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Calculation of the reflected tsunami wave front kinematics using the grid-based algorithm

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Abstract. There are many methods for computing the tsunami kinematics directly and inversely. The direct detection of waves in the deep ocean makes it possible to establish the tsunami source characteristics and origin. The present study proposes a modification in the methodology of determining tsunami travel times and wave front positions of direct and reflected waves. The initial ray approximation can be optimized with the use of an algorithm that calculates all potential variations and applies the corrections to travel time values. Such an algorithm was tested in an area with model bathymetry and compared with a non-optimized method. The modified algorithm for the calculation of travel times of reflected waves was also tested on the model bottom topography.

1. Travel time computations in non-homogeneous areas

The numerical modeling of the tsunami wave propagation can provide a better understanding of this phenomenon. The tsunami wave propagation and travel times can be calculated for different time intervals (isochrones), and to this end various numerical methods have been developed [1]. For the areas with complicated geomorphology and bathymetry (islands and narrow straits), the travel time computations based on the methods that use the Huygens principle are more effective than others [2]. The basic premise in using this principle requires that all the points of the source area, where the tsunami wave was generated, be sources of omnidirectional wave energy radiation. The algorithm for such a computation searches through all the adjacent grid points to the generated wave front and calculates a minimum of the whole tsunami travel time interval through these points. The sum total of all travel times from the source point to the destination point is a minimum for all possible wave travel paths. Such an algorithm was proposed more than 40 years ago [3], and was dedicated to some optimization problems. Rectangular computational grids with known ocean depth values are used normally for travel times calculations from the source region to all terminal points. Figure 1 shows a grid fragment of such a computational area. The small black squares designate the points on the grid, where the wave from the initial tsunami source has arrived and the tsunami travel times to these grid points are known.

We need to find the travel time from the source up to the point A. Relative to the point A, the neighboring points, where the travel times are

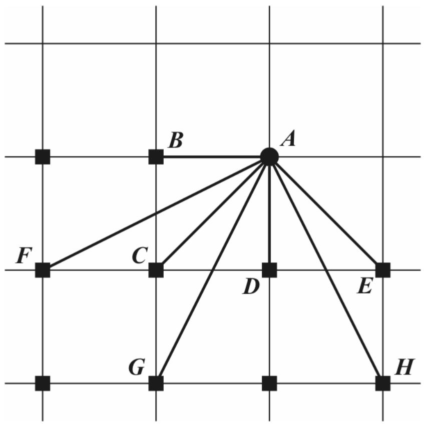

Figure 1. The scheme of travel-time determination using the Huygens principle

known, there will be the points B, C, D, E, F, G, and H. Let the travel times up to them be equal to $T_B, T_C, T_D, T_E, T_F, T_G$, and T_H, respectively. We assume that between the adjacent grid points the depth is linearly varying. To determine the tsunami travel time, we take the distance between two neighboring grid points L, and a depth value varying from H_1 up to H_2. If the angle of bottom declination is an auxiliary value α, then $H_2 = H_1 + L \operatorname{tg} \alpha$, and the travel time T can be designated as

$$\begin{aligned} T &= \int_0^L \frac{dl}{\sqrt{g(H_1 + l \operatorname{tg}\alpha)}} = \frac{1}{\sqrt{g \operatorname{tg}\alpha}} \int_0^L \left(l + \frac{H_1}{\operatorname{tg}\alpha}\right)^{-1/2} d\left(l + \frac{H_1}{tg\alpha}\right) \\ &= \frac{2}{\sqrt{g \operatorname{tg}\alpha}} \left(l + \frac{H_1}{\operatorname{tg}\alpha}\right)^{1/2} \Bigg|_0^L = \frac{2}{\sqrt{g \operatorname{tg}\alpha}} \frac{\sqrt{H_2} - \sqrt{H_1}}{\sqrt{\operatorname{tg}\alpha}} \\ &= \frac{2}{\sqrt{g} \operatorname{tg}\alpha} \frac{H_2 - H_1}{\sqrt{H_2} + \sqrt{H_1}} = \frac{2L}{\sqrt{gH_2} + \sqrt{gH_1}}. \end{aligned} \tag{1}$$

Therefore, the tsunami travel time between the neighboring grid points is equal to a distance between them divided by the arithmetically averaged velocity of the tsunami at these grid points. Thus, in order to find the travel time from the source up to the point A (Figure 1), it is necessary to find a minimum of seven time values:

$$\begin{aligned} T_1 &= T_B + \frac{2\Delta x}{\sqrt{gH_B} + \sqrt{gH_A}}, & T_2 &= T_C + \frac{2\sqrt{(\Delta x)^2 + (\Delta y)^2}}{\sqrt{gH_C} + \sqrt{gH_A}}, \\ T_3 &= T_D + \frac{2\Delta y}{\sqrt{gH_D} + \sqrt{gH_A}}, & T_4 &= T_E + \frac{2\sqrt{(\Delta x)^2 + (\Delta y)^2}}{\sqrt{gH_E} + \sqrt{gH_A}}, \\ T_5 &= T_F + \frac{2\sqrt{(2\Delta x)^2 + (\Delta y)^2}}{\sqrt{gH_F} + \sqrt{gH_A}}, & T_6 &= T_G + \frac{2\sqrt{(\Delta x)^2 + (2\Delta y)^2}}{\sqrt{gH_G} + \sqrt{gH_A}}, \\ T_7 &= T_H + \frac{2\sqrt{(\Delta x)^2 + (2\Delta y)^2}}{\sqrt{gH_H} + \sqrt{gH_A}}, \end{aligned} \tag{2}$$

where Δx, Δy are steps of the grid in horizontal and vertical directions and H_A, H_B, H_C, H_D H_E, H_F, H_G, H_H are the depth values at the corresponding points. A minimum of the values T_i $(i = 1, \ldots, 7)$ will give us the tsunami travel time from the source up to the point A. In such a way it is possible to find point by point the travel times to all the points of a computational grid. As for the neighboring grid points, the following can be stated. Let us introduce the so-called sixteen-dot (sixteen-radial) stencil (Figure 2) which shows a set of grid points which are regarded as neighboring ones to the grid point A.

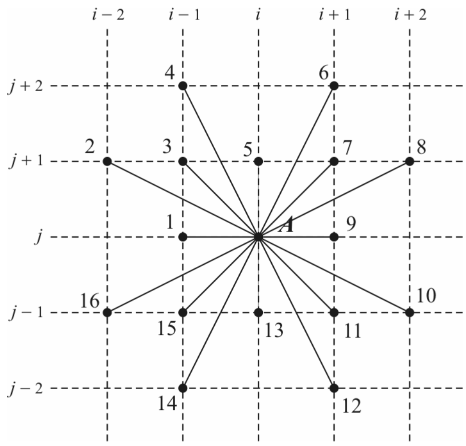

Figure 2. The sixteen-dot stencil for tsunami travel time calculations on a rectangular computational grid

In this stencil, the neighbors to the point A, that have the grid coordinates (i, j), are the grid points whose indices differ from the grid coordinates of the point A not only by one, but also by some grid-points whose coordinates differ by two (see Figure 2). In this case, if at any of these 16 points the tsunami travel time is already known, it is possible to find the wave travel time to the point A (at the center of the stencil).

2. The accuracy improvement of the tsunami travel time computations

A limited number of ray segment directions cause errors in the determination of the travel time values at grid points. These travel time mismatches can be easily seen in Figure 3, where isolines for travel time computations in an area with a flat bottom are displayed. The dimension of the computational area is $1{,}000 \times 1{,}000$ points with grid steps equal to 1 km in both directions, the ocean depth being 1,000 m. When the tsunami travel times are determined correctly, the shapes of isochrones must be close to circles. However, in actuality, instead of circles after carrying out the computations we have 16-angle polygons (Figure 3).

One of the ways to improve the accuracy of travel times obtained is to include into the calculation a bigger number of points. For example, it is possible to use a 32-dot stencil, but in this case the total number of arithmetical operations increases at a two fold rate. In addition, long distances between some points of the stencil and its center (up to 3 grid-steps) will cause errors in the resulting travel times.

It is possible to significantly improve the approximation quality by using the technique of variations. Briefly, this means the search for new locations of points along the wave ray. These new points do not have to be grid

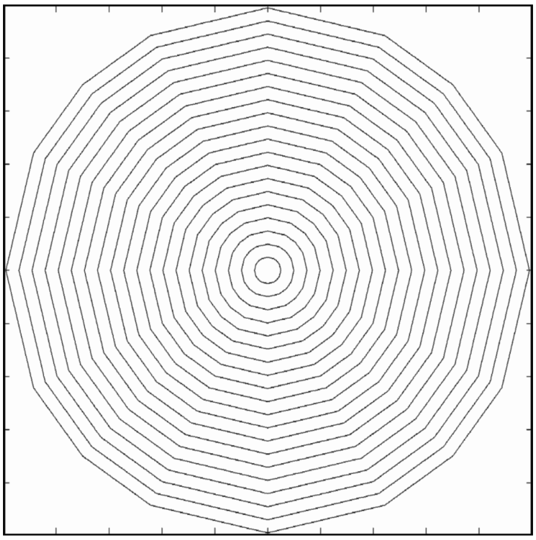

Figure 3. The test isochrones computation in the constant depth area using a 16-dot stencil (the area size 1,000 × 1,000, isochrones from 0 to 5,000 seconds with step of 250 s)

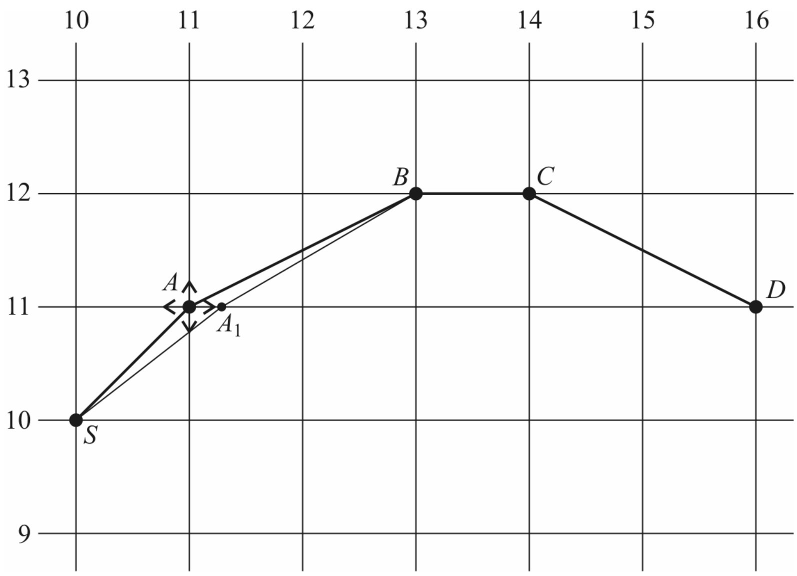

Figure 4. The scheme of the ray-trace optimization procedure

points. Within every grid-point included into the calculation the wave ray trace advances with very small incremental steps (1/10 of the grid) in different directions along the grid lines until a new location provides a shorter propagation time along the wave ray path. To describe the optimization algorithm, a special attention is given to a small segment of the computational area (Figure 4).

Let us look at the resulting approximation of the wave ray (the broken line $SABCD$ in Figure 4), which consists of a number of straight segments. In the process of the travel time calculation if the line $SABCD$ is designated to be the wave-ray trace between the source S and the target point D, the path of the ray between the source point and any other grid point can be restored. The procedure for obtaining more precise travel time values by relocation of the points B, C, and D is as follows. In the course of the computation the algorithm will estimate the travel time at the point B, and then the correctional procedure is applied. The grid point A, which is situated on the ray trace between the points S and B, must be relocated along the horizontal and vertical axes within a small step (about 1/10 of a grid step). At every new position of this intermediate point A_1 a travel time along the trace SA_1B will be calculated, based on the assumption that the depth between two neighboring grid points linearly varies.

A minimum travel time value will be accepted as the corrected travel time at the point B. It is of great importance that for further computations a corrected travel time value at the point B be used. When the general algorithm (based on the 16-dot stencil) gives a preliminary travel time value at the point C, the correction procedure must be repeated again for the other three grid points (A, B, and C). As a result, a new corrected value of travel time at the point C will be defined. Such a correctional procedure will

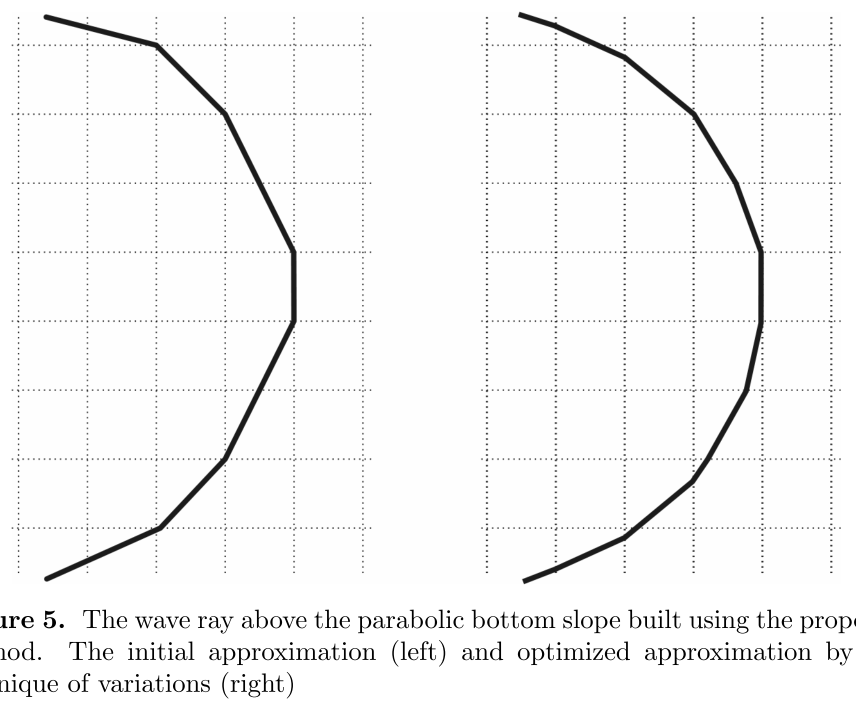

Figure 5. The wave ray above the parabolic bottom slope built using the proposed method. The initial approximation (left) and optimized approximation by the technique of variations (right)

be applied to all grid points along the ray path. Figure 5 shows the result of the wave ray optimization with the ray trace indicated above the parabolic bottom slope. In this computational area the depth increases as a squared distance from the left boundary. In the left part of the figure the ray before optimization is displayed, and in the right part — after optimization. The optimized ray trace is much smoother than the initial one.

The shape of the optimized ray (the right drawing in Figure 5) shows a good correlation with the analytical solution, being a segment of the circle [4]. As far as travel times are concerned, such a type of optimization can be applied to the process of travel times computations. This procedure for the travel time correction is schematically presented in Figure 6.

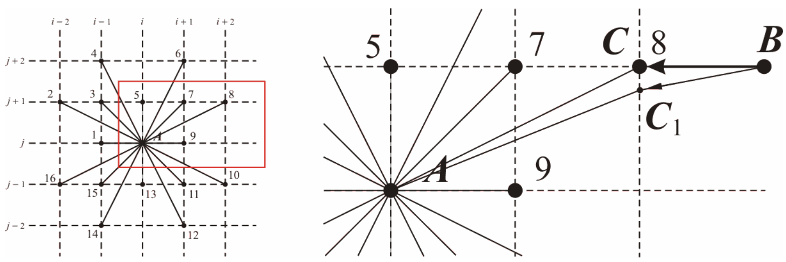

Figure 6. The scheme of determination of the optimized travel time at the center of the 16-dot stencil (point A)

A segment of the whole 16-dot stencil is shown in Figure 6. The travel time value is to be determined at the grid-point A. A minimum travel time value at this point is obtained within the travel time at the neighboring grid point C. In the previous computations, the tsunami travel time value at the point C was determined using the time value at the grid point B. This can be taken from analysis of the source grid-points used for previous travel time calculations. So, the value at the point A is calculated as the sum of the travel time value at the point B plus the wave travel times along the segments BC and CA. In this case the possibility of obtaining a more optimal trajectory of the wave ray must be examined. By relocating the point C it is possible to find a new location of the intermediate point C_1 which can give a minimum travel time between the point B and the grid point A. Finally, this travel time (along the trace BC_1A) is to be accepted as the corrected travel time at the grid point A. Using such an optimization procedure, the test travel time computations were carried out for a constant water depth area. A small round tsunami source is situated at the center of the area defined by 1,000 × 1,000 grid-points. After determination of the travel times at all the grid points, tsunami isochrones for every 250 s of the propagation were drawn. The shapes of isochrones by this method are much smoother than for the test without optimization (see Figure 3). At some points of the computational domain, the travel-time values differ

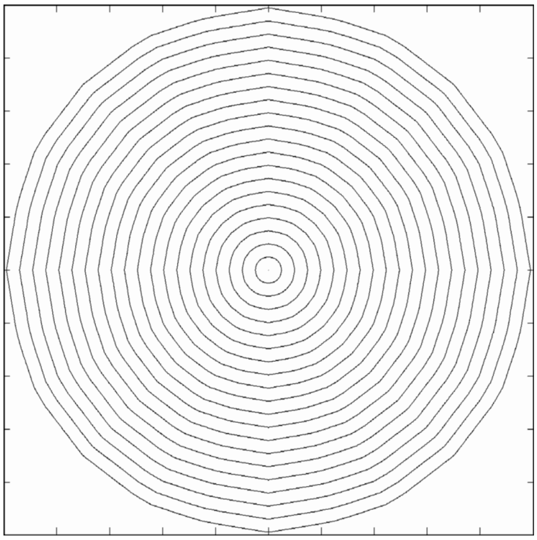

Figure 7. Isolines of the travel times array calculated by the optimized algorithm within the uniform depth

by up to one minute for every one hour of the tsunami wave propagation time. So, for distant tsunamis this error can be significant, which means that the tsunami impact can occur at least a few minutes earlier than that predicted by conventional methods. The precision of the method proposed can be improved by relocation not only of the point C, but, also, of the intersection point of the segment AC_1 and the segment connecting the 7th and the 9th points of the 16-dot stencil (see Figure 6).

3. Calculation of the reflected tsunami wave travel times

In the case of a real tsunami due to the interference of direct and reflected waves, the amplitude of the secondary tsunami wave can be high enough to cause some damage to people and infrastructure. So, it is of great importance to calculate the reflected wave kinematics. Figure 8 demonstrates the reflecting ability of the ocean coasts. Here one can see the direct and reflected tsunamis of 11.03.2011 at the eastern coast of Japan.

The height of the reflected off the shore tsunami is amounting to the impacting wave. So, it is very important to determine the kinematics of the wave after its reflection off the coastline. For this purpose the method described in the first part of the paper can be easily modified. All the grid points are separated into three categories: points, where no wave has arrived, points, where the leading tsunami wave has already arrived and points, where the secondary (reflected) wave has arrived. All the grid points along the reflecting boundaries (coastlines), where the direct tsunami has arrived, are regarded as the secondary wave source. In this case the method, described in the first part of this paper, starts working again in the area,

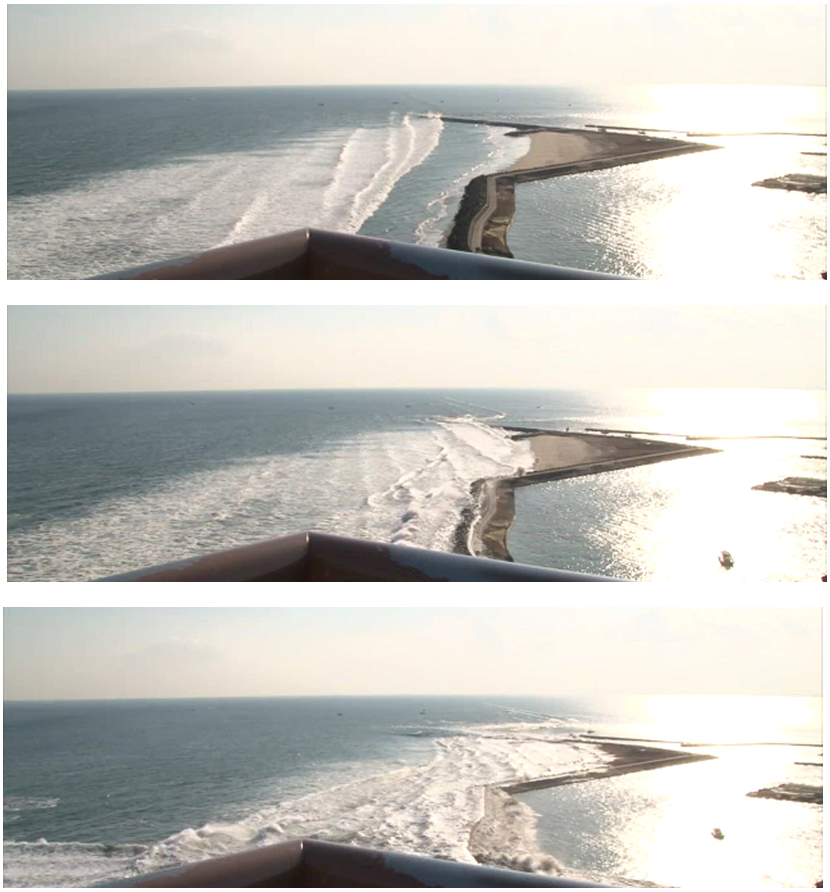

Figure 8. The tsunami reflection of the shoreline during the Great Tohoku tsunami of 11.03.2011

where the first wave travel times have been determined. Figure 9 shows isochrones of the tsunami generated by a small rounded source having the center located at the point S. Here the depth linearly increases in the right direction starting from the value D_0 at the straight reflective shoreline A–B.

For testing the algorithm proposed, let us consider the kinematics of a tsunami generated by the rounded source (colored blue in Figures 10–12) located not far from the reflective boundary in the area of the uniform depth. The tsunami wave front after reflection moves back to the right from this boundary (Figure 10). In order to show that the reflection was simulated correctly, the isochrones from Figure 10 were divided into the direct wave isochrones and the reflected ones (Figure 11).

Then we draw all the isochrones again in one figure, but the reflected wave isochrones are mirrored relatively the reflective boundary A–B (Figure 12).

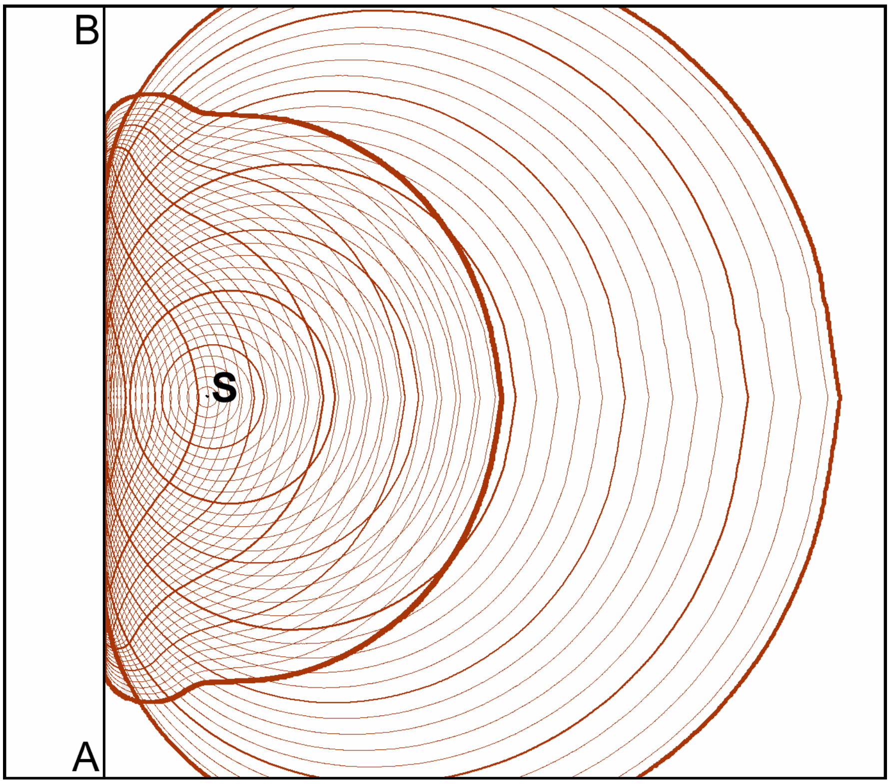

Figure 9. Tsunami isochrones of direct and reflected waves above the sloping bottom

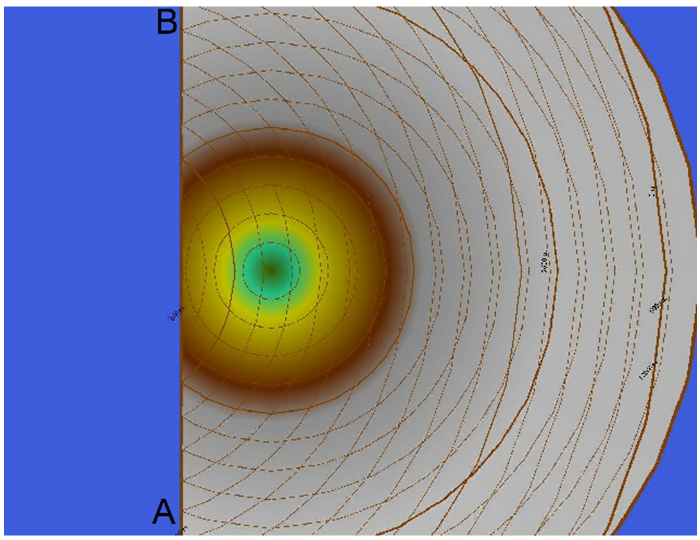

Figure 10. Tsunami isochrones of direct and reflected waves above the uniform depth

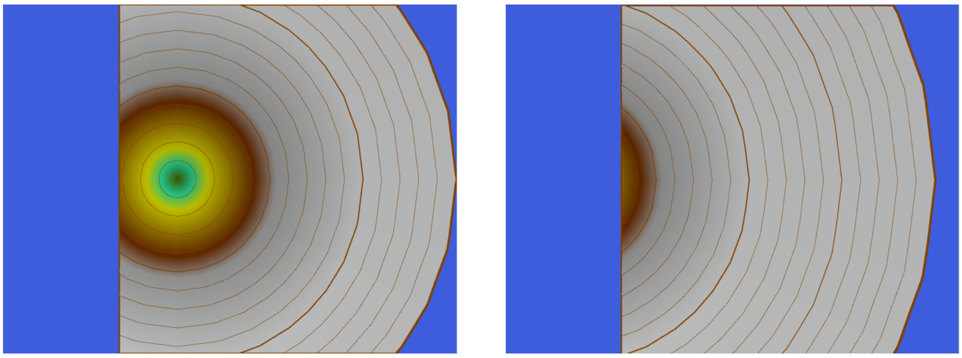

Figure 11. The separate visualization of the direct and reflected wave isochrones

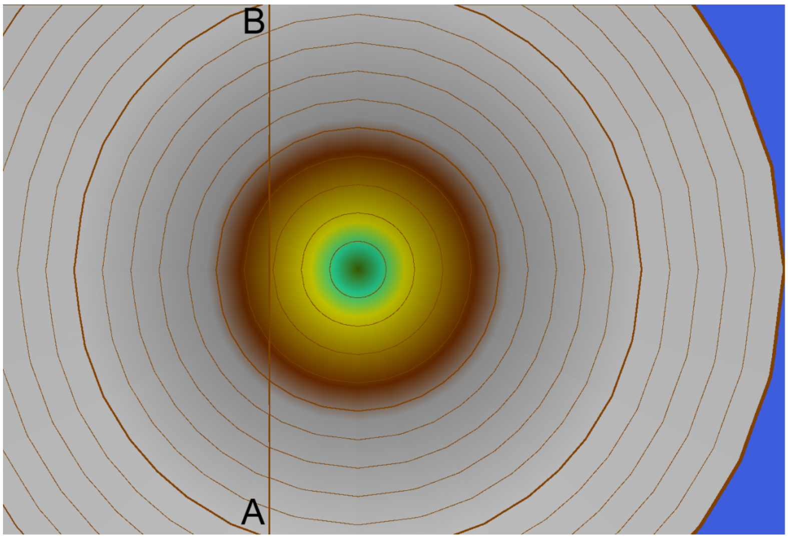

Figure 12. The mirrored visualization of the reflected wave isochrones for testing the algorithm

As is seen from Figure 12, all the isochrones are approximately round-shaped that corresponds to the exact analytical solution for such a bottom topography. It means that the kinematics of the reflected tsunami wave was correctly simulated, and this algorithm can be used for modeling real tsunamis.

Conclusion

The efficient method for calculating the tsunami wave kinematics on computational grids has been proposed and tested. This method based on the Huygens principle determines the wave travel-times from a tsunami source to all other grid points. It was optimized and tested with the known exact analytical solutions. Above the flat ocean bottom the time difference between the conventional 16-dot stencil method and the modified one is estimated to be up to one minute for every one hour of tsunami wave propagation. The method proposed is able to determine the reflected wave kinematics.

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