Bull. Nov. Comp. Center, Comp. Science, 44 (2020), 1–8

Calculation of a uniform gas flow from the heated tungsten plate surface*

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Abstract. At INP SB RAS, various mechanisms for the erosion of tungsten samples during the pulsed heating were studied. Data were obtained that made it possible to analyze the experimental results of the surface temperature dynamics taking into account the cooling due to the vaporation in vacuum. A numerical model was developed for the effect of a powerful pulsed electron beam on tungsten, which includes the Stefan problem for determining the temperature distribution in a sample and the system of gas dynamics equations. The gas dynamics system is implemented by the two methods of first order of accuracy: the upwind scheme and Belotserkovsky's coarse particle method. For the program verification, a test solution of the gas decay was used. In the process of calculating the problem in a complete statement, an analysis of the residual was carried out.

Introduction

At the BETA experimental bench created in the INP SB RAS, the results of heating a tungsten target with a powerful submillisecond electron beam were obtained [1]. Mathematical modeling of the tungsten erosion will give required and important results for the development of the ITER and other experimental thermonuclear reactors [2]. A diverter is needed to remove the outer layers of the plasma cord. Part of the particles from the walls of the vacuum chamber inevitably falls into the cord. This is undesirable for the two reasons. The plasma is cooled by the emission of impurities, and the reactor wall is overheated by additional radiation. It is necessary to show that this material absorbs the heat well, it is insignifcantly sprayed by particles from the plasma, it accumulates some hydrogen, does not break down mechanically, does not melt and does not spray when the pulsed effects of powerful particles and energy flows are expected in a tokamak. Mathematical modeling, as replacement for an expensive physical experiment with a computational experiment, is especially relevant for studying the thermophysical properties of materials in extreme conditions. The tungsten evaporation model is based on solving the gas dynamics equations with complex

*Model development was funded by RFBR according to the research project No. 19-01-00422. The reported study (computational experiment) was carried out under state contract with ICMMG SB RAS (0315-2019-0001).

boundary conditions. This paper deals with the model calculation of the gas flow from the sample surface using two finite difference schemes.

1. Problem definition

A tungsten plate located in vacuum is heated by a powerful laser pulse. Evaporation of metal begins at temperatures above 4,000 degrees Kelvin. A one-dimensional gas flow is simulated from the right boundary with a given linear temperature increase, a given density and velocity flow — to the left boundary, where the gas freely exits. The mathematical model of the gas flow from the sample surface is based on the solution of the system of gas dynamics equations [5]:

$$\begin{aligned} &\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \\ &\frac{\partial (\rho u)}{\partial t} + \frac{\partial (u(\rho u))}{\partial x} + \frac{\partial P}{\partial x} = 0, \\ &\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + \frac{2T}{3}\frac{\partial u}{\partial x} = 0, \\ &u = \frac{(\rho u)}{\rho}, \quad P = \frac{R}{M}\rho T, \end{aligned} \tag{1}$$

where ρ is the gas density, u is the gas velocity, P is the gas pressure, T is the temperature, R is the gas constant, M is the molar mass. As an equation of state, the relation for an ideal gas is chosen.

This system of equations can be rewritten in the divergent form. The equation for temperature was replaced with the equation

$$\begin{aligned} &\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \\ &\frac{\partial (\rho u)}{\partial t} + \frac{\partial (u(\rho u) + P)}{\partial x} = 0, \\ &\frac{\partial \rho E}{\partial t} + \frac{\partial (u((\rho u) + P))}{\partial x} = 0, \\ &\rho T = \frac{2}{3}\Big(\frac{R}{M}\Big)^{-1}\Big((\rho E) - \frac{\rho u^2}{2}\Big), \\ &u = \frac{(\rho u)}{\rho}, \quad T = \frac{(\rho T)}{\rho}, \quad P = \frac{R}{M}(\rho T). \end{aligned} \tag{2}$$

Systems (1) and (2) of equations are equivalent. The internal energy ε according to the Mendeleev–Clapeyron law for a monatomic ideal gas is representable in the form $\varepsilon = \frac{2}{3}\frac{M}{R}T$.

To solve the difference problem, we turn to dimensionless quantities. The numerical values for system (1) of the parameters are given in the table:

Parameter	Typical value	Units
r_0	10^{-1}	mm
t_0	10^{-4}	μs
T_0	10^3	K
R	$8.31 \cdot 10^{-6}$	$\text{mm}^2 \cdot \text{kg}/\mu\text{s}^2 \cdot \text{mol} \cdot \text{K}$
M	0.18384	kg/mol
a_1	26.191	
a_2	8.39713	K
P_0	10^{-9}	$\text{kg}/\text{mm} \cdot \mu\text{s}^2$

2. Numerical simulation

We use a uniform rectangular grid for the spatial variables (r_i, z_k). The required flow parameters are comparable to mesh analogues. The required flow parameters ρ, ρu, u, ρE, ρT, P, T are comparable to the mesh analogues:

$$f_i^n = f(x_i, t^n): \ \rho_i^n, \ (\rho E)_i^n, \ (\rho T)_i^n, \ P_i^n, \ T_i^n;$$
$$f_i^n = f(x_{i+\frac{1}{2}}, t^n): \ (\rho u)_i^n, \ u_i^n;$$
$$i = 2, \ldots, N_x - 1, \quad n = 0, \ldots, N^t - 1.$$

To carry out the model calculations, we assume that the temperature of the sample linearly increases. The initial data used are the following:

$$P_i^0 = 10^{-4}, \quad T_i^0 = 0.3, \quad \rho_i^0 = \frac{R}{M}\frac{P_i^0}{T_i^0},$$
$$(\rho E)_i^0 = \frac{3}{2}\frac{R^2}{M^2}P_i^0, \quad (\rho T)_i^0 = \frac{R}{M}P_i^0, \quad (\rho u)_i^0 = 0, \quad u_i^0 = 0.$$

As a result of the computational experiments [3, 4], two satisfactory types of boundary conditions for the rate of emission of tungsten vapor were obtained: the Dirichlet condition with the use of the estimate

$$u|_\gamma = \sqrt{\frac{5}{3}\frac{R}{M}}\sqrt{T|_\gamma}$$

or the Neumann homogeneous condition giving a rougher estimate. For the density, it is necessary to use the Dirichlet condition with the use of the estimate

$$\rho|_\gamma = \frac{1}{2}\Big(\frac{R}{M}\Big)^{-1}\frac{1}{T|_\gamma}\exp\Big(a_1 - \frac{a_2}{T|_\gamma}\Big).$$

For the solution, we will use the upwind scheme [6] for system (1) and Belotserkovsky's coarse particle method [7] to solve the divergent system (2). A simplest stable scheme of the first order of accuracy is a scheme with directional differences depending on the speed sign:

$$f_i^{n+1} = f_i^n - \frac{\tau}{h}\Big(u^+(f_i^n - f_{i-1}) - u^-(f_{i+1}^n - f_i^n) - \frac{a}{2} f_i^n (u_{i+1}^n - u_{i-1}^n) - \frac{1}{2}(p_{i+1}^n - p_{i-1}^n)\Big),$$

$f_i^n = (\rho_i^n, \rho u_i^n, T_i^n)$, $\quad p_i^n = (0, P_i^n, 0)$, $\quad a = (0, 0, 2/3)$, $\quad u^{\pm} = (u_i^n \pm |u_i^n|)/2$.

To solve the equation in the divergent form (3), we will use a coarse particle method. According to this method, the initial system of the gas dynamics equations can be divided into two stages based on splitting to physical processes. At the Euler phase, gas is considered to be stationary. The system of equations of the Euler phase is obtained from the original system of equations if in them the divergent terms of the mass flux density, momentum component, and total energy are omitted. This system of equations describes the process of changing the gas parameters in an arbitrary flow domain due to the action of the pressure forces, as well as due to the potential difference:

$$\frac{\partial \rho}{\partial t} = 0, \quad \frac{\partial (\rho u)}{\partial t} + \frac{\partial P}{\partial x} = 0, \quad \frac{\partial \rho E}{\partial t} + \frac{\partial P u}{\partial x} = 0.$$

The system of equations of the Lagrangian phase contains divergent terms and is responsible for the advective transfer of the gas-dynamic quantities. At the Lagrangian phase of the scheme, the transport effects are calculated that take into account the exchange between cells when they are re-arranged to the previous Eulerian grid. For writing the finite difference scheme of the Eulerian phase, we can use the central differences or linearize the equations of the Eulerian phase and approximate the derivatives using the pressure and velocity values obtained from the exact solution of the linearized problem.

In the Lagrange phase, gas is transferred to the neighboring cell with flows. The part moved in the directions x of the physical quantity can be written as $\left|\frac{\tau u}{h}\right|$. The mass flow is determined by the formulas of the first order of accuracy

$$\Delta M_{i+1/2}^n = \begin{cases} \tilde{f}_i^n \dfrac{u_i^n + u_{i+1}^n}{2} h\tau, & u_i^n + u_{i+1}^n > 0; \\ \tilde{f}_{i+1}^n \dfrac{u_i^n + u_{i+1}^n}{2} h\tau, & u_i^n + u_{i+1}^n < 0. \end{cases}$$

During the time τ, we calculate the mass flows ΔM^n through the boundaries of the Euler cells. A fraction moved along the axis of the physical quantity $f_i^{n+1} = (\rho_i^{n+1}, (\rho u)_i^{n+1}, (\rho E)_i^{n+1})$ can be written down as

$$f_i^{n+1} h^2 = \tilde{f}_i^n h^2 - \Delta M_{i+1/2}^n + \Delta M_{i-1/2}^n.$$

In one time step, the stored values are transferred to no more than only one cell. At each time step, the solution of the equations of continuity,

motion, and total energy is reduced to the consistent implementation of the Euler and Lagrangian phases. As the initial condition for the Eulerian phase, the value of functions from the previous time is taken, for the Lagrangian phase, the initial condition is the solution from the Eulerian phase. A modification of this method allows one to accurately describe the expansion of gas to vacuum [8, 9]. The gas-dynamic model is verified on a series of the Toro analytical tests [6].

3. Simulation results

The residual was calculated for $\tau = 10^{-11}$ and $h = 10^{-4}$ with a sequential increase in the number of points. The norms in C and in L_2 were applied to evaluate the result of the calculations. Figure 1 presents the results for the

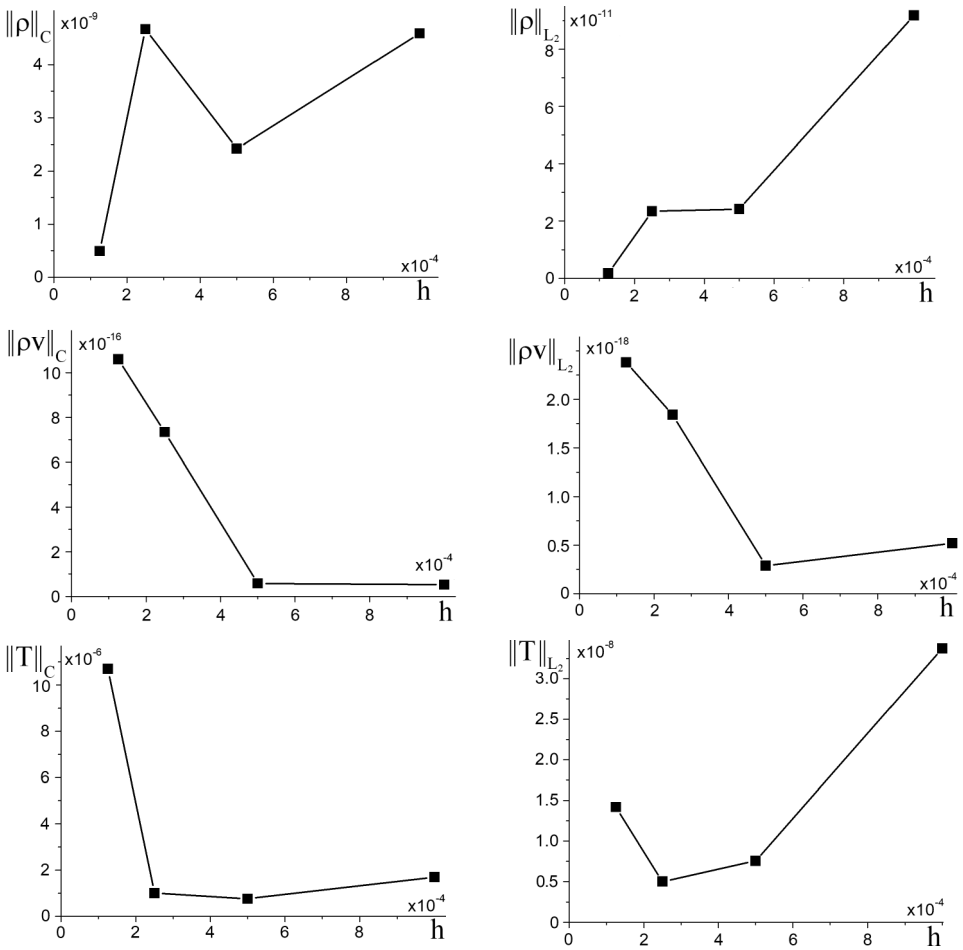

Figure 1. The results for the upwind scheme

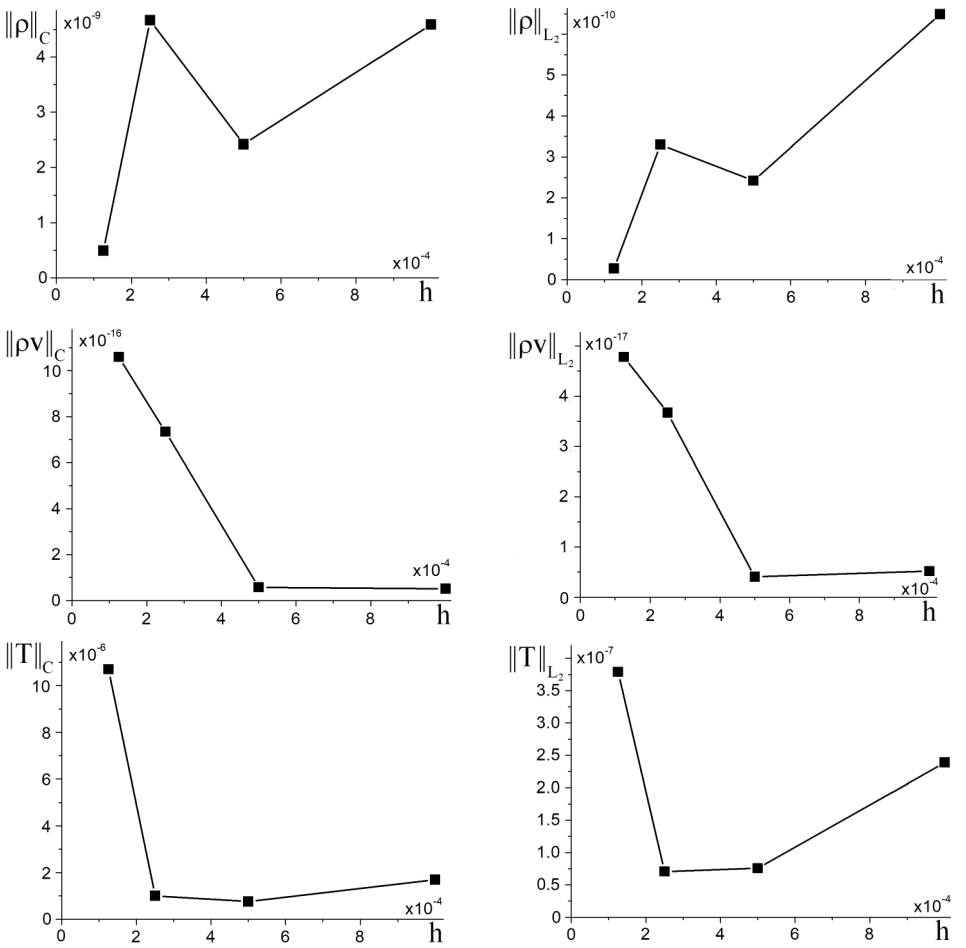

Figure 2. The results for the "Coarse particle" method

upwind scheme, Figure 2 — for the "Coarse particle" method. The residual slightly increases during the solution due to the accumulation of errors, but does not exceed the error of the schemes.

Figure 3 shows the temperature graph for directional and Belotserkovsky's differences. A thermal wave propagates into vacuum. The presence of a "hump" at the beginning of the wave is due to the density and temperature of the technical vacuum and its temperature in the installation. The magnitude of the "hump" is proportional to the magnitude of this density. Ideally, the gas must be calculated using the two-phase model. Now we assume that a highly rarefied gas in the installation and the tungsten vapors are one and the same substance with the same parameters. The specific total energy smoothly propagates into vacuum along with the mass of gas.

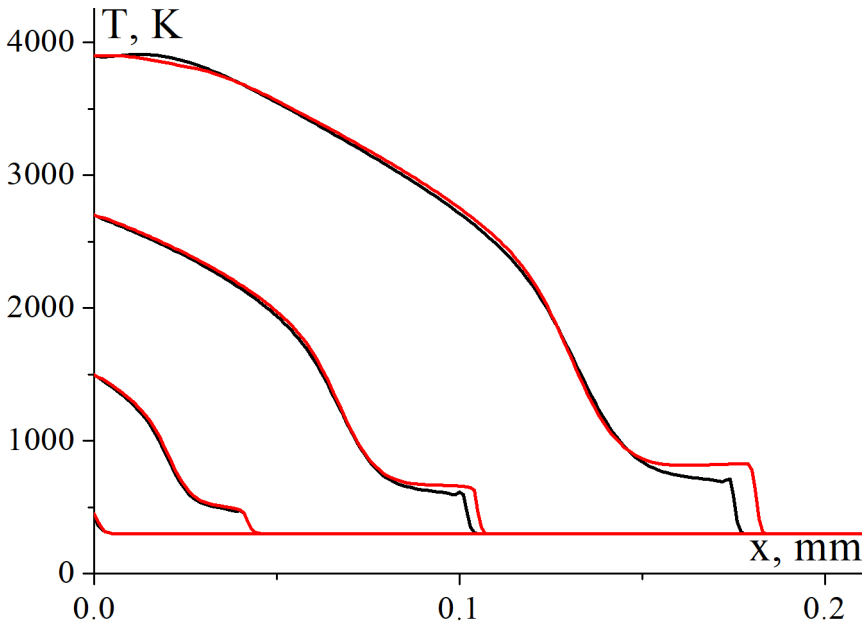

Figure 3. The temperature graph is for directional differences (black), Belotserkovsky's (red)

Conclusion

For the calculation of the tungsten evaporation dynamics under the influence of thermal pulsed loads, we have developed a model and a program. The gas dynamics system is implemented by the two methods of the first order of accuracy: the upwind scheme and Belotserkovsky's coarse particle method. For the program verification, a test solution of decay of the gas was used. In the process of calculating the problem in the complete statement, an analysis of the residual was carried out. The residual slightly increases in the course of the solution due to the accumulation of errors, but does not exceed the error of the schemes.

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