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The use of PS-nets in the design technology for distributed applications

N.G. Markov, E.A. Miroshnichenko, A.V. Sarajkin

1. Introduction

Formal models of parallel computations are widely used in the design of parallel and distributed software (PDS) for multiprocessor computer systems (MCS) [1]. The choice of a model is decisive for efficiency of the solution of design problems. The necessity of such a choice is due to a splash of interest to high-level models of parallel processes. However, not all of them completely fit the problems of PDS design. In [2], the apparatus of PS-nets is proposed. On the one hand, it contains high-level means to describe interaction between and properties of the parallel processes. On the other hand, it is substantiated with a rigorous mathematical theory. Various stages of PDS design with the use of PS-nets are considered in the paper.

2. The fundamentals of PS-nets

The fundamentals of the PS-net theory can be found, for example, in [2].

Definition 1. A PS-net N is a tuple $N = \langle V, D, T, R, F, P, A, S\rangle$, where $V=\{v_i\}$ is the set of vertices; $D=\{d_i\}$ is the set of arcs; $T=\{\tau_i\}$ is the set of durations of arc activations; $R=\{r_i\}$ is the set of resources; $F=\{\Delta r_{ij}^-, \Delta r_{ij}^+\}$ is the set of functions of the resource usage, and Δr_{ij}^- (Δr_{ij}^+) is the value of a decrease (increase) in the volume of the resource r_j by turning the arc d_i on (off); $P = \{p_i\}$ is the set of classes of precedence, and p_i is the ordered pair (d_j, d_k) such that the n-activation of d_k is possible only if d_j has been activated more then n times; $A = \{a_i\}$ is the set of classes of alternatives (mutual exclusion), and a_i is the pair of arc sets: the set of alternate entrances In_i and the set of exits Out_i such that if any of alternate entrances is turned on, than turning any of alternate entrances on is forbidden until one of the exits is activated; $S = \{s_i\}$ is the set of classes of synchronism, and s_i is the set of arcs which should be turned on only simultaneously.

Definition 2. A PS-net N in the matrix form is a tuple

$N = \langle D^+, D^-, R^+, R^-, A^+, A^-, P^+, P^-, S, T\rangle$,

where

$D^+ = [d_{ij}]_{m\times n}$, $m = |D|$, $n = |V|$, $d_{ij} = \begin{cases} 1, & \text{if the arc } d_i \text{ goes into } v_j;\\ 0, & \text{otherwise;}\end{cases}$

$D^- = [d_{ij}]_{m\times n}$, $d_{ij} = \begin{cases} 1, & \text{if the arc } d_i \text{ goes from } v_j;\\ 0, & \text{otherwise;}\end{cases}$

$R^+ = [r_{ij}]_{m\times l}$, $l = |R|$, $r_{ij} = \Delta r_{ij}^+$;

$R^- = [r_{ij}]_{m\times l}$, $r_{ij} = \Delta r_{ij}^-$;

$A^+ = [a_{ij}]_{m\times f}$, $f = |A|$, $a_{ij} = \begin{cases} 1, & \text{if the arc } d_i \in In_j;\\ 0, & \text{otherwise;}\end{cases}$

$A^- = [a_{ij}]_{m\times f}$, $a_{ij} = \begin{cases} 1, & \text{if the arc } d_i \in Out_j;\\ 0, & \text{otherwise;}\end{cases}$

$P^+ = [p_{ij}]_{m\times b}$, $b = |P|$, $p_{ij} = \begin{cases} 1, & \text{if } p_j = (d_i, d_k);\\ 0, & \text{otherwise;}\end{cases}$

$P^- = [p_{ij}]_{m\times b}$, $p_{ij} = \begin{cases} 1, & \text{if the arc } p_j = (d_k, d_i);\\ 0, & \text{otherwise;}\end{cases}$

$$S = [s_{ij}]_{m\times c},\ c = |S|,\ s_{ij} = \begin{cases} 1, & \text{if } d_i \in s_j; \\ 0, & \text{otherwise}; \end{cases}$$

$$T = [t_{ij}]_{m\times m},\ t_{ij} = \begin{cases} \tau_i, & \text{if i=j}; \\ 0, & \text{otherwise}. \end{cases}$$

Definition 3. *A marking M of the PS-net N in the matrix form is a quadruple* $M = \langle M^V, q, g, M^R\rangle$, where

$M^V = [m_i]_{1\times n}$, m_i is the number of markers in the vertex v_i;

$g = [g_i]_{1\times m}$, g_i is the number of steps to turning the arc d_i off;

$q = [q_i]_{1\times m}$, q_i is the counter of activations of the arc d_i;

$M^R = [m_i]_{1\times l}$, m_i is the volume of the resource r_i.

For the zero moment of time *the initial marking* M^0 is preset.

Let E and $e = (1, 1, .., 1)$ be an identity matrix and identity line, respectively.

One can see that the matrix representation is the set of incidence matrices which determines interrelations between the sets of PS-nets. The rows of matrices correspond to PS-net arcs, and the columns correspond to vertices, resources, classes of alternatives, classes of precedence, and classes of synchronism. The row-vectors of matrices, which are appropriate to the arc d_i and define incidence on the elements of sets are defined as follows: $D^+_{i,*} = (d_{i1}, d_{i2}, \ldots, d_{in})$, $\ldots$, $S_{i,*} = (s_{i1}, s_{i2}, \ldots, s_{ic})$. Similarly, the column-vectors appropriate to the j-elements of the sets, are defined as follows: $D^+_{*,j} = (d_{1j}, d_{2j}, \ldots, d_{mj})$, $\ldots$, $S_{*,j} = (s_{1j}, s_{2j}, \ldots, s_{mj})$.

In addition to usual operations of linear algebra on vectors and matrices, the following operations are proposed. For integer matrices X and Y of sizes $l \times m$ and $m \times n$, respectively, the following operations are defined:

$$Z = X \otimes Y = [z_{ij}]_{l\times n},\ z_{ij} = \max(x_{ik} \times y_{kj}),\ k \in 1, ..., m;$$

$$Z = X^B = [z_{ij}]_{l\times m},\ z_{ij} = \begin{cases} 1, & \text{if } x_{ij} = 1; \\ 0, & \text{otherwise}. \end{cases}$$

Let α and β be row-vectors with integer elements of dimensions m and n, respectively. We define the following operations of comparison:

$$\alpha < \beta = \begin{cases} \text{“}true\text{”}, & \text{if } \forall i : 0 \le i \le l \ \ \alpha_i \le \beta_i \text{ and} \\ & \exists j,\ 0 \le j \le l : \alpha_j < \beta_j,\ l = \min(m, n); \\ \text{“}false\text{”}, & \text{otherwise}; \end{cases}$$

$$\alpha \le \beta = \begin{cases} \text{“}true\text{”}, & \text{if } \forall i : 0 \le \ i \le l \ \ \alpha_i \le \beta_i; \\ \text{“}false\text{”}, & \text{otherwise}; \end{cases}$$

$$\alpha = \beta = \begin{cases} \text{“}true\text{”}, & \text{if } \forall i : 0 \le \ i \le l \ \ \alpha_i = \beta_i; \\ \text{“}false\text{”}, & \text{otherwise}; \end{cases}$$

Operations $\alpha > \beta$ and $\alpha \ge \beta$ are defined in the same way.

Definition 4. A class of synchronism s_n is called *excited* in the marking M, if the following conditions are satisfied:

1. There are markers in all initial vertices of arcs of s_n: $M(V) - [S_{*,n}]^T \otimes D^- \ge 0$.
2. No arc of s_n is turned on: $g(D) \otimes S_{*,n} = 0$.
3. For all arcs of s_n the conditions of precedence are satisfied: $g(D)(P^+ - P^-) - [S_{*,n}]^T \otimes P^- \ge 0$.
4. The conditions of mutual exclusion are satisfied for all arcs of the class s_n: $[S_{*,n}]^T \otimes A^+ \otimes [q(D)(A^+ - A^-)]^T \le 0$.
5. Turning the arcs of s_n on will not decrease the volume of any resource down to the negative value: $M(R) - [S_{*,n}]^T R^- \ge 0$.

Definition 5. *A front of excitement* $I(M) \subseteq S$ is the set of all classes of synchronism excited in the marking M.

Definition 6. *A front of turning on* $G(M) \subseteq I(M)$ is any maximal subset of the front of excitement $I(M)$ such that the following conditions are satisfied:

1. The total decrease of any resource with arcs of all classes from $G(M)$ does not exceed its volume: $M(R) - [G]^T R^- \geq 0$.

2. There are no two arcs in all classes from $G(M)$ included in the same set of alternate entrances: $[G]^T A^+ \leq 1$.

If there are some fronts of turning on, one of them is to be chosen nondeterministically.

Let $\widehat{G}(M)$ denote the set of all arcs included in the classes of the front $G(M)$.

In the moment t, we select the set of arcs $Q(M^t)$ which will be turned off at one step: $Q(M^t) = \{d_i : (g_i = 1) \vee (d_i \in \widehat{G}(M^t) \& (\tau_i = 1))\ \}$.

Definition 7. Let the state of the PS-net be determined by the marking M^t in the moment t. Then the marking M^{t+1} is calculated by the following rules called *the rules of marking change*:

1. The rule of redistribution of markers in vertices:
$M^{t+1}(V) = M^t(V) - [G(M^t)]^T \otimes D^- + [g^t(D)]^B \otimes D^+$.

2. The rule of changing the resource volumes:
$M^{t+1}(R) = M^t(R) - [G(M^t)]^T R^- + [g^t(D)]^B R^+$.

3. The rule of counting the arc activations: $q^{t+1}(D) = q^t(D) + [g^t(D)]^B$.

4. The rule of redistribution of markers on arcs:
$g^{t+1}(D) = g^t(D) + [G(M^t)]^T (T - E) + [e - g^t(D)] - e$.

3. The stages of the PDS design technology

The proposed technology can decide the primal problems of the PDS design, such as stratification, decomposition, determination of the PDS structure as a whole and at all hierarchical levels, as well, transformation of algorithms into the parallel form, tentative estimation of MCS resources, evaluation of separate tasks' durations, the choice of the process interaction discipline and, at last, the PDS realizability on the selected MCS. The selected parallelizing mode and the MCS homogeneity degree are taken into account. Let us describe the most important stages of the technology.

Detalization and decomposition of a PDS. On initial stages of the development of a PDS model, as well as of any complicated system, it is necessary to make clear the purposes of simulation so as to fulfill orientation, stratification, detalization and localization of the model.

An essential point of constructing a PDS model is a correct choice of the level of detalization. When solving this problem, we should consider a possibility of developing a sequence of models. It means that, in the beginning, it is necessary to design and use a PS-net of the first order of complexity, which is adequate to the PDS model only in the first approximation. Then, on this basis, we construct a PS-net of the second order of complexity which has a deeper level of detalization and a large number of components and parameters. If necessary, we create a PS-net of the third order of complexity. The proposed method of subnets integration is the main tool which allows us to modify the level of detalization. The process of developing the sequence is to be continued until the most appropriate model is obtained. On choosing the level of detalization, it is necessary to study the possibility of partitioning the PDS into separate subsystems and creating the independent models for them. In the general case, the problem of the system decomposition is not trivial. The separation of a system should be performed under the condition of functional isolation of subsystems or minimum functional connections. In the latter case, we mean not only the amount, but also the direction and potentiality of connections. The principle of decomposition allows us to fulfill the process of simulation in parallel.

The proposed principle assumes moving down through layers. If the main descending design is accompanied by the development of program components from bottom to top, then there is a possibility of additional clarification of the project using of the method for clarification of model (PS-nets) parameters of the PDS being designed with the help of half-scale experiments on a uniprocessor PC. The clarification of parameters of the top-level projects can initiate revision and modification of the lower-level projects, which leads to redesign of the model. Such a cycle is completed if there is no need in any significant modifications of the project for the next clarification of parameters.

Parallelizing of the sequential algorithms. If some sequential algorithm should be realized in the structure of the designed PDS, it is necessary to decide a problem of its analysis and transformations for making its parallel execution possible. To do this, we have to apply the developed methodology of the algorithm simulation using graphs of information-logical relations. As a result, the model (PS-net) of an algorithm expressing intrinsic parallelism is created. The obtained model should then be corrected at the stages of taking into account the number of MCS processors and the degree of the MCS homogeneity.

Taking into account the MCS resources. The next important stage of the PDS design is the stage of taking into account the MCS resources, for which it is necessary to use means of PS-nets. Each of the considered MCS resources should be added to the PS-net resource list; the rules of using it are determined by the functions of resource usage of the PS-net. To evaluate the initial volume of the resource, one should take into account a priori information about the target MCS. The number of processors, as the most important MCS resource, should be taken into account by the proposed method.

Taking into account the degree of the MCS homogeneity. If the target MCS is homogeneous, the models obtained at the given stage are used without modifications at all levels of hierarchy. If not, the models should be corrected using the developed method of arc duplication. The correction of models is carried out for those levels, on which the possibility of parallel execution of the processes on various MCS processors is stipulated.

Realizability of a PDS on the selected MCS. The designed PDS cannot be realized on the selected MCS, if there is a deficit in the MCS resources for its working. To evaluate the amount of resources available in the MCS, it is necessary to analyze the interpreted protocols of the resource usage. If the maximum consumption of the MCS resources does not exceed their available volume, then the designed PDS can be realized on the selected MCS, and cannot otherwise. At the same time, it is impossible to ignore the problems of the PDS efficient execution under the restrictions of the MCS. If these restrictions are too strong for the designed PDS, its functioning will be ineffective. Since MCS processors are the resource of great influence on efficiency of the PDS functioning, the developed method should be applied to evaluate their necessary amount.

The correctness analysis of the designed PDS. Our research allowed us to obtain some additional results. So, two advanced mutually completing approaches have been proposed.

The first approach assumes that an initial PS-net is analyzed using original methods [2] in order to reveal local and potential deadlocks in it.

Definition 8. *A local deadlock* is a subset P' of the sets of relations P specified on a subset S' of the set of classes S, such that it determines a directed cycle of expectation of one class from S' by the other.

Definition 9. *A potential deadlock* is a subset P' of the sets of relations P specified on a subset S' of the set of classes S, such that it determines a directed cycle of expectation of one class from S' by the other for marking M^t reachable from M^0.

Within the first approach, the transformations of PS-nets based on their matrix representation have been developed. This allows us to reduce all relations of a PS-net to the relations of precedence (P-relation) and simultaneity (S-relation). Such a transformed PS-net was called the base one (BPS-net). The transition from a PS-net to a BPS-net makes it possible to derive the matrix of a contiguity Sc of a special kind on which the search for expectation cycles is carried out:

$$S^c = S^T \otimes \left[P^+ \otimes [P^-]^T\right] \otimes S.$$

The second approach is based on usage of extensive buildups in the theory of Petri nets (PN) [3] for the analysis of behavioral properties of PDS [4, 5]. To realize this approach, we have developed the system of matrix transformations (the so-called split-transformation) which can be represented as a PN with the help of a BPS-net. Some lemmas and theorems about correctness of these transformations have been proved on the basis of two key statements, namely: 1) any parallel part of calculations can be executed sequentially, and the order of execution is of no importance; 2) any n-place choice (alternative) can be replaced with a sequence of $n-1$ of two-place choices. It has been shown that the time complexity of the transformation of a BPS-net into a PN, as well as its dimension, do not exceed a polynomial of $2n$-degree, where n is the size of the initial BPS-net.

The transformations developed within the PS-net theory allow one to formulate the problem of reachability for PS-nets and for PNs, either [3]. To solve this problem, the available methods can be applied, in particular, those based on matrix equations.

To simulate and analyze parallel software, the simulation system has been created on the basis of PS-nets. It allows one to describe a PS-net in a special language, to work with a textual and graphic representation of a model, to reveal local and potential deadlocks in a model, to run it in various modes, to represent the protocol of simulation in forms of the time diagrams of execution and resource usage, etc. As a tool especially intended to support the PDS design within the proposed technology, a subsystem of the project store has been developed. It allows one to create and use the project databases containing the explanatory texts, models and results of simulation.

4. Conclusion

This paper presents a brief outline of some important results of research of a new model called PS-nets.

It should be noted that all the proposed methods and tools have been successfully applied to the design of the PDS for some real-life systems.

References

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