Bull. Nov. Comp. Center, Comp. Science, 25 (2006), 25–36

The LGA models simulating sound wave propagation in a pipe*

Valentina Markova

Abstract. The Lattice Gas Cellular Automata (LGCA) models can be considered as an alternative to the conventional approach to the spatial dynamics simulation. The LGCA is based on a microscopic model of a physical process being simulated. Here we consider two simple LGCA models: HPP and HPPRP. They are based on a square lattice, whose nodes can be occupied by the moving particles in the HPP-model, and the moving and the rest particles in the HPPRP-model. In this paper, the possibility of a simple LGA models to simulate sound wave process in a pipe having regard to the wall features (moving obstacle and wall absorption) are investigated.

1. Introduction

The Lattice Gas Cellular Automata (LGCA) models or the Lattice Gas Automata (LGA) models [1–8] are powerful instruments for the simulation of complex events (sound waves, hydrodynamics of multi-phase and multi-component fluids, electromagnetic fields in inhomogeneous media, high-viscous flows, and chemical reactive flows) due to the following:

- *Preservation of conservation mass and momentum laws.* This means that the LGA evolution of events very closely coincides with a real mechanism of event under simulation.
- *Inherent spatial parallelism.* It allows ideally realize them on computers with massively parallel processors.
- *Locality of operational interactions.* This suggests that programming will be fairly simple.
- *Absolute computational stability due to the absence of a round-off error.* The LGA algorithms are free of a round-off error because the LGA cell states are Boolean vectors.
- *Simplicity of boundary conditions.* The boundary conditions are represented by specialized cells. The specialized cells behavior differs in the collision step. Such a technique for the CA boundary conditions representation gives a possibility for simulating dynamics of an event with complicated boundary geometries.

*Supported by the Russian Academy of Sciences, Basic Research Program of No. 13.15 (2006) and the Grant of Rosobrazovanie, contract RNP.2.2.1.1.3653.

In the LGA, dynamics of an event is described by a set of hypothetical particles, which have moved through space and collided with each other and with obstacles. The space is represented as a regular lattice whose nodes can contain a quantity of hypothetical particles. Each lattice node is assigned to a LGA cell. As opposed to the classical cellular automaton, an initial state of the LGA cell is determined by a set of some particles, locating in the cell at this time moment. There are two types of particles: the moving particles and the rest particles. The moving particles have the some mass (equal to one unit) and equal absolute velocity (equal to one unit). This allows them to move from one lattice node to its nearest neighbor in discrete time steps. The neighbor coordinates are indicated by a velocity vector direction of a moving particle. An exclusion principle is imposed on the moving particles: no two particles may sit simultaneously on the same node if their direction is identical. The rest particles have the same velocity (equal to zero) and a different mass.

Interactions between particles are simple. Each interaction consists of two successive steps: collision and propagation. The collision rules are chosen in such a way that the mass and moment conservation laws are satisfied. The collision rules determine the LGA cell transition table. All cells update their own states simultaneously and synchronously. An iterative change of the LGA global state (evolution of the LGA) describes the dynamics of an event on microscopic level.

In order that a modeling process be observed in the usual fashion of a physical event, averaged values of particles density and velocities for each LGA cell are calculated in some averaging area. Automata noise arises for small values of the averaging radix. This is the main disadvantage of the LGA models. Automata noise cannot be eliminated, but its effect can be reduced by increasing the averaging radix.

In this paper, two LGA models on a 2D lattice with four neighbors are used. The first model, the HPP model, proposed by Hardy, de Pazzis, and Pomeau [3], is fully deterministic "one-speed" lattice gas model. It has been introduced to analyze mechanical properties of 2D fluids in homogeneous media, such as the divergence of 2D transport coefficients. The second model, the HPPRP model [5], is an extension of the HPP model: certain rest particles (RP) are incorporated within cells of the lattice. As opposed to the HPP model, the HPPRP collision rules can be deterministic or nondeterministic. In [5] it is shown that the HPPRP model corresponds to the wave equation. One can also specify that certain regions of the 2D lattice have different rest particle numbers. The energy exchanges between moving and rest particles in the regions are thus different, and media with different sound speeds can be realized. The possibility of the "multi-speed" model have been demonstrated by the examples of simulation of electromagnetic fields and sound wave propagation in the inhomogeneous media in [5] and [9], respectively.

In this paper, the possibility of a simple LGA models to simulate sound wave process in a pipe having regard to the wall features (moving obstacle and wall absorption) are investigated.

2. The HPP simulating sound wave propagation in a pipe with a moving obstacle

2.1. The HPP model. In the HPP model, each cell (HPP cell) contains only the moving particles. The HPP cell state is determined by the velocity vector $v = (v_1, v_2, v_3, v_4)$ at this time moment. The l-th digit value of the vector, $l = 1, 2, 3, 4$, shows the presence ($v_l = 1$) or the absence ($v_l = 0$) of particle in the direction to the l-th neighbor. The HPP cell with the velocity vector $v = (0, 1, 1, 0)$ is shown in Figure 1. (Arrows in the cell show the directions of the velocity vector particles.)

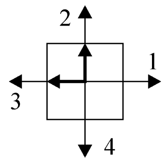


Figure 1

The evolution of the HPP cell consists of two steps:

- *Head on collision.* Two moving particles, arrive at a cell with the opposite direction of their velocity vector (head on collision), escape from the cell, changing the direction of their velocity vector by 90 degrees (Figure 2).
- *Propagation.* The moving particles move from their node to the nearest neighbor in the direction of their velocity vector.

Figure 3 illustrates one iteration of evolution of a cellular array with size of 4×4 cells.

The HPP model has superfluous laws of conservation: the total mass and moment are conserved along each space axis individually. In addition, the HPP model does not satisfy all the conditions of isotropy. That is the

Figure 2. Collision rules for the HPP model

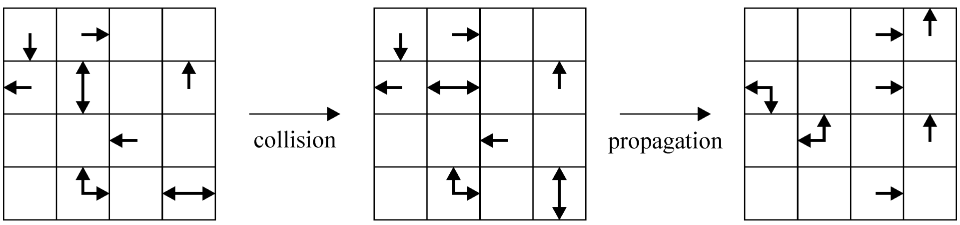


Figure 3. One iteration of the HPP evolution

reason that the HPP model has limited usefulness in the physical modeling. In [4], as an example, a capability of simulating two dimensional electromagnetic fields has been demonstrated. The HPP simulation of sound wave propagation was taken in [9].

2.2. Simulating sound wave propagation in a pipe with a moving obstacle. In Figure 4, a pipe (the simulation space) is represented as a 2D cellular array W with the size of $M \times N$ cells. We will distinguish three kinds of cells:

- *Source cells* are HPP cells. They generate moving particles with some probability at each iteration for a time. The density of an initial flow is given by the cell generation probability. It is evident that the mass conservation is violated. In our case, the sound wave propagation inside the cellular array from left to right. Because of this, source cells are accommodated in the 0-th row of the array W.
- *Ordinary* (*work*) *cells* are HPP cells.
- *Wall cells*. Wall cell collision rules are realized by the bounce-back scheme: a moving particle colliding with a wall cell simply reverses its momentum. The bounce-back rule is shown in Figure 5 (rules obtained from the presented rule by symmetrical transformations, are not shown). In the wall cell, moment conservation laws are not satisfied. Sound speed is close to zero near the wall.

In experiment, sound wave process is represented by a cellular array with the size of 1500×400 HPP cells. In the initial state, the array cells except the source and wall cells, contain two moving particles. The source cells

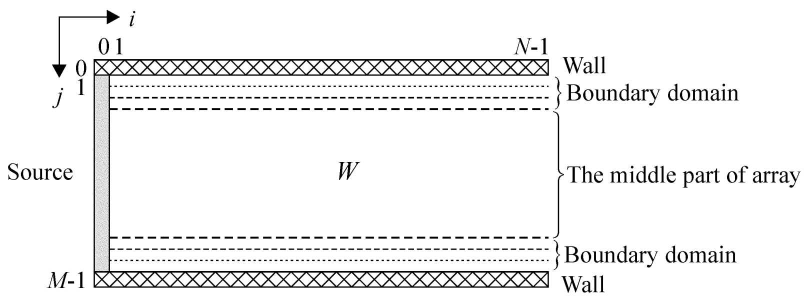


Figure 4. Cellular array for simulating 2D sound wave process

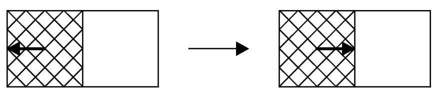
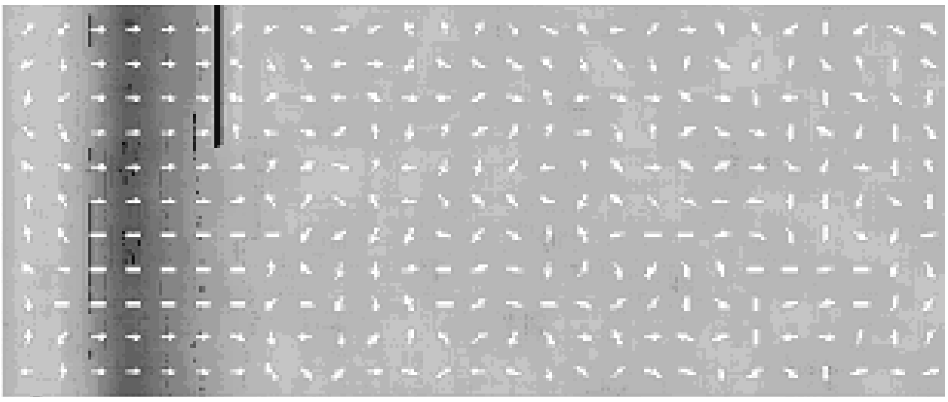
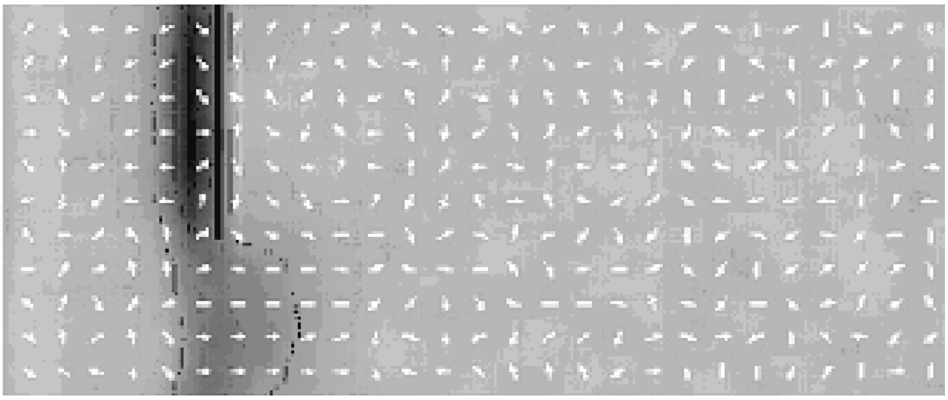


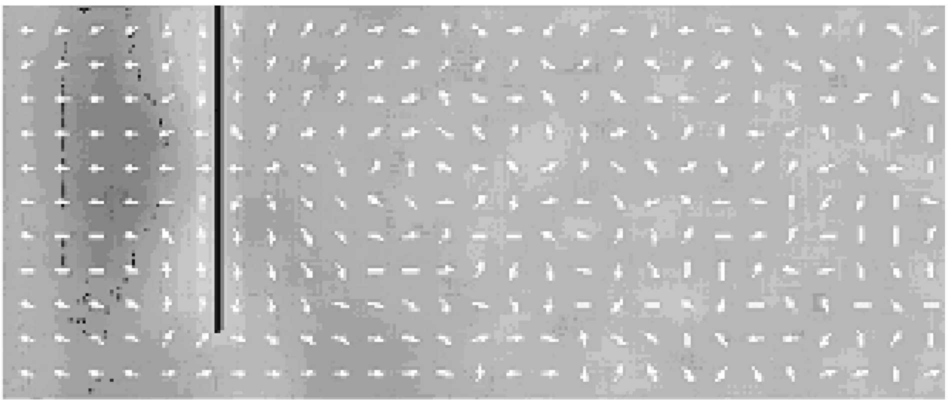
Figure 5. Bounce-back rule



$t = 200$



$t = 350$



$t = 500$

Figure 6. Velocity vector fields of the HPP evolution simulating wave process in a pipe with a moving obstacle

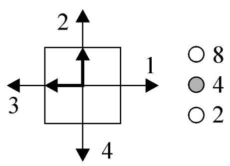
generate moving particles with some probability 0.7 at each iteration in the interval from 0 up to 100.

Moving obstacle is generated as follows. From some one cell of the wall, the moving obstacle begins to grow perpendicular to the wall: the next work cell on the way of obstacle lowering is declared to be a wall cell after several iterations. It is evident that the mass conservation law is not satisfied in this cell. In the worse case, two moving particles can be lost in this cell.

Figure 6 shows the velocity vector fields for different steps of the HPP evolution that simulates the sound wave process in a pipe with the moving obstacle. Here the averaging radix equals 20 cells, the gray color intensity determines the concentration of particles in cells: the white color of a cell denotes the absence of particles, and the black color of a cell corresponds to the presence of four moving particles. From the figure it will be noticed that the top part of a wave after colliding with obstacle bounces back. The lower part of the wave gets around an obstacle and propagates into the pipe. After a time, the pipe is filled with the sound wave.

3. The HPPRP simulation of sound wave propagation in a pipe with wall absorption

3.1. The HPPRP model. As opposed to the HPP cell, each cell of the HPPRP model (HPPRP cell) can contain the moving particles and the rest particles. The HPPRP cell state is defined by the two vectors: velocity vector v and mass vector m (Figure 7). The rest particles have the zero velocity and a different mass. Here we will consider the rest particles with the masses equal to 2, 4, and 8. It is evident that two mass 2 rest particles are equal to one mass 4 rest particle, four mass 2 rest particles are equal to one mass 8 rest particle. The length of the mass vector $m = (m_1, m_2, \ldots, m_r)$ does not depend of the space structure and is equal to the number of the rest particles. The k-th digit value of the vector, $k = 1, 2, \ldots, r$, determines the presence ($m_k = 1$) or the absence ($m_k = 0$) of a rest particle with mass 2^k in a cell. The rest particles are created (annihilated) with a certain probability p_k, $k = 1, 2, \ldots, r$, in so doing, the limitations $p_{k+1} \geq p_k$ and $\sum_{k=1}^{r} p_k \leq 1$ are satisfied. So, the state of a HPPRP cell is represented by an $(r + 4)$ long Boolean vector, where r is the number of the rest particles.



$v = (0, 1, 1, 0)$,
$m = (0, 1, 0)$

Figure 7

In the HPPRP model, the propagation step is similar to the HPP propagation step, the collision rules differ from the HPP collision rules. In addition to head on collision of moving particles, the energy exchange between the moving and the rest particles occurs. In response to this exchange, either a rest particle is created and moving particles are annihilated or a rest particle is annihilated and moving particles are created. In the general case, the collision rules are deterministic or non-deterministic. They can be divided into 3 groups:

1. *Head on collision.* The moving particles collide with each other according to head on collision rule (see Figure 2) independent of the presence or the absence of the rest particles.
2. *Rest particle creation.* If two moving particles collide with each other and there is initially no mass 2 rest particle, then moving particles will be annihilated and a mass 2 rest particle will be created.
3. *Rest particle annihilation.* If a mass 2 rest particle already exists in the cell, and there are no two moving particles, then two moving particles will be created after the collision step, and the rest particle will be annihilated.

The result of the application of the rest particle creation rule to the HPPRP cell with the state defined by the vectors $v = (1,1,1,1)$ and $m = (0,0,0)$, is shown in Figure 8. Three variants of this rule application are possible. In the first and the second variants, one pair of moving particles collides with each other. The collided particles are annihilated, and a mass 2 rest particle is created after the collision step. In the third variant, four moving particles collide with each other. As a result, a mass 4 rest particle is created and four moving particles are annihilated.

The result of the application of the rest particle annihilation rule to the HPPRP cell with the state defined by the vectors $v = (0,0,0,0)$ and $m = (0,0,1)$, is shown in Figure 9.

As opposed to the HPP cell, in the HPPRP cell, the size of automaton transition table is 2^{r+4}. Moreover, several rules with a similar or a dissimilar

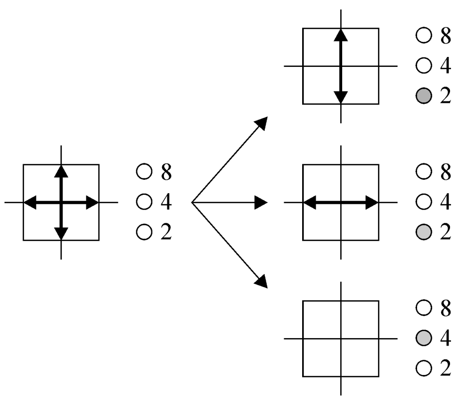


Figure 8. Rest particle creation rules

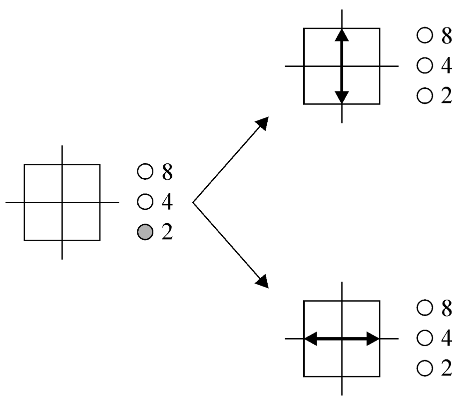


Figure 9. Rest particle annihilation rules

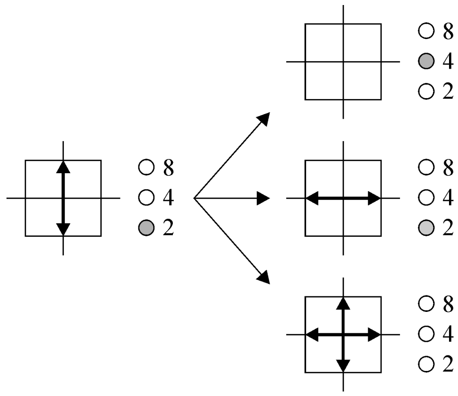


Figure 10. The collision step in the HPPRP cell

probability can be inserted into each row. Figure 10 shows the collision step in the HPPRP cell. For simplicity, collision rules are equiprobable.

The collision rules show the possibilities of the HPPRP model to simulate media with different sound velocity. The relationship between the sound velocity and the rest particles numbers has been experimentally investigated in [9]:

LGA model	Relative speed
HPP	1.0
HPPRP ($m_1 = 2$)	0.8
HPPRP ($m_1 = 2$, $m_2 = 4$)	0.7
HPPRP ($m_1 = 2$, $m_2 = 4$, $m_3 = 8$)	0.5

3.2. The HPPRP simulation of sound wave propagation in a pipe with wall absorption. Absorption of sound by a wall means the decrease of the sound wave propagation velocity along the cellular array wall (see Figure 4) as compared to the sound wave propagation speed in the middle part of the array. The effect of wall absorption is realized due to formation of the so-called *boundary domain* allocated along the whole length of the wall. A boundary domain contains several boundary layers (Figure 11).

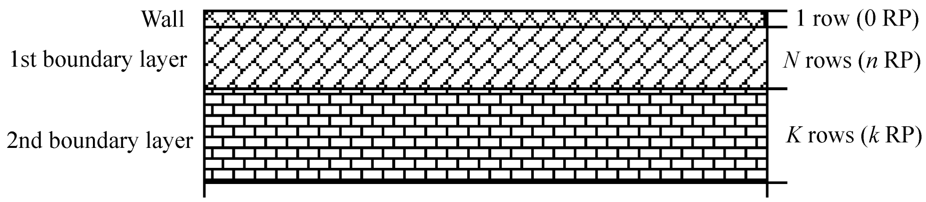


Figure 11. The boundary domain structure

For simplicity, two boundary layers are used here. A boundary layer combines several rows including the HPPRP cells. The number of rows can be arbitrary. In both layers, the HPPRP cells incorporate n and k rest particles with masses equal to $2^1, 2^2, \dots, 2^n$ and $2^1, 2^2, \dots, 2^k$ respectively. In the middle part of the array, the HPPRP cells include l rest particles with masses equal to $2^1, 2^2, \dots, 2^l$.

Based on the multi-speed HPPRP model property, the following relationship

$$n > k > l \geq 0,$$

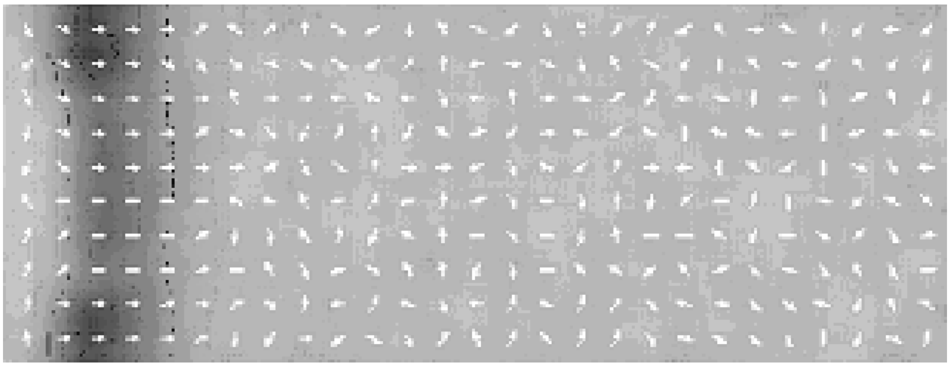
is the condition of the wall absorption effect modeling. It is evident that increasing the rest particles number in the first layer or both in the first and in the second layers results in a rise of the absorption factor.

In the experiment, the sound wave process is represented by a cellular array with 240×600 HPPRP cells. Here, both boundary layers consist of 20 rows. In the first layer, the HPPRP cells have three rest particles with the masses equal to 2, 4, and 8. In the second layer, the HPPRP cells have two rest particles with the masses equal to 2, and 4. In the middle part of the array, the HPPRP cells have one rest particle with the mass equal to 2. In the initial state, the array cells except the source and wall cells contain two moving particles. The source cells generate moving particles at each iteration in the interval from 0 up to 100. Figure 12 shows the velocity vector fields (the averaging radix equals to 20 cells) of the HPPRP evolution, simulated wave propagation in the pipe with the mentioned rest particles distribution. Here in the middle part of the array the sound wave runs with a greater velocity than along the array wall.

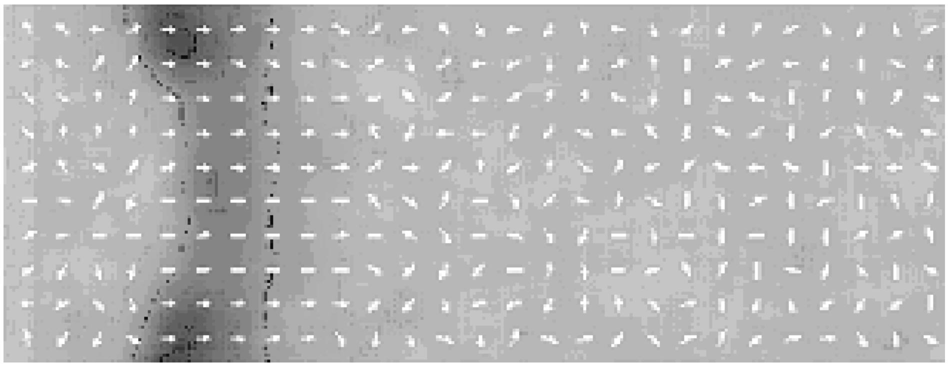
Figure 13 shows the HPPRP evolution, simulating the sound wave propagation in the pipe with such rest particles distribution for which the condition $n > k > l$ is not met. There are: one rest particle with the masses equal 2 in the first-layer cells, two rest particles with the mass equal 2 and 4 in the second-layer cells and three rest particles with the mass equal 2, 4, and 8 in the middle part of the array. In this case, in the middle part of the array the sound wave velocity less than the sound wave velocity along the array wall.

4. Conclusion

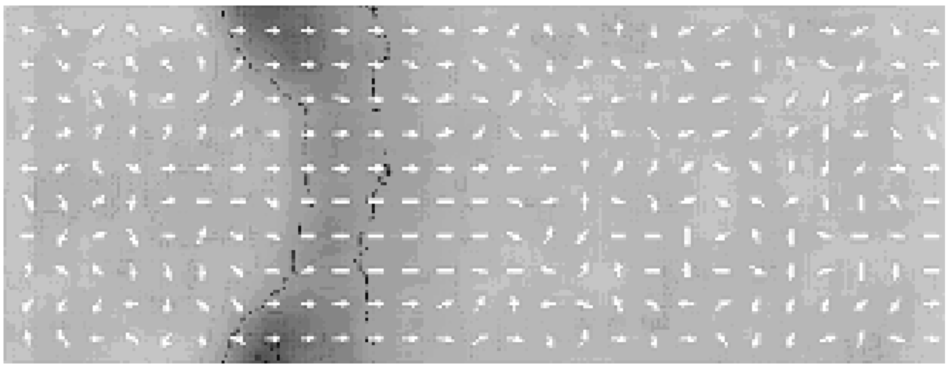
In this paper, the possibility of a simple LGA models to simulate sound wave process in a pipe taking into consideration the wall features (moving obstacle and wall absorption) are investigated. Two LGA models on a 2D lattice with four neighbors (the HPP, and the HPPRP) are used here. The moving obstacle is generated as follows. From some cell of the wall, the moving obstacle begins to grow perpendicular to the wall: the work cells on the way of movement of an obstacle are replaced by the wall cells after several



$t = 100$

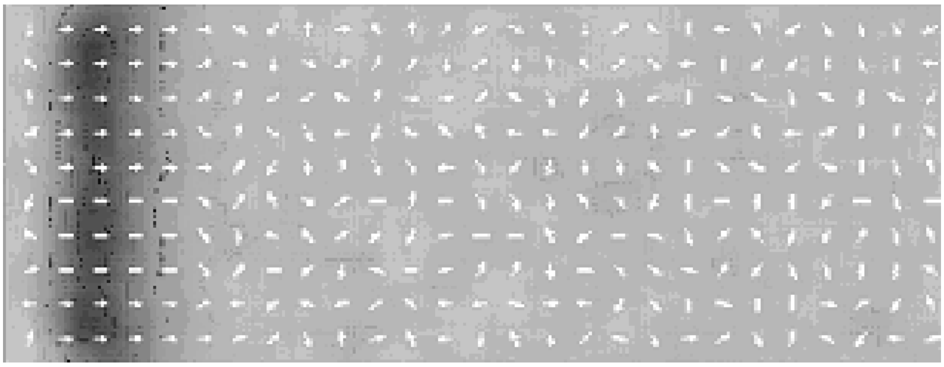


$t = 200$

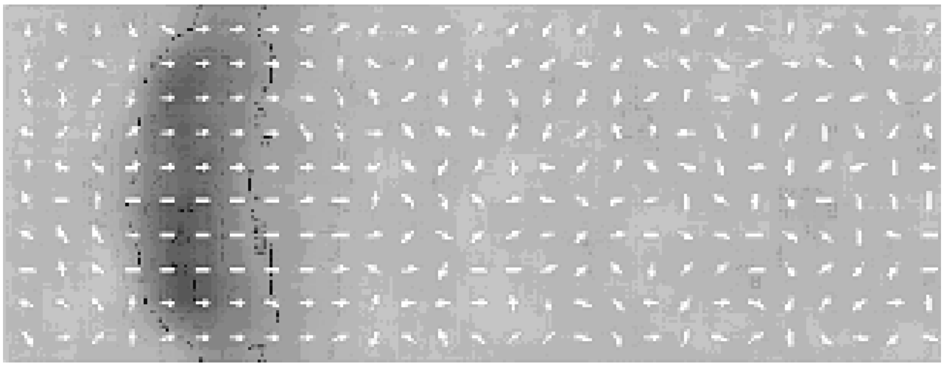


$t = 450$

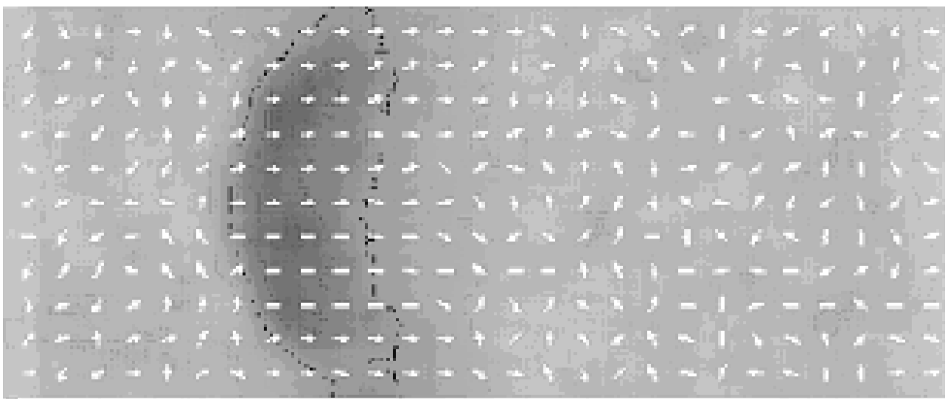
Figure 12. Velocity vector fields of the HPPRP evolution simulating wave process in a pipe with wall absorbtion



$t = 100$



$t = 200$



$t = 450$

Figure 13. Velocity vector fields of the HPPRP evolution simulating wave process in a pipe without wall absorbtion

iterations. The idea of simulating a wall absorbtion is based on using the HPPRP cells with different rest particle numbers. The condition of the wall absorption effect modeling is given. The experiments have been shown that an appropriate combination of boundary layers and rest particles numbers makes possible to simulate sound wave process in a pipe having regard to the wall features.

References

[1] Frisch U., Crutchfield J.P., Hasslacher B., Lallemand P., Rivet J.-P. Lattice gas hydrodynamics in two and three dimensions // Complex Systems. — 1987. — Vol. 1. — P. 649–707.

[2] Rothman D.H., Zaleski S. Lattice-Gas Automata. Simple Models of Complex Hydrodynamics. — London: Cambridge Univ. Press, 2001.

[3] Hardy J., Pomean Y., de Pazzis O. Time evolution of a two-dimensional model system // J. Math. Physics. — 1973. — Vol. 14. — P. 1746–1752.

[4] N.R. Simons, G.Bridges, B. Podaima and A. Sebak. Cellular automata as environment for simulating electromagnetic phenomena // IEEE Microwave and Guided Wave letters. — 1994. — Vol. 4. — P. 247-249.

[5] Zhang M., Cule D., Shafai L., Bridges G., Simons N. Computing electromagnetic fields in inhomogeneous media using lattice gas automata // Proc. Symposium on Antenna Technology and Applied Electromagnetics, Aug. 14–16, 1998, Ottawa, Canada.

[6] Bandman O. Computational Properties of Spatial Dynamics Simulation by Probabilistic Cellular Automata // Future Generation Computer System. — 2005. — Vol. 21. — P. 633–644.

[7] Medvedev Yu.G. Gas-lattice simulation in high viscous flows // Bull. Novosibirsk Comp. Center, Ser. Computer Science. — Novosibirsk, 2002. — Iss. 17. — P. 63–73.

[8] Medvedev Yu.G. Three-dimensional cellular automata model of viscous fluid flow // Avtometriya. — 2003. — Vol. 39, No. 3. — P. 43–50 (In Russian).

[9] Markova V., Deykun M. Experimental investigation of cellular automata for wave processes simulation // Avtometriya. — 2006. — Vol. 42, No. 3. — P. 13–20 (In Russian).