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Diffusion effect in lattice gas automata waves*

Valentina Markova

Abstract. It is known [1] that the Lattice Gas Automata (LGA) models simulate a sound wave process. Moreover, in [2], it is proved that the LGA model corresponds to the hyperbolic equation. The computer simulation show that a wave profile varies with time: the profile amplitude reduces, the profile width increases. This suggests that the particle interaction rules simulate a diffusion process in addition to the wave process. In this paper, the availability of the diffusion effect is demonstrated by computational experiments with a simple LGA model.

1. Introduction

In the Lattice Gag Automata (LGA) models [1–5], the dynamics of an event is described by a set of hypothetical particles, which move in space and collide with each other and with obstacles. The space is represented as a regular lattice whose nodes contain a quantity of hypothetical particles. Each lattice node is assigned to a LGA cell. Interactions between particles are simple. Each interaction consists of the two successive steps: collision and propagation. The collision rules are chosen in such a way that the mass and momentum conservation laws are satisfied. The collision rules determine the LGA cell transition table. All the cells update their own states simultaneously and synchronously. An iterative change of the LGA global state (evolution of the LGA) describes the dynamics of a complex event [1–7].

An appreciable interest to the LGA modeling wave process is explained by the following. First, the LGA models are absolutely computationally stable due to the absence of a round-off error. Second, these models include an inherent spatial parallelism by definition. And basically, the LGA models allow to simulate wave process in the inhomogeneous medium without additional conditions on the boundary of density discontinuity as opposed to partial differential equations. This follows from the discreteness property. An appropriate experience of the LGA modeling of electromagnetic fields in an inhomogeneous medium is depicted in [4].

Our experience of modeling sound wave process [7, 8] has demonstrated that the LGA wave profile, first, differs in the form from a conventional wave profile and, second, it is washed out with time, the mass and momentum

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conservation laws being satisfied. This has suggested that the HPP model captures a diffusion effect as opposed to the hyperbolic wave equation.

In the paper, the availability of a diffusion effect is demonstrated by computational experiments with a simple LGA model, namely, HPP model (the model on a 2D lattice with four neighbors). This paper is organized as follows. After Introduction, in the second section, the main concepts of the HPP model are given. The third section is concerned with an experimental study of 1D unit wave propagation in the HPP medium. In order that a modeling wave process be observed in the usual (discrete) fashion, a twice averaging technique of the HPP cellular automaton global states (Boolean states) is considered in detail in order that a modeling wave process be observed in the usual fashion. In the third section, the procedure for revealing a diffusion effect in the HPP waves is given. The procedure consists in transformation of each CA evolution iteration in a wave profile for a given number of iterations using the twice averaged cell state and the comparison of the obtained parameters of wave profiles at each step.

2. The HPP cellular automaton for simulating wave propagation

2.1. The basic definitions. Formally, the HPP cellular automaton is defined by the set $\mathcal{N}=\langle M, A, \Theta\rangle$, where M is a set of coordinates in a discrete space, A is a cell state alphabet, Θ is a set of transition functions.

The space M is represented as a regular two-dimension four-neighbor Euclidean lattice $M = \{(i, j) : \ i = 0, 1, \ldots, I, \ j = 0, 1, \ldots, J\}$, the lattice nodes (i, j) being the cell *names*. Each cell can contain several particles. All particles have unit mass and unit speed. No more than one particle may occupy a given lattice site or move in a given direction at a given time.

An alphabet A is a set of Boolean vectors (states) $\boldsymbol{a} = (a_1, a_2, a_3, a_4)$, $a_k \in \{0, 1\}$. The k-th digit value of the vector $\boldsymbol{a}$, $k = 1, 2, 3, 4$, shows the presence ($a_k = 1$) or the absence ($a_k = 0$) of particles in the direction to the kth neighbor. A pair $(\boldsymbol{a}, (i, j))$, $\boldsymbol{a} \in A$, $(i, j) \in M$ is called a *cell*. Each cell is assigned to a finite state automaton. A set of cells, in which all cells have different names, Ω forms a *cellular array*. A set of the state of a cellular array at the time t is called a *global state* $\Omega(t)$ of the cellular array.

The set of transition functions $\Theta = \{\theta_s\}$ is defined by functioning rules of the HPP cellular automaton. The CA operates synchronously: all cells update their own states simultaneously at each iteration step according to transition functions. As a result, the cellular automaton changes a current global state $\Omega(t)$ to a new global state $\Omega(t + 1)$. Each step consists of the two successive phases: *propagation* and *collision*. This means that the transition function is a composition of the propagation (θ_1) and the collision (θ_2) functions: $\theta(\boldsymbol{a}) = \theta_2(\theta_1(\boldsymbol{a}))$.

Figure 1. Collision rules for the HPP cellular automaton

Both the functions are constructed in such a way that the laws of conservation of mass and momentum for all the cells from the array $\Omega = \{(\boldsymbol{a}, (i,j))\}$ should be satisfied.

In the *propagation phase*, in each cell each particle moves to the kth neighboring cell if $a_k = 1$.

In the *collision phase*, two particles arrived at a cell with the opposite direction, collide with each other (collisions are elastic) and change their direction by 90 degrees (Figure 1).

In spite of some inherent to HPP anisotropy, an iterative change of the CA global states (*evolution* of the HPP) describes the wave process dynamics on microscopic level.

2.2. Averaging cell states. In the process of the evolution of the HPP CA, the states of cells are Boolean vectors, and having no physical significance. In order that a modeling process be observed in the usual fashion of a physical event, the cell states from $\Omega(t)$ are averaged over some *averaging area* $Av(i,j)$. This means that for each cell $(\boldsymbol{a}, (i,j))$ a new state is calculated as follows

$$\langle (\boldsymbol{a}, (i,j)) \rangle = \frac{1}{|Av(i,j)|} \sum_{(l,m) \in Av(i,j)} \rho(l,m) = \mathcal{P}(i,j).$$

Here $|Av(i,j)|$ is the number of the cells situated in $Av(i,j)$ (the area $Av(i,j)$ can have different forms: a square, a column, a row), $\rho(l,m) = \sum_{k=1}^{4} a_k(l,m)$, $\rho(l,m)$ is called a *model density* of the cell with the name (l,m), $\mathcal{P}(i,j)$ is an averaged density value (a new state) of a cell with the name (i,j).

3. The sound wave propagation simulation

In the experiments carried out, the propagation of 1D unit sound wave propagation process is presented by evolution of a cellular array of size 50000×4000 cells. The initial states of the array cells are given by a random number generator. Further a cellular array with a given density will be referred to as the *model medium*. In the experiments, a medium model density is $\rho = 2$ (the random number generator with probability 0.5 produces four particles in all the cells of the array). A source for generation of initial momentum is located in the center of the array and represents a

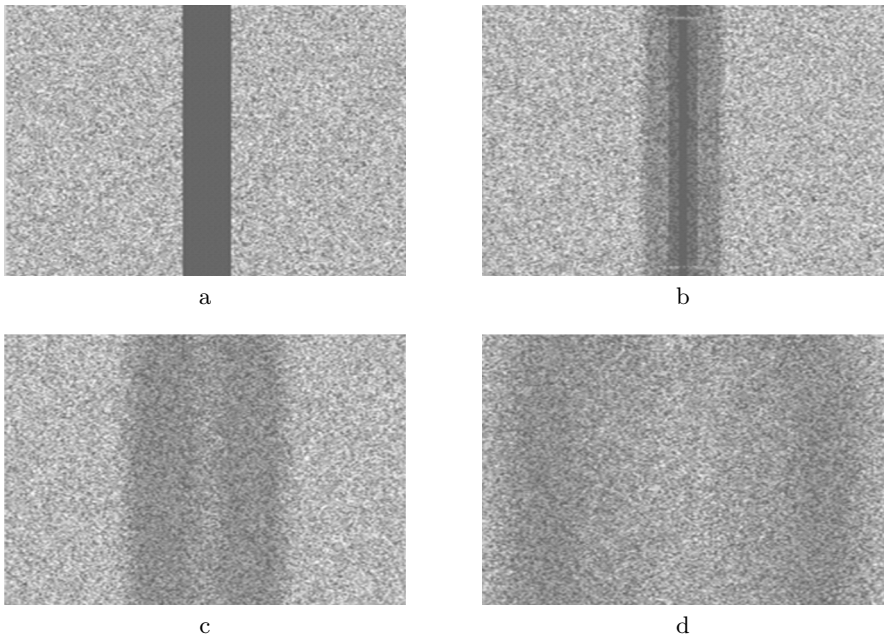

Figure 2. The HPP cellular automaton evolution

subarray of size 50×4000 cells. Each source cell generates with probability 1 four particles within one iteration. The boundary conditions are periodical. Figure 2 presents the wave profiles at different iterations. The initial state of the cellular array to be investigated is shown in Figure 2a.

Two compact subareas are formed by the evolution of this cellular automaton. Those subareas move from the source in the opposite directions with a certain velocity, thus simulating a unit lengthwise wave propagation in medium.

In our experiments, the global states of the cellular array are twice averaged. First, a value of particles density for each cell from a global state $\Omega(t)$ is averaged over a square 21×21 cells. Second, the obtained values from $\langle\Omega(t)\rangle$ ($\mathcal{P}(i,j;t)$) are averaged over the array column of size 4000 cells

$$\Omega(t) \xrightarrow{\text{square}} \langle\Omega(t)\rangle \xrightarrow{\text{column}} \langle\langle\Omega(t)\rangle\rangle.$$

The result of a twice averaged global state $\Omega(t)$ represents the following set $R(t) = \{R_0(t), R_1(t), \ldots, R_J(t)\}$, where $R_j(t)$ is the averaged density value over the jth column of the cellular array:

$$R_j(t) = \frac{1}{|Av(j)|} \sum_{(i,j)\in Av(j)} \mathcal{P}(i,j;t), \qquad |Av(j)| = I.$$

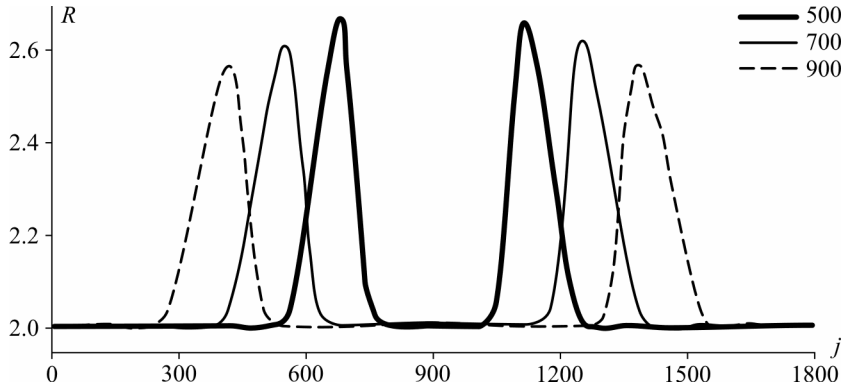

Figure 3. Wave profiles at different iterations of unit wave propagation in HPP medium with $\rho = 2$

Further, a twice averaged global state of the cellular array at the tth iteration of the cellular automaton evolution will be called a *wave profile* at the tth iteration of the unit wave propagation.

From Figure 3 follows that the HPP wave profile differs, first, in the form from the conventional wave profile, and, second, it is washed out with time, the mass and momentum conservation laws being satisfied. This confirms that the HPP model captures the diffusion effect as opposed to the hyperbolic wave equation.

4. Revealing the diffusion effect in the HPP waves

In this paper, the revealing the diffusion effect in the HPP wave is shown on the basis of computational experiments. The experiments consist in simulation of the HPP cellular automaton evolution for a given time interval, and in the comparison of unit wave profiles.

The procedure is very simple, it is as follows.

1. Let $\widehat{\Omega} = \{\Omega(k), \Omega(k+1), \ldots, \Omega(n)\}$ be a set of cellular array global states on a given interval $T = \{k, k+1, \ldots, n\}$. It is not necessary the $\Omega(k)$ be 0th global state. According to the averaging procedure, a unit wave profiles set $\widehat{R} = \{R(k), R(k+1), \ldots, R(n)\}$ is calculated.
2. For each wave profile $R(t)$, $t \in T$, two values are calculated (Figure 4)—wave *amplitude* $h(t)$ and wave *median* $m(t)$. A value of wave amplitude is $h(t) = h_{\max}(t) - \rho$, where $h_{\max}(t)$ is a maximum value of the wave profile $R(t)$ and ρ is an initial model density of the medium. The value of a wave *median* is equal to the distance between the points on the height $(h_{\max} + \rho)/2$ at the iteration t.
3. Two functions characterizing the diffusion effect are calculated on $\widehat{R}$—the wave amplitude $H = h(t)$ and the wave median $M = m(t)$, $t \in T$.

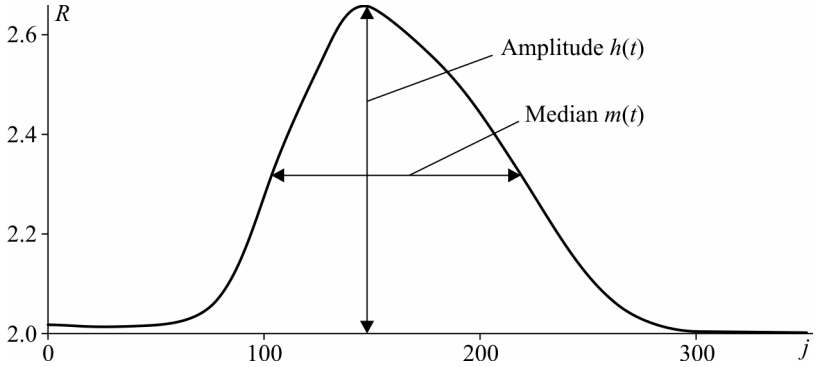

Figure 4. Amplitude and median of wave profile

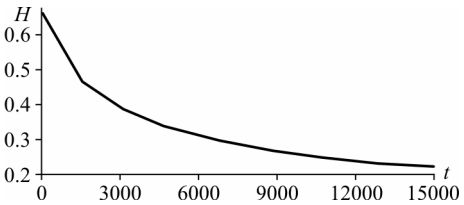

Figure 5. The function H

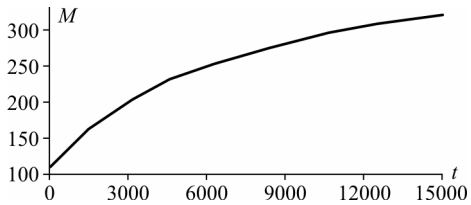

Figure 6. The function M

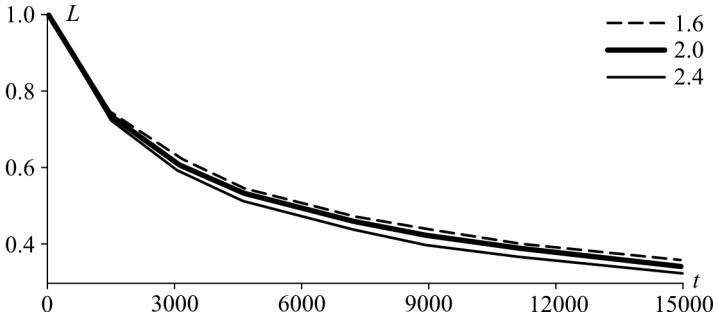

Figure 7. The function L

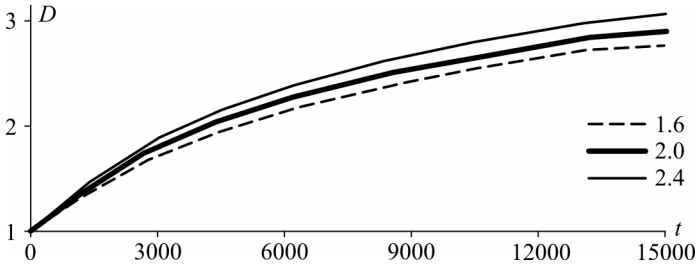

Figure 8. The function D

In Figures 5 and 6 the functions H and M are shown. Both curves point to the fact that the diffusion effect in this wave process diminishes with time. These results coincide well with the physics of a wave process.

For comparison of the diffusion effect in media with different model densities, functions of a relative change in the wave profile parameters are calculated:

- the function of a relative change of the wave amplitude value on the time $L = l(t)$, where $l(t) = \frac{h(t)}{h(0)}$, $h(0)$ is the wave amplitude value for $t = 0$.
- the function of a relative change of the wave median value on the time $D = d(t)$, where $d(t) = \frac{m(t)}{m(0)}$, $m(0)$ is the wave median value for $t = 0$.

It can be seen in Figure 7 and 8 that a diffusion effect in the wave is enhance as HPP density increases.

5. Conclusion

In the paper, the existence of a diffusion effect is demonstrated on computational experiments with a simple LGA model (HPP LGA model). For revealing a diffusion effect the following procedure is considered:

- Simulating the HPP CA evolution for the given time interval.
- Transformation each CA evolution iteration into wave profile using twice averaging cell state from global state of cellular array.
- Comparison of the obtained parameters of wave profiles at each iteration.

Moreover, wave process is investigated in media with different density. Experiments point to the fact than that CA diffusion process is more complex that the classic diffusion process.

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