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Realization and testing a wave analog of the common depth point method on synthetic and field data*

A.N. Kremlev, V.I. Priimenko, S.A.M. Oliveira, R.M. Missagia

Realization of the Wave Analog of the Common Depth Point Method (WCDP) is described. This method is one of numerous employed in seismic data processing and is based on the "rigorous" mathematical solution of the inverse scattering problem in linear approximation by multiple overlapping data. The WCDP method is tested on synthetic data corresponding to typical geological objects (inclined reflecting areas, point reflectors, salt-dome model) and field data (profile with a wave field anomaly in the Deryugin basin in the Okhotsk See). The results have shown a high quality of the WCDP profiles and stability of this method to the choice of a wave velocity model.

1. Introduction

Presently, the Common Depth Point (CDP) method [1, 2] and different modifications of the wave migration method are the basic ones for multichannel seismic data processing [3, 4]. The WCDP method differs from other migration methods by the fact that it is based on the rigorous mathematical solution of the inverse scattering problem of acoustic waves in linear approximation by multichannel overlapping data. Such an approach enables us to take into account all wave processes of reflection and diffraction of seismic waves. Using the multifold input data in the algorithm increases the signal/noise ratio in the resulting stack and allows, like the CDP method, making wave velocity analysis of a medium.

In our work, we describe the realization of the WCDP method and present some results of its testing on synthetic and field data.

2. Statement of the problem and summation formula

The inverse problem of acoustic waves diffraction consists in determination of the function $a(x, z)$, describing heterogeneities of a medium by the wave field $u(x, x_0, t)$, recorded for different positions of the source (x_0) and the

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receiver (x) located on the surface. This wave field satisfies the Cauchy problem [5]

$$\frac{\partial^2 u}{\partial t^2} = c^2[1 + a(\vec{x})]\nabla^2 u + \delta(t)\delta(\vec{r} - \vec{r}_0), \qquad u|_{t<0} \equiv 0, \tag{1}$$

where c is the background velocity of acoustic waves, $(\vec{r}, \vec{r}_0, t) \in R^2 \times R^2 \times R$, $\vec{r} = (x, z)$, $\vec{r}_0 = (x_0, z_0)$. The result of the problem solution in linear approximation is the following focusing operator [6–8]

$$\alpha(\vec{r}) = \int \frac{d\omega}{2\pi}\frac{d\chi}{2\pi}\frac{d\chi_0}{2\pi} e^{i(\chi+\chi_0)x}\Phi_v(z, \chi, \chi_0, \omega)\hat{u}(\chi, \chi_0, \omega) + \text{c.c.}, \tag{2}$$

which allows us to calculate the visualization function $\alpha(\vec{r})$ being the local average of the required function $a(\vec{r})$ over the domain with the size on the order of a sounding signal wavelength. In formula (2), the function $\hat{u}(\chi, \chi_0, \omega)$ is the spectrum of a recorded field,

$$\begin{aligned}\Phi_v(z, \chi, \chi_0, \omega) \; = \; & \Theta\Big(\frac{\omega^2}{v^2} - \chi^2\Big)\Theta\Big(\frac{\omega^2}{v^2} - \chi_0^2\Big)\frac{v}{\omega} \times \\ & \left(\frac{\omega^2}{v^2} + \chi\chi_0 + \Big(\frac{\omega^2}{v^2} - \chi^2\Big)^{1/2}\Big(\frac{\omega^2}{v^2} - \chi_0^2\Big)^{1/2}\right)^{1/2} \times \\ & \exp\left\{-iz\left(\Big(\frac{\omega^2}{v^2} - \chi^2\Big)^{1/2} + \Big(\frac{\omega^2}{v^2} - \chi_0^2\Big)^{1/2}\right)\right\}\end{aligned} \tag{3}$$

is the kernel of the focusing operator. Here $\Theta(\cdot)$ is the Heavyside function and v is *a priori* stacking velocity, the symbol "c.c." means the complex conjugated term to the previous one. The exponential factor in formula (3) provides a phase shift corresponding to the wave propagation time and similar to the phase shift in coordinates of a source and receiver in migration by Gazdag [9] and Stolt [10], but at the same time their multipliers are the result of exact problem solution.

The focusing operator (2)–(3) is a basis of the WCDP method.

3. Summation formula in CMP-offset coordinates

In order to realize the WCDP method in stacking formula (2)–(3), let us turn to the "Common Mid-Point offset" coordinates:

$$m = (x + x_0)/2, \qquad l = x - x_0. \tag{4}$$

Let us denote by μ and ν the frequency variables corresponding to m and l. Because of invariance of the wave phase we have $\chi x + \chi_0 x_0 = \mu m + \nu l$. Using relations (4) we obtain

$$\chi = \mu/2 + \nu, \qquad \chi_0 = \mu/2 - \nu. \tag{5}$$

Let $U(m, l, t)$ be a wave field in (m, l)-coordinates and $\hat{U}(\mu, \nu, \omega)$ be its spectrum. From the obvious equality $u(x, x_0, t) = U(m, l, t)$ and formulas (4) we arrive at

$$\begin{aligned} u(x, x_0, t) &= U((x + x_0)/2, x - x_0, t), \\ U(m, l, t) &= u(m + l/2, m - l/2, t). \end{aligned} \tag{6}$$

From the Fourier integral

$$\hat{u}(\chi, \chi_0, \omega) = \int dx\, dx_0\, e^{-i(\chi x + \chi_0 x_0)} \cdot u(x, x_0, \omega) \tag{7}$$

and using formulas (4), (6) we come to

$$\begin{aligned} \hat{u}(\chi, \chi_0, \omega) &= \hat{U}(\chi + \chi_0, (\chi - \chi_0)/2, \omega), \\ \hat{U}(\mu, \nu, \omega) &= \hat{u}(\mu/2 + \nu, \mu/2 - \nu, \omega). \end{aligned} \tag{8}$$

By introducing the new variable

$$q = v\left(\left(\frac{\omega^2}{v^2} - \chi^2\right)^{1/2} + \left(\frac{\omega^2}{v^2} - \chi_0^2\right)^{1/2}\right) \tag{9}$$

and using formulas (8), it is possible to obtain the final variant of the summation formula (2)–(3):

$$\alpha_v(m, t) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} e^{-iqt} \int_{-\infty}^{\infty} \frac{d\mu}{2\pi} e^{i\mu m} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \tilde{\Phi}_v(m, t, \mu, \nu, q). \tag{10}$$

Here the kernel $\tilde{\Phi}_v$ is defined by the formula

$$\tilde{\Phi}_v(m, t, \mu, \nu, q) = \hat{\Phi}(vt, \chi, \chi_0, \omega), \tag{11}$$

where $\chi = \mu/2 + \nu$, $\chi_0 = \mu/2 - \nu$, and $\omega = \omega(q)$ is a solution to equation (9). For the numerical construction of the WCDP profile by formulas (2), (3) or (10), (11), the corresponding integrals are changed by the integral sums with finite summation limits both with respect to the time variable and with respect to the spatial coordinates. There are two important advantages of formulas (10), (11).

First, summations with respect to the time frequency q and the spatial frequency μ have the forms of Fourier sums, and we can use the efficient fast Fourier transform (FFT) for their calculations.

The second difference is associated with the choice of aperture summation. Figure 1 represents a generalized seismic plane.

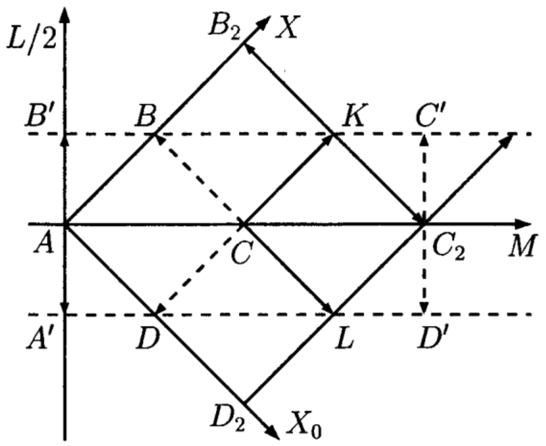

Figure 1. The WCDP aperture

Each seismic trace of a 2D profile is characterized by the coordinates (x, x_0) or (m, l), and can be represented as a point on the plane. Moreover, all the traces are located within the band $|l| \leq l_{\max}$, where $l_{\max}$ is the largest distance between the source and the receiver.

The square $ABCD$ in Figure 1 corresponds to the aperture in (x, x_0)-coordinates, its diagonal AC forming a summation basis. The choice of a summation basis is determined by depth of disposition and inclination of reconstructed boundaries. The size of this basis is also constrained by the volume of computer memory, which is connected with a possibility of efficient reconstruction of a spatial spectrum of a wave field with the help of the FFT algorithm.

Increasing the summation basis in (x, x_0)-coordinates results in appearance in the aperture of the domains (the triangles BB_2K and DD_2L), having no real seismic traces. For using the FFT algorithm we need to fill in these domains with zero traces. This brings about an increase in the volume of calculations but does not increase the volume of information about the profile under study.

In the "CMP-offset" coordinates, the aperture of summation is a rectangle, whose summation basis is parallel to Om-axes. The rectangle $A'B'C'D'$ corresponds to the same basis of summation AC_2, shown in Figure 1. Such a rectangle has only real recorded seismic traces or traces to be reconstructed from them by the reciprocity theorem.

4. Testing the WCDP method on synthetic data

Inclined reflected areas and point diffractors are typical subjects in seismic prospecting and their reconstruction determinates the efficiency of seismic data processing.

In Figure 2, the reconstruction results of inclined reflected surfaces under different inclination angles for the cases when (a) the migration velocity v coincides with the background velocity and when (b) these velocities are differ by 25% are presented. It is necessary to mark that in spite of consid-

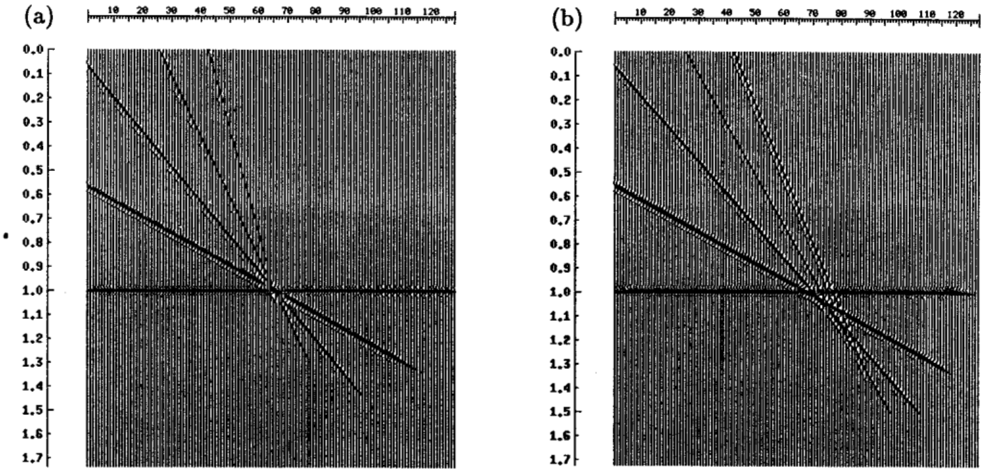

Figure 2. Dipped reflectors reconstruction (angles of 0, 15, 30, 45, and 60 degrees): (a) $V_{\text{media}} = V_{\text{migration}} = 2000$ m/s; (b) $V_{\text{media}} = 2000$ m/s, $V_{\text{migration}} = 1500$ m/s

erable difference between these velocities in the case (b) we have a stable reconstruction of inclined boundaries but with some displacement from their real position corresponding to such a difference.

Figure 3 presents the results of reconstruction of point reflectors located at different depths and different positions with respect to the summation basis. The high quality of reconstruction of such subjects should be noted, their reconstruction precision being close to a theoretical limit defined by the sounding wavelength.

The model of a salt-dome structure shown in Figure 4 is typical for many coastal sea-bed areas of the Atlantic Ocean. Synthetic data for such a model were calculated for 521 explosion points by solution to the acoustic equation with variable velocity and medium density with the help of the

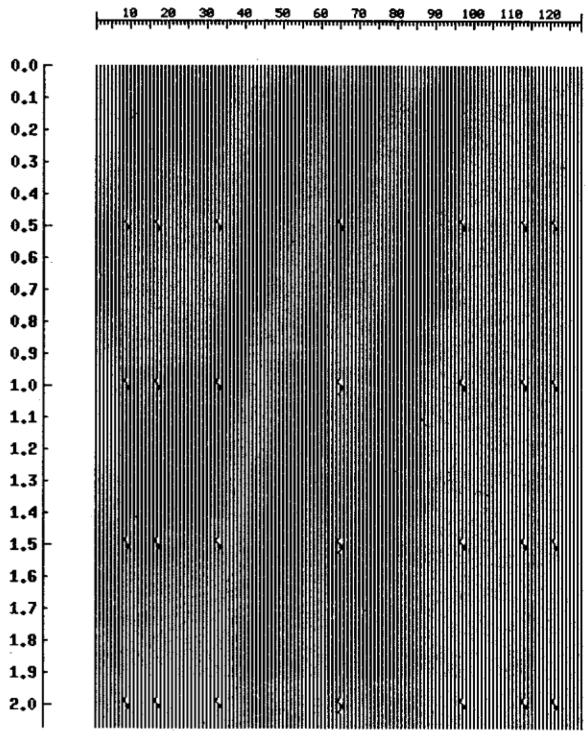

Figure 3. Diffractors reconstruction

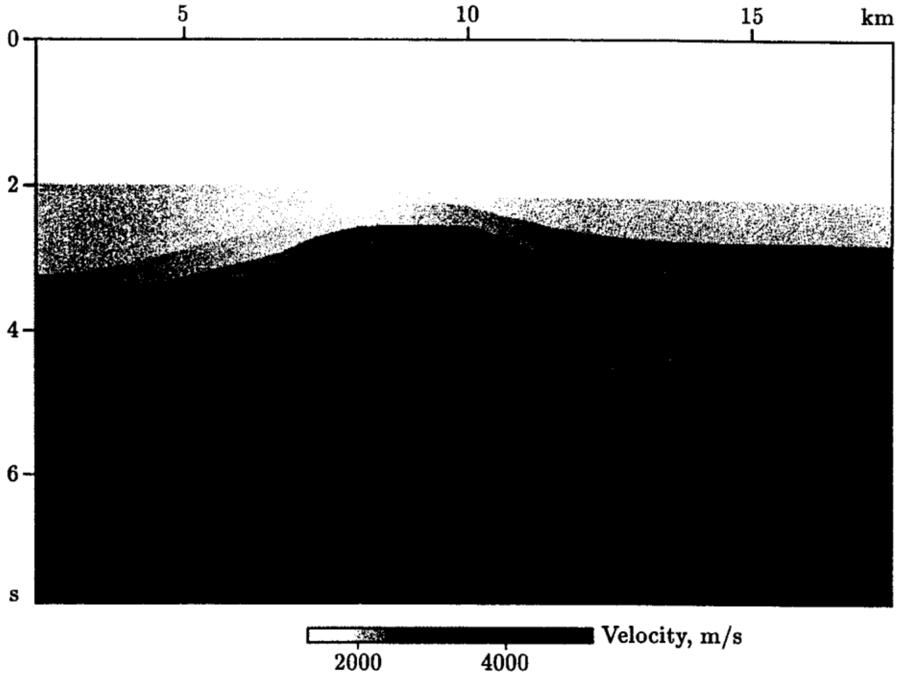

Figure 4. Model

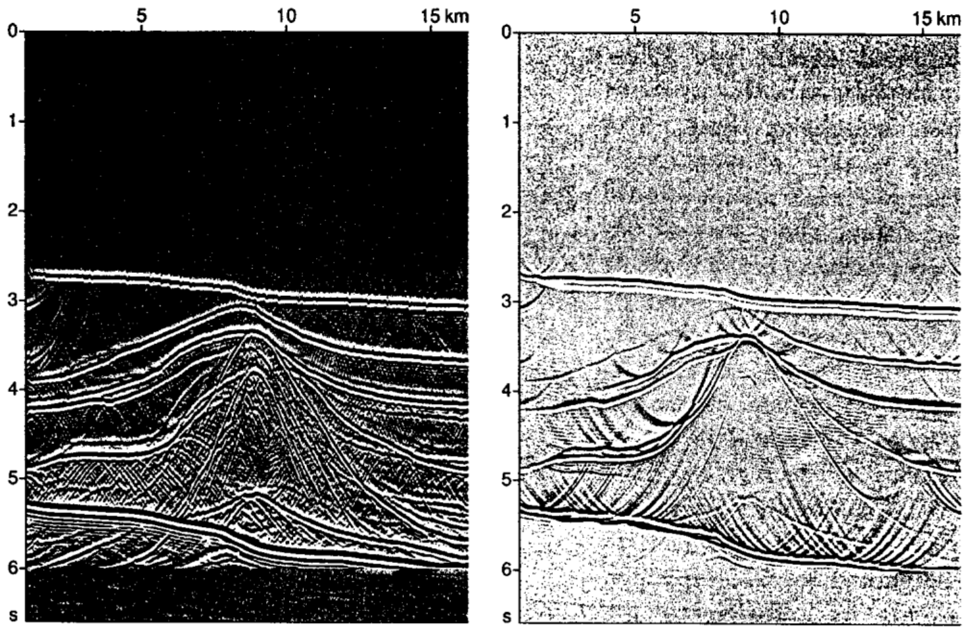

Figure 5. The WCDP reconstruction:
$V_{\text{migration}} = 1500$ m/s (left), $V_{\text{migration}} = 1600$ m/s (right)

finite difference approach described in [11]. The distance between explosion points and receivers was equal to 30 m, the number of sources being equal to 96. The results of data processing are shown in Figure 5 for the summation velocities of 1500 and 1600 m/s. All the reflecting surfaces including inclined and subsalt reflectors are well reconstructed. It is necessary to mark that dislocation of a subsalt reflector with respect to its real disposition is connected with a strong literal velocity variation in the upper layers that does not taken into account in time migration. In addition, a strong multiple reflection is observed near to this boundary indicating to the necessity of preliminary data processing for its removal.

5. Testing the WCDP method on real data

There are shown fragments of the WCDP and the CDP time profiles situated in the southern part of the Deryugin cavity. The wave field on the CDP profile (Figure 6(a), the upper part) is one of the noise-type kind called "gaping zone" [12].

Processing of such a profile by the WCDP method enables us to understand the structure of offshore sediments in that area and to discover the nature of the CDP wave field anomaly. It is shown in Figure 6(b) that the reflecting boundaries in the upper part of profile are divided into the elements having different orientation and separated one from another by depth dislocations. Such an acoustic heterogeneity of the medium brings about the multiple seismic waves diffraction. Diffracted waves are "spread" in summation by the CDP method resulting in the noise-type profile. Processing by the WCDP method focuses such waves ensuring that we can obtain more information about the profile under study. The attention should be given to the system of faults in the upper part of the WCDP profile and to the faults located on the time 3.2 s in its left part being absent on the CDP profile.

6. Conclusion

The use of the strict solution of the inverse acoustic problem in linear approximation with multifold data enables us (1) to take into account all wave singularities of seismic waves reflections and diffractions in the most useful way, and (2) to ensure useful signal accumulation. The realization of the WCDP method and its testing on synthetic and field data show a high quality of reconstructed stacking sections and good stability of the method of choosing *a priori* velocity model. Conservation of real amplitudes and undistorted wavelets on the WCDP stacks holds the greatest promise for their use in investigation of dissipating properties of geological media.

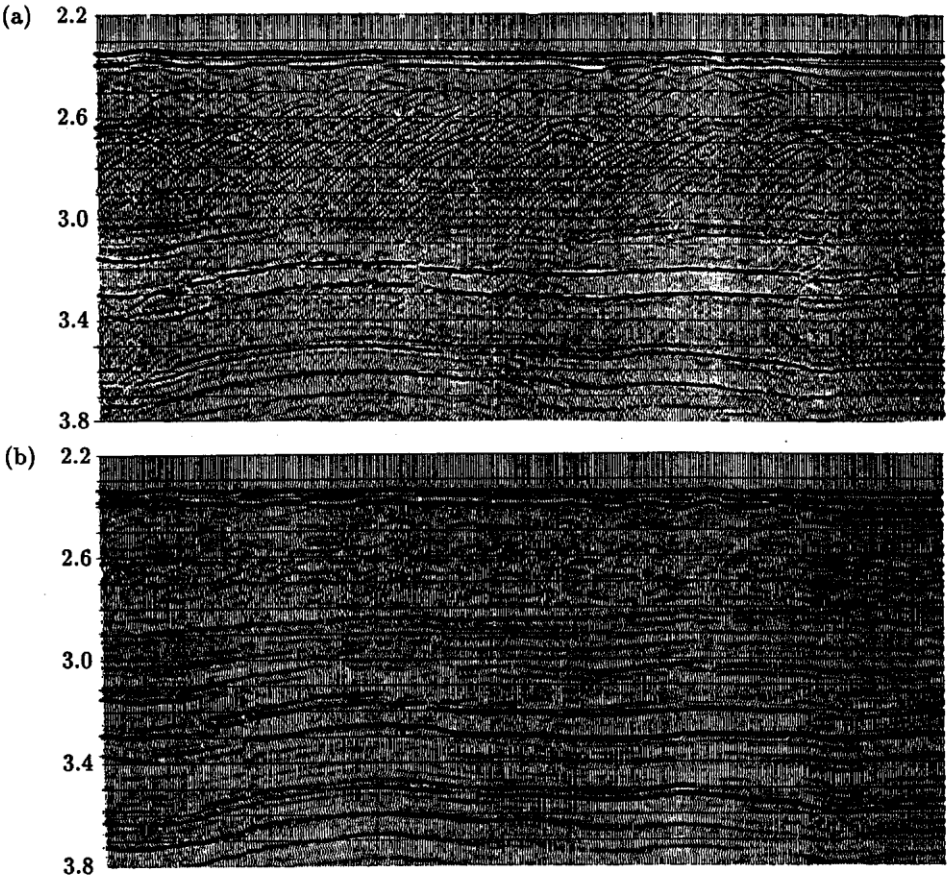

Figure 6. The Deryugin Basin of the Okhotsk Sea:
(a) the CDP stack and (b) the WCDP stack

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