Bull. Nov. Comp. Center, Num. Model. in Atmosph., etc., 6 (2000), 35–42

An axially symmetric numerical model for the emergence of a tornado in a mesoscale cyclone*

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Spouts and tornadoes are phenomena that are especially dangerous. They are accompanied by very strong winds and cumulonimbus, shower, hail, and charge clouds. The vertical dimensions of a tornado are, as a rule, several fold larger than its horizontal dimensions, since intensive motion usually begins in the cloud layer and reach the Earth very quickly. Typical features of spouts and tornadoes are not only extremely high wind velocities, but also very great velocity gradients. For instance, a downdraft of tens of meters per second at the center of a powerful tornado, and an updraft of the same intensity at a distance of only several tens of meters from the center are often observed [5–7, 17, 18]. In accordance with the existing theory, a plane-parallel flow even with smaller velocity gradients is hydrodynamically unstable.

There exist several models that explain the stabilizing effect of rotation at mesoscale vortices [1, 15–17]. In all these models, the so-called gyroscopic effect, which is due to rotation of a mechanical system, in particular, in liquids and gases, is described differently. The reasons for the existence of rather powerful convective formations which can form the basis for the formation of mesoscale vortices were investigated in detail in [2, 3, 5–7]. These are the existence of large thermals in the lower part of a convective layer and intensive cloud convection in its upper part. These structures can merge at certain conditions, for instance, at rotation of the cloud under study. There is no, however, consensus of opinion regarding the reasons for the existence of torque in clouds. For instance, the authors of papers [2, 5, 16–18] consider the intensive mesoscale cyclone that often accompanies a tornado as the reason for the torque. Nevertheless, some powerful tornadoes with anti-cyclonic rotation are known. It was shown in [2] that the torque concentrated in the single convective cell that is formed by merging of several clouds is the reason of reverse rotation of such tornadoes. The authors of [4, 11] and [8, 10, 12] explained the mechanism of concentration of torque at the axis of a cell. They have shown that a vortex is formed from a

*Supported by the Russian Foundation for Basic Research under Grant 00-15-98543 and 99-05-64678.

spontaneous convective cell in the presence of torque in the atmosphere. First, the torque is concentrated in a local area under the influence of air convergence in the lower and middle parts of the cell. Then, the structure of the cell itself changes considerably as follows: a descending rotating jet appears at the center of the cell, and the motion is further concentrated at the axis of rotation. Note that there exist other theoretical explanations of tornadoes, and so far there is no consensus of opinion regarding the reasons of its occurrence [17].

The main purpose of this paper is to describe an axially symmetric model of mesoscale vortex in order to estimate some micro, meso-, and macrophysical factors of the tornado mechanism. Note that such a very simplified axially symmetric model is a preliminary step in the development of a spatial model that could simulate many meteorological situations in which tornadoes occur. At the present time, such models are not available.

1. Mathematical formulation of the problem

Let us consider a mesoscale vortex that is axially symmetric, and its axis is vertical and fixed. We assume for simplicity that this vortex is at the center of a mesoscale cyclone. Also, we assume that the time of rotation of each air particle in the central part of the cyclone about the fixed axis is the same, i.e., the internal part of the cyclone rotates as a solid body: $V_c = kr$. This rotation causes a pressure field $P_c(r) = P_0 - kr^2/2$. Then, the following system of equations of deep convection in a cylindrical system of coordinates that rotates together with the central part of the cyclone can be used to describe the vortex:

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial r} + W\frac{\partial U}{\partial z} - \frac{V^2}{r} - 2kV = -R\theta\frac{\partial}{\partial z}\frac{p'}{P} + \frac{\partial}{\partial r}\frac{\nu}{r}\frac{\partial Ur}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial U}{\partial z},$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial r} + W\frac{\partial V}{\partial z} + \frac{UV}{r} + 2kU = \frac{\partial}{\partial r}\frac{\nu}{r}\frac{\partial Vr}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial U}{\partial z},$$

$$\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial r} + W\frac{\partial W}{\partial z}$$
$$= -R\theta\frac{\partial}{\partial z}\frac{p'}{P} + g\left(\frac{\vartheta}{\theta} + 0,61q - q_L\right) + \frac{1}{r}\frac{\partial}{\partial r}\nu r\frac{\partial W}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial W}{\partial z},$$

$$\frac{\partial \vartheta}{\partial t} + U\frac{\partial \theta}{\partial r} + W\frac{\partial \theta}{\partial z} = -W\frac{\partial \theta}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\nu r\frac{\partial \vartheta}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial \vartheta}{\partial z} + \Phi_1,$$

$$\frac{\partial q}{\partial t} + U\frac{\partial q}{\partial r} + W\frac{\partial q}{\partial z} = -W\frac{\partial Q}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\nu r\frac{\partial q}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial q}{\partial z} + \Phi_2,$$

$$\frac{\partial q_c}{\partial t} + U\frac{\partial q_c}{\partial r} + W\frac{\partial q_c}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\nu r\frac{\partial q_c}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial q_c}{\partial z} + \Phi_3,$$

$$\frac{\partial q_r}{\partial t} + U\frac{\partial q_r}{\partial r} + W\frac{\partial q_r}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\nu r\frac{\partial q_r}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial q_r}{\partial z} + \Phi_4,$$
$$\frac{\partial q_i}{\partial t} + U\frac{\partial q_i}{\partial r} + W\frac{\partial q_i}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\nu r\frac{\partial q_1}{\partial r} + \frac{\partial}{\partial z}\mu\frac{\partial q_i}{\partial z} + \Phi_5;$$

$$\begin{aligned}
\Phi_1 &= \frac{L}{c_p}(CR - ER_c - ER_r - P_9) + \frac{L_s}{c_p}(P_4 - P_8),\\
\Phi_2 &= -CR + ER_c - ERr + P_9 - P_4 + P_8,\\
\Phi_3 &= -k_1(q_c - a) - k_2 q_c q_r^{0.88} - k_2 q_c q_i^{0.88} + CR - E - R_c,\\
\Phi_4 &= \frac{1}{\rho}\frac{\partial}{\partial z}(\rho V_r q_r) + k_1(q_c - a) + k_2 q_c q_r^{0.88} - ER_r - P_3 + P_5,\\
\Phi_5 &= \frac{1}{\rho}\frac{\partial}{\partial z}(\rho V_i q_i) k_2 q_c q_i^{0.88} + P_3 + P_4 - P_5 - P_8 - P_9;
\end{aligned}$$

$$\begin{aligned}
ER_r &= -\frac{1}{\rho}\frac{c(\frac{Q}{Q_n} - 1)(\rho q_r^{0.88})}{5.48\cdot 10^5 + 4.1\cdot 10^6/e_n},\\
Q_n &= 3.8p^{-1}\exp\left(\frac{17.5(T-273)}{T-36}\right),\\
e_n &= 6.11\exp\left(\frac{17.5(T-273)}{T-36}\right),\\
c &= 1.6 + 5.7\cdot 10^{-4} V_r^{1.5},\\
V_r &= 3634(\rho q_r)^{0.1364},\\
P_4 &= \frac{1}{\rho}\frac{(\frac{Q}{Q_{is}}(\rho q_i^{0.525})^{-0.42} f_0}{7\cdot 10^5 + 4.1\cdot 10^6/e_{is}},\\
Q_{is} &= 3.8p^{-1}\exp\left(\frac{21.64(\theta-273)}{\theta-8}\right),\\
e_{is} &= 1.6 Q_{is} p,\\
P_5 &= \begin{cases} 0 & \text{at } T \le 273\text{ K},\\ c\cdot 2.27\cdot 10^{-6}(T-273)(\rho q_i)^{0.525}\rho f_0^{-0.42} & \text{at } T > 273\text{ K},\end{cases}\\
P_8 &= -\frac{1}{\rho}\frac{(\frac{Q}{Q_{is}} - 1)c(q_i\rho)^{0.525} f_0^{-0.42}}{7\cdot 10^5 + 4.1\cdot 10^6/e_{is}},\\
P_9 &= -\frac{1}{\rho}\frac{(\frac{Q}{Q_n})c(q_i\rho)^{0.525} f_0^{-0.42}}{5.4\cdot 10^5 + 4.1\cdot 10^6/e_n},\\
V_i &= 3.12\cdot 10^3(\rho q_i)^{0.125} f_0.
\end{aligned}$$

Here t is the time; r and z are the radial and vertical coordinates, respectively; U, V, and W are the radial, vertical, and tangential velocity components; ϑ, q, and p' are deviations of temperature, humidity, and pressure from their values $\Theta(z)$, $Q(z)$, and $P(z) + P_c(r)$ in an unperturbed atmosphere; $r(z)$ is the density value in the unperturbed atmosphere; q_c, q_r, and q_i are the water contents of cloud droplets suspended in the air, rain droplets, and ice particles; $q_L = q_c + q_r + q_i$; R is the gas constant of dry air; f is the Coriolis parameter; g is the gravitational acceleration; Φ_i, $i = \overline{1.5}$ take into account various mechanisms of moist convection; ν is the kinematic turbulence coefficient; ν_1 is the kinematic turbulence coefficient for rain droplets and ice particles; $\alpha = \partial\theta/\partial z$; $q_l = q_r + q_c + q_i$; L is the specific heat of evaporation; L_s is the specific heat of sublimation; λ_s is the specific heat of water crystallization; c_p is the specific heat of air at constant pressure; V_r and V_i are the fall speeds of rain droplets and ice particles; Q_n is the saturation specific humidity; T is the background temperature; e_n is the saturation vapor pressure; c is the ventilation coefficient.

2. Calculation domain, boundary conditions

The calculation domain is a cylinder with the following parameters:

$$0 \le z \le H, \quad 0 \le r \le R, \quad \text{where} \quad H = 10 \text{ km}, \quad R = 1000 \text{ m}.$$

The power of the computer used made it possible to take 400 nodes in the vertical line and 128 nodes along the r-axis.

The boundary conditions are conditions of symmetry of flow pattern with respect to the cyclone axis:

- at $r = 0$: $U = V = \dfrac{\partial W}{\partial r} = \dfrac{\partial \vartheta}{\partial r} = \dfrac{\partial q}{\partial r} = \dfrac{\partial q_c}{\partial r} = \dfrac{\partial q_r}{\partial r} = \dfrac{\partial q_i}{\partial r} = 0$;
- at a sufficient distance from the axis conditions of vanishing fluxes across the boundary are used:
- at $r = R$: $W = \dfrac{\partial Ur}{\partial r} = \dfrac{\partial Vr}{\partial r} = \dfrac{\partial \vartheta}{\partial r} = \dfrac{\partial q}{\partial r} = \dfrac{\partial q_c}{\partial r} = \dfrac{\partial q_r}{\partial r} = \dfrac{\partial q_i}{\partial r} = 0$;
- at the underlying surface, non-slip conditions for the velocities, vanishing perturbations of the humidity and cloud moisture, zero fluxes of precipitation droplets, and daily temperature variation are specified;
- at $z = 0$:

$$U = V = W = 0, \quad q = q_c = 0, \quad \frac{\partial q_r}{\partial z} = \frac{\partial q_i}{\partial z} = 0, \quad \vartheta = \vartheta_0 \sin(\omega t);$$

- at the upper boundary of the calculation domain, conditions at vanishing perturbations are used;

- at $z = H$: $$\frac{\partial U}{\partial z} = \frac{\partial V}{\partial z} = W = 0, \quad q = q_c = q_r = q_i = \vartheta = 0.$$

Microphysical processes $\Phi 1$, $\Phi 2$, $\Phi 3$, $\Phi 4$, and $\Phi 5$ are parameterized by using a scheme proposed by Kessler and modified to some extent by Y. Ogura and T. Takahashi [13]. The main microphysical processes are condensation, coagulation, sublimation, evaporation, melting, and autoconversion (automatic transition of "mature" cloud droplets into precipitation moisture). The dimensions of cloud particles are not calculated. It is considered that the cloud moisture is transported with the air and the rain droplets and crystals fall with respect to the air at a velocity that depends on the water and ice content of precipitation.

3. Results of calculations

Two series of numerical experiments have been performed. In the first series, an optimal numerical scheme for stable calculation at large wind velocities and large velocity gradients was chosen. A monotone implicit scheme used together with a coordinate splitting method turned out to be such a scheme. In the second series of experiments, the role of individual factors in the vortex formation mechanism was verified. The time of vortex formation was found to be proportional to the rotation velocity of the mesoscale cyclone under study. Thus, at $k = 0.5 \cdot 10^{-2}$ s^{-1} this time is $T \approx 2.5$ hours, and at $k = 0.2 \cdot 10^{-1}$ s^{-1}, $T \approx 40$ minutes. The values $k > 0.2 \cdot 10^{-1}$ s^{-1} correspond to cyclones with unreal intensities, and the time $T > 2.5$ hours is larger than the time of existence of real convective cells. The vortex intensity in the calculations turned out to be proportional to the total thickness of the layer with unstable stratification. This layer must be originated at the Earth. When there were no thermals in the lower layer, there was no vortex formation even in the presence of powerful clouds. In the case of "dry" convection (i.e., in the absence of cloudiness), vortices of small intensity emerged. These vortices extended from the Earth to heights from several tens of meters to 1.2 km, with rotation velocities $v < 25$ m/s. Vortices of such type are similar to dust vortices which are often visible due to solid particles they contain. At sufficient air humidity and powerful convection in the surface layer, the largest thermals reach the condensation level and give birth to convective clouds. If the cloud layer thickness constitutes several kilometers, conditions for the formation of powerful vortices with velocities of 30–40 m/s are created. The formation of such a vortex at $k = 0.5 \cdot 10^{-2}$ s^{-1} lasts about 2–3 hours. It begins with a relatively slow rotation of the cloud. Then, as the velocities increase, a descending jet appears in the central part of the vortex. It approaches the Earth, and the surface layer becomes involved into the rotation. Unfortunately, as velocities larger than 40 m/s are reached, the calculation becomes unstable. It seems that a scheme with a more detailed space-time resolution is needed to model very intensive vortices, such as tornadoes.

Let us give a brief description of the vortex structure at the stage of maximal development calculated at the following parameter values: $k = 0.5 \cdot 10^{-2}$ s^{-1}, $\nu = 20$ m^2/s, $\mu = 20$ m^2/s. It was assumed that at $t < 0$ (i.e., when there is no vortex) the temperature of the atmosphere linearly decreases with height from T=20°C at $z = 0$ to $T = -50$°C at $z = 10$ km. A diurnal temperature variation with an amplitude $\vartheta_0 = 10$°C is specified at the underlying surface. The relative humidity decreases linearly with height from 100% at $z = 0$ to 70% at $z = 10$ km.

At these parameter values, a circulation of the following type was formed by the beginning of the third hour: the air motion in a vertical cross-section of the atmosphere seemed to consist of two cells of different sizes, intensities, and directions of circulation. The internal cell of smaller size provided relatively powerful downdrafts at the axis of the vortex and weaker updrafts at a distance $r > 20$ m from the vortex axis near the Earth. The area with downdrafts expanded as z increased, and reached maximal values at about 400 m at a height of 1300 mm. The maximal velocity of downdrafts was $w = -27$ m/s at $z = 100$ m. As z increases, the intensity of downdrafts decreases, and at $z > 1400$ m they vanish at the vortex axis. Observational data show that downdrafts at the center are typical for real large vortices, such as tornadoes. This area has, both in theory and experiments, the shape of a funnel broadening upwards. The circulation direction of the external, larger, cell is reverse. The areas of updrafts of both cells merge. The maximal $w = 21$ m/s at $z = 75$ m, $r = 47$ m. The tornado formation was initiated by rotation of the central part of a powerful shower cloud which extended from $z = 900$ m to $z = 9200$ m at the stage of its maximal development. In this case, the precipitation intensity constituted 45 mm/hour. The horizontal dimensions of the cloud were about 2 km. It was assumed that a cloud is a part of space with cloud droplets suspended in the air. The rotation velocity of air particles increased as the funnel descended, and reached a maximum $v = 33$ m/s at $z = 40$ m, $r = 16$ m. This pattern of motion is typical for tornadoes of medium intensity (see [2–4, 6, 7]) and spouts (see [4, 8]).

Now let us focus our attention on empirical data. In accordance with [12], the main characteristic of the hazard of a spout is its intensity class K. Here, the average velocity of rotational motion v_k and the width of the descending funnel L_k are determined from the following relations:

$$v_k = 6.3(K + 2.5)^{1.5} \text{ m/s}, \quad L_k = 1.609 \cdot 10^{0.5(K+1.5)} \text{ m}, \quad K = 0, 1, \ldots, 6.$$

Substituting $K = 1$ into these relations, we have $v_k = 38$ m/s, $L_k = 38$ m, and in accordance with the existing theory, $v = 33$ m/s at $z = 40$ m,

$r = 16$ m. The width of the descending jet changes from 16 m at the first calculation level to 64 m at $z = 250$ m. Thus, the above results of calculations correspond to a vortex with intensity of the first class.

Conclusions

The model presented shows that the following conditions must be satisfied for the emergence of powerful mesoscale vortices, such as spouts and tornadoes:

- A convective layer that extends from the underlying surface to heights of several kilometers;
- Strong atmospheric turbulence in the convection zone.

The same conditions were obtained in [4, 8, 10–12] with the help of simpler models.

The available data of observations are in qualitative agreement with these conclusions. For instance, tornadoes occur most often in south-eastern states of USA close to the Atlantic coast. They frequently take place when tropic cyclones from the Caribbean Sea penetrate the mainland. In this case, a mesocyclone may occur that can be a source of a torque for the formation of a tornado. Additional conditions for convection come into existence in the lower layers. These are due to the fact that the moist air coming from the ocean is, as a rule, much colder than the warm land: tornadoes occur almost always during warmer seasons. Nevertheless, the conclusions obtained with the help of this model are of qualitative character: the assumption about the axial symmetry of motion decreases the degrees of freedom of the system, which can affect the final result. In other words, a vortex may not occur at the same conditions in a three-dimensional formulation.

In conclusion, I would like to note that this model corroborates the conclusions concerning the mechanism of occurrence of mesoscale vortices obtained by L.N. Gutman in the late fifties on the basis of simpler models.

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