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Averaging methods with isotropy conservation in the FHP-GP CA model*

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Abstract. This paper presents the results of a research into the anisotropy factor introduced by the hexagonal structure of a cellular array in a discrete model of a gas-powder flow with an integer alphabet and a hexagonal neighborhood structure at the stage of calculating the average values of pressure and flow velocity. The averaging method to obtain isotropic velocity and pressure fields with an arbitrary slope angle to the coordinate axes is formulated. The conclusions reached are supported by the results of the computational experiments conducted with the help of a specialized software package on a computing cluster.

1. Introduction

The research into using the cellular automata for modeling complex nonlinear processes of spatial dynamics has been conducting for half a century [1–3]. The FHP-GP model [4] used in the software package [5] is a parallel composition [6] of two-dimensional cellular automata [7] with Boolean and integer alphabets and a hexagonal neighborhood structure.

The final stage of the simulation consists in the conversion of the discrete values of the concentration of model particles in cells with discrete coordinates to continuous values of the gas pressure at any given point of a plane, and the discrete values of the velocity of these particles — to continuous values of the projections of the flow velocity on the coordinate axes.

The aim of the paper is to formulate the principles for constructing gas pressure fields and flow velocity in the Cartesian system with a constant value step, as well as profiles of these quantities at an arbitrary angle to the coordinate axes for the FHP-GP two-dimensional cellular automata model of a gas-powder flow with a hexagonal cellular array.

2. Problem statement

The main objective of this study is to provide the means of locating the points in which the averaged values of the velocity and concentration of particles are computed with a constant step both along each of the coordinate axes, and for an arbitrary angle with one of the coordinate axes.

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*Computational experiments have been carried out on a cluster of JSCC RAS.

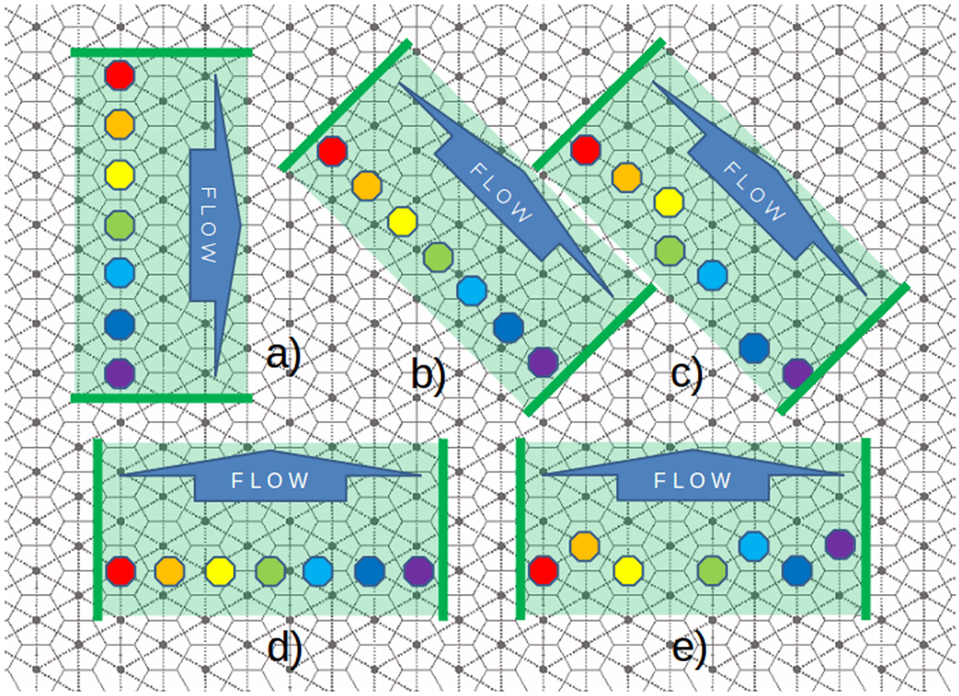


Figure 1. Construction of the velocity profile using symmetric and accidental averaging vicinities: a) 0°, symmetric & accidental; b) 45°, accidental; c) 45°, symmetric; d) 90°, accidental; and e) 90°, symmetric

In the process of computation of the averaged values of velocity and pressure, the need for transition to continuous coordinates is due to the impossibility of placing on the hexagonal structure of the averaging vicinity with an equal step along both coordinate axes and, especially, the one directed at a random angle to the axes, in such manner that the center of each of the vicinities coincides with the center of a hexagon (Figure 1).

3. Methods of symmetric vicinity and of vicinity with a random center

Some examples of constructing a symmetric vicinity and two particular cases of asymmetric vicinities of averaging by a radius of three cells are represented in Figure 2. All symmetric vicinities of a given radius consist of the same number of cells, because they are constructed from the center of the cell. Asymmetric vicinities, despite of the same radius, can contain a different number of cells, because their centers may be at different distances from the center of the nearest cell.

In Figure 3, the dimensions of the simulated pipe are represented for three experiments with the angle of 0, 45, and 90 degrees to the axis x,

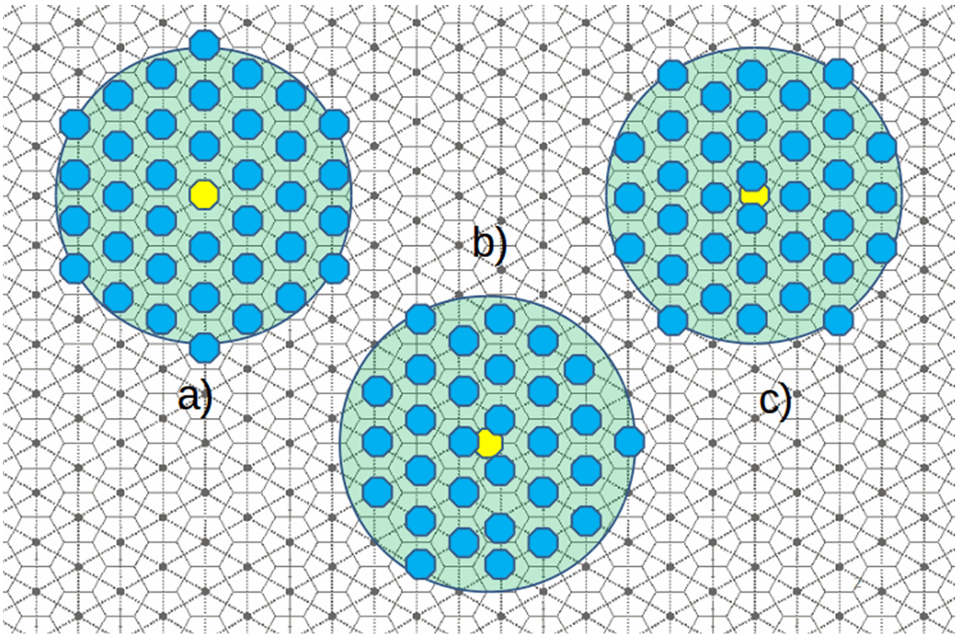
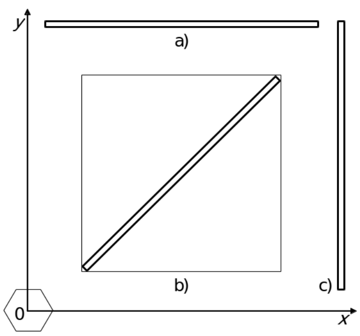


Figure 2. Types of averaging vicinities: a) symmetric, 37 cells; b) asymmetric, 30 cells; and c) asymmetric, 34 cells



Angle, degrees	Size x, metrical	Size y, metrical	Size x, columns	Size y, lines
0	1600	40	1848	40
45	$1640/\sqrt{2}$	$1640/\sqrt{2}$	1339	1159
90	40	1600	46	1600

Figure 3. Dimensions of the simulated pipe in the cell array: a) 0°, 1848×40 cells; b) 45°, 1339×1159 cells; c) 90°, 46×1600 cells

respectively. For each of these directions, a series of experiments was carried out, in which the pressure of the pipe outlet is varied in the range from 2 to 100 model units. At the inlet, the pressure is greater than at the outlet by two model units. The velocity profile was built in the middle of the pipe in order to eliminate the edge distortion.

4. Simulation results

4.1. Symmetric and asymmetric vicinities. The profiles with a symmetric and asymmetric averaging vicinity are shown in Figure 4. As far as it is possible to construct a symmetric vicinity only from the center of a cell, the cell that is the closest to a required point was chosen. This approximation gives specific increments of the expected Poiseuille parabola at all pressures in sections with identical coordinates.

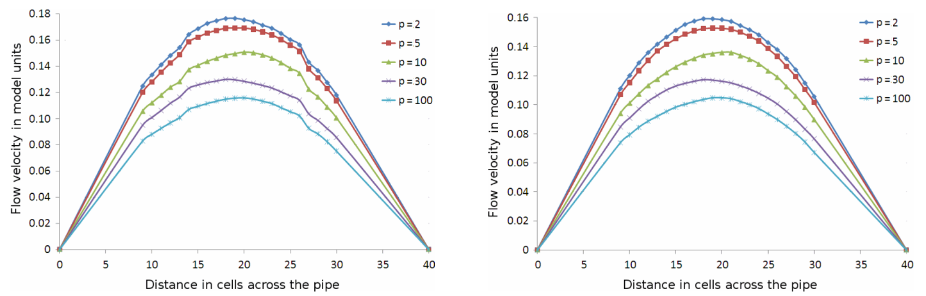


Figure 4. Velocity profiles at outlet pressure from 2 to 100 model units for symmetric (left) and asymmetric (right) vicinities

When using an asymmetric vicinity, its center is located at a point of space in which it is required to obtain the value of the flow velocity. Therefore, despite of different powers of the vicinities at the required points, the parabolas without steps were obtained.

4.2. Different directions of a flow. When calculating the projections of the particle vectors onto one of the Cartesian axes, a maximum possible value is greater than that of the projection onto the other axis. Therefore, the second task was to search for a correction coefficient, in which orthogonally located flows produced by an equal difference in pressure would have the same velocity profile.

The nature of the discrepancy between the projections is simple: the maximum possible projections of the model particles velocities in a cell onto the coordinate axes (Figure 5) do not coincide because one of the six non-zero projections gives all six velocity vectors, and the others — only four vectors. The projections differ by a factor of $\sqrt{3}$.

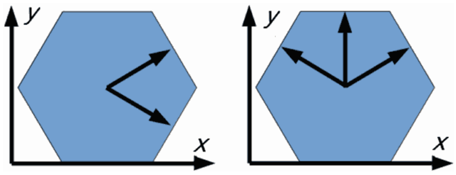


Figure 5. Particles involved in calculating a maximum value of the flow velocity along the axis x (left) and y (right)

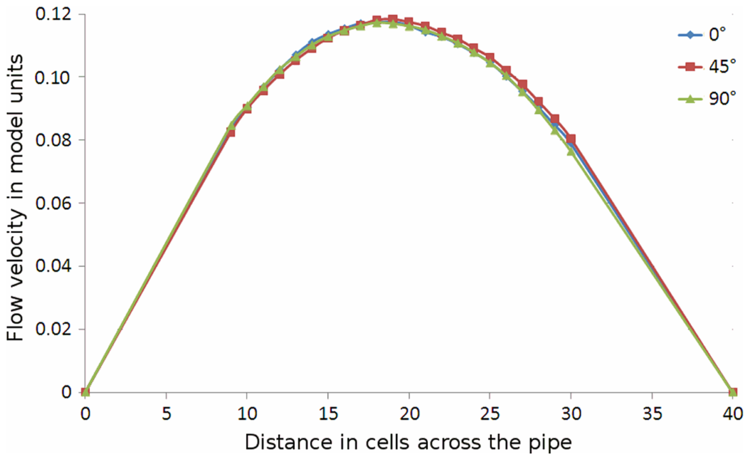


Figure 6. Matching the velocity profiles after the correction for directions. The flow propagates at angles of 0, 45, and 90 degrees to the axis x

This correction for the flow rate was experimentally confirmed (Figure 6). Its application entails equal values of the flow velocity regardless of the direction of its movement.

5. Interpretation of the results

The results of the experiments have shown the following:

1. The power of the symmetric averaging vicinity depends only on its radius; such vicinities give a less noisy result.
2. Asymmetric averaging vicinities bring a result precisely at a required location, and they allow one to obtain the velocity and pressure fields with a constant step between values.
3. This is a criterion for choosing the type of vicinity: if the step between the values of velocity and pressure is much greater than the distance between the neighboring cells, it is advisable to use a symmetric averaging vicinity; otherwise asymmetric one is preferred.
4. While averaging, a correction for the direction to a projection of the flow velocity should be applied in order to ensure the isotropy of the fields to be sought for.

5. The introduction of the correction for the direction of the flow makes it possible to reproduce more reliably the chemical transformations of the reagents moving in gas jets at different angles to the coordinate axes.

Conclusion

The anisotropy factor introduced by the hexagonal structure of the cellular array is investigated. The averaging method of the obtaining the isotropic velocity and pressure fields with an arbitrary slope angle to the coordinate axes is formulated. The conclusions reached are supported by the results of the computational experiments.

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