psi\Big) + R\Delta_H\Psi - \mu\Delta\Delta_H\Psi = F, \tag{1}$$

$$\Delta_H\Psi = \Big(\frac{1}{H}\Psi_x\Big)_x + \Big(\frac{1}{H}\Psi_y\Big)_y, \quad F = \frac{1}{\rho_0}\,\mathrm{rot}\,\frac{\tau}{H};$$

$$\Psi|_{\Gamma_0} = 0, \quad \frac{\partial\Psi}{\partial N}\Big|_{\Gamma_2,\Gamma_4} = 0, \quad \frac{\partial\Psi}{\partial l}\Big|_{\Gamma_2,\Gamma_4} = 0, \quad \int_{\Gamma_i}\frac{\partial\Psi}{\partial N}\,dl = Q_i, \;\; i = 1, 3, 5;$$

$$\Delta_H\Psi|_{\partial\Omega} = 0, \quad \Psi|_{t=0} = 0.$$

Here Ψ is the stream function, $H(x, y) \geq H_0 > 0$ is the depth, f is the Coriolis parameter, p is the pressure, $\rho = \rho_0 = \mathrm{const}$ is the density, μ is the horizontal turbulent viscosity, R is the bottom friction coefficient, and τ is the wind friction tension.

In terms of the vorticity $\zeta = \Delta_H\Psi$, equation (1) can be rewritten in the form:

$$\zeta_t - \mathrm{rot}\Big(\frac{1}{H}\zeta\nabla\Psi\Big) - \mathrm{rot}\Big(\frac{f}{H}\nabla\Psi\Big) + R\zeta - \mu\Delta\zeta = F, \tag{2}$$

$$\zeta|_{\partial\Omega} = 0, \quad \zeta|_{t=0} = \Delta_H\Psi^0(x, y).$$

2. Construction of schemes

In this paper, we employ a scheme for which the splitting combined with a finite element method is used. In this case, splitting is carried out at different steps of constructing a numerical model including both splitting in terms of physical processes allowing linearization of the initial problem and further splitting with respect to time of one of the FEM operators obtained. For $t \in [t_n, t_{n+1}]$, $n = 0, \dots, N_t - 1$:

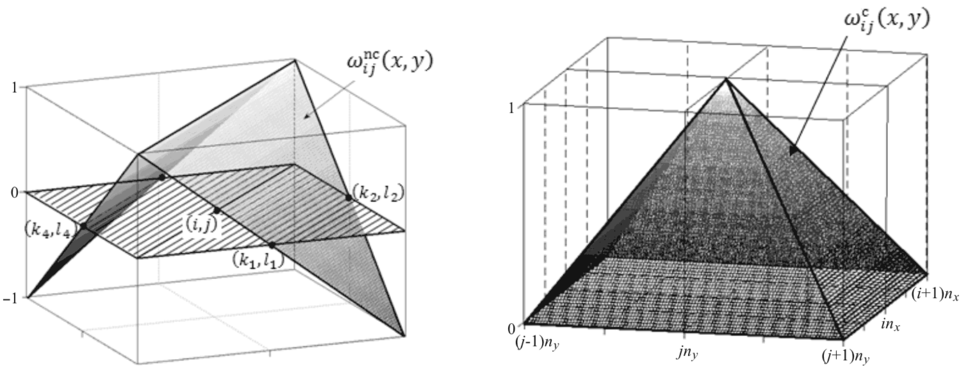
$$\zeta_t - \operatorname{rot}\Big(\frac{1}{H}\zeta \cdot \nabla\Psi|_{t=t_n}\Big) - \mu\Delta\zeta = 0, \tag{3}$$
$$\zeta|_{\partial\Omega} = 0, \quad \zeta|_{t=t_n} = \Delta_H \Psi|_{t=t_n};$$

$$(\Delta_H \Psi)_t - \operatorname{rot}\Big(\frac{f}{H}\nabla\Psi\Big) + R\Delta_H\Psi = F, \tag{4}$$
$$\Psi|_{\Gamma_0} = 0, \quad \frac{\partial\Psi}{\partial N}\Big|_{\Gamma_2,\Gamma_4} = 0, \quad \frac{\partial\Psi}{\partial l}\Big|_{\Gamma_2,\Gamma_4} = 0,$$
$$\int_{\Gamma_i} \frac{\partial\Psi}{\partial N}\, dl = Q_i, \;\; i = 1, 3, 5, \quad \Delta_H\Psi|_{t=t_n} = \zeta|_{t=t_{n+1}}.$$

At the first step (3) corresponding to the vorticity advection and diffusion the non-conforming finite elements (Figure 3a) are used. These are piecewise linear functions determined by values at midpoints of sides of triangles in the following way:

$$\omega_{ij}^{\mathrm{nc}}(x_k, y_l) = \begin{cases} 1, & (k,l) = (i,j), \\ 0, & \text{otherwise.} \end{cases}$$

Here (x_k, y_l) is a midpoint of a side of a certain triangle of the grid. Finite elements of such a type were introduced in [2] for solving the stationary Stokes



a b

Figure 3. The non-conforming (a) and conforming (b) basis functions

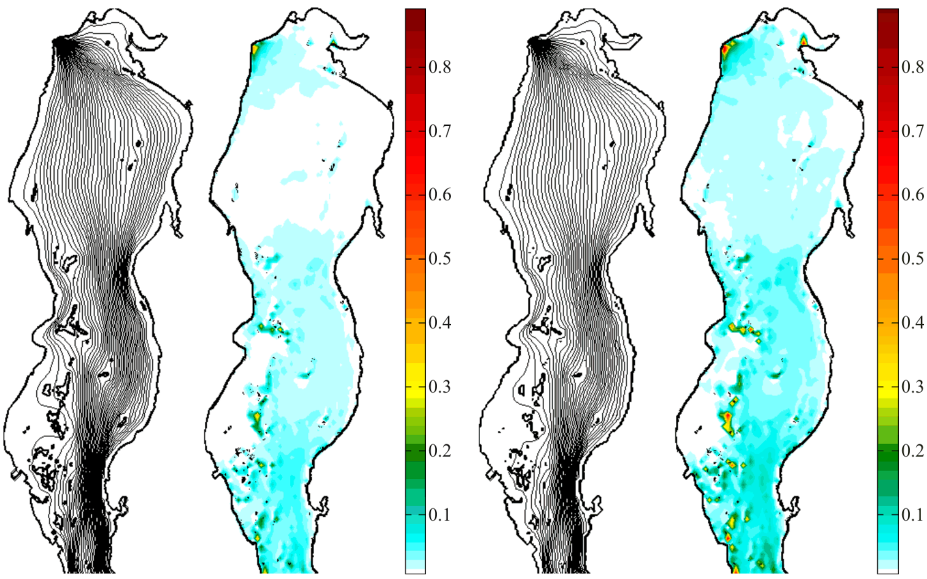
equations. Later these elements were used in [3] for obtaining a noise-free scheme for the two-layer shallow water equations. Because of orthogonality we can avoid the lumping procedure when splitting the FEM operator with respect to time. Also, in comparison with the case of conforming finite elements, a smaller number of grid points is used in the FEM scheme obtained [4].

At the second step (4), for solving the linear stream function equation, the conforming piecewise linear finite elements determined by values at vertices of triangles (see Figure 3b) are used. Hence, it appears possible to essentially reduce the number of grid points in the numerical scheme when passing from one splitting step to another.

The detailed description of the model is presented in [5].

3. Test results

Figure 4 represents the results obtained for the south-east wind with the speed 4 m/s and for the weak and the strong inflows. The increment between isolines of the stream function $\Delta\Psi$ is equal to 50 m^3/s for version (a) and 125 m^3/s for version (b). The flow along the flooded old river channel, which is specific for the Novosibirsk reservoir, is clearly seen. In the wide part and near to the left bank of the reservoir the flows are comparably moderate, that is, the speed is about 0.1 m/s.



a) $Q_1 = 2000$ m^3/s, $Q_3 = 15.3$ m^3/s b) $Q_1 = 4300$ m^3/s, $Q_3 = 146$ m^3/s

Figure 4. The isolines of the stream function and distribution of the velocity module (m/s) for weak (a) and strong (b) inflows

Currently, the model of pollution transfer is being developed, for which the calculated stream function involved in input data is used. This model is based on the monotonous scheme built with the help of the non-conforming finite element method applied to the advection–diffusion equation [4]:

$$C_t + uC_x + vC_y + \mu\Delta C = f,$$
$$C|_{\Gamma_i} = C_i(x), \;\; i = 1, 3, \quad \frac{\partial C}{\partial N}\Big|_{\Gamma_5} = 0.$$

Here C is the pollution concentration, f is the function describing the intensity and location of the pollution, and C_i are certain default values of the pollution concentration.

Conclusion

The flows of the Novosibirsk reservoir are defined with the help of the scheme based on the splitting in terms of physical processes and with respect to time and on a finite element method as applied to a 2D nonlinear vorticity equation. The results obtained correlate well with available data and can be used for different practical applications, for example, for the pollution transfer simulation.

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