An approach to the utility network design^{*}

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Abstract. In this study, the problem of the utility network design is treated by the hypernet approach according to the compatibility of different types of resources with allowance for their laying in the same track. Also, we study the reliability aspect of the designed utility network for obtaining the optimal laying of a sufficient reliable utility network on a given area with minimal costs. The performed numerical experiments show how the method proposed works.

Keywords: multilayer network, utility network, deployment area, track, graph, hypergraph, hypernet, connectivity, network reliability

1. Introduction

Currently, there is a significant number of published works related to design and construction of networks for various purposes [1-13]. In particular, in [3, 4] the authors consider the search for the optimal routes for the laying of the utility networks. The task of locating the construction objects on a given territory is solved in [5, 6]. In [7, 8], the possibility of applying GIS-technologies in tracing and placing of linear objects is studied. An application of splines in the optimization of the car roads tracing is treated in [9]. Moreover, problems of the utility networks optimization also arise in other adjacent areas, such as the placement of logistics facilities [10] and power supply systems [11], design and reconstruction of transport networks [12], and routing of service networks [13].

Based on the hypernet approach [14], we solve the task of providing the connectivity of given consumers with given sources on a given area with minimization of the laying and maintenance costs and within given constraints. As a result, we propose a new method for the selection of routes for the laying of the utility networks taking into account the compatibility of different types of resources for placing them in the same track. The joint laying of gas pipelines and pipelines with combustible substances is not permitted due to the fact that the distance between communications of different purposes is normalized according to building codes and regulations. Our previous results on the hypernet using for the utility network design are presented in [15, 16].

^{*}Supported by the Basic Research Program of the ICM&MG SB RAS No. 0315-2016-0006.

Also, we introduce an algorithm for obtaining not only the cheapest solution, but reliable enough as well. Note that the reliability analysis of hypernets is not a new task, see, for example [17, 18]. As a reliability measure, the hypernet analogue of the 2-terminal reliability is considered, under assumption that failures occur in the primary network, and nodes in the secondary network should be connected.

2. Definitions and notations

We use the classical definitions from the hypernet theory [14]:

Hypernet HN = (X, V, R; F) consists of a set of nodes $X = (x_1, \ldots, x_n)$, a set of branches $V = (v_1, \ldots, v_g)$, and a set of edges $R = (r_1, \ldots, r_m)$; PN = (X, V) is a graph of primary network; WN = (X, R) is a graph of secondary network; $F : R \to 2^V$ is a mapping associating each item $r \in R$ with the path $F(r) \subseteq V$.

In the primary network PN = (X, V) for $v \in V$ we assume: $\rho(v)$ is the length of v, a(v) is the unit cost of land (rent, taxes, etc.), b(v) is the unit cost of construction (excavation) work for the type of linear structures, γ_1 is the discount rate on construction costs (this factor is for efficient indicators from different years comparable to the time values).

In the secondary network WN = (Y, R) for $r \in R$ we assume: $\rho(r) = \sum_{v \in F(r)} \rho(v)$ is the length of r, c(r) is the cost of r including its installation and laying between the corresponding elements of $Y \in D$, γ_2 is the discount factor cost equipment, $d_v(r)$ is the linear operating costs constructions in plots $v \in V$ of a chosen route F(r), T is a set of all types of utility resources. Thus, each edge r has a type $t(r) \in T$.

As was said above, different resource types may be incompatible with each other for their laying in the same track. For the description of the compatibility of different types of resources, we introduce the following notations.

A binary relation $CT \subset T \times T$ is defined by the rule: if $(t_1, t_2) \in CT$, then these types of resources can be placed in the same track, i.e. both of them can include the same branch.

Let $MinCT(t_1, \ldots, t_h)$ be the minimum number of disjoint subsets into which a subset of the types $\{t_1, \ldots, t_h\}$ can be divided.

3. Statement of the problem

In the general case, the task of a utility network design can be formulated as a problem of providing the connectivity between given objects Y_{source} and Y_{consumer} which are placed on a two-dimensional area D with minimization of the laying and maintenance costs. The obtained configuration determines the topology of the designed utility network. In terms of the hypernet theory the problem is to find the hypernet HN, i.e. to embed each path $r \in R$ in WN into the edges of the graph PN in such a way that all conditions and restrictions imposed on the utility network be met, and the functional below would take the minimum value:

$$Q(\text{HN}) = \sum_{v \in V'} \left(a(v) + \gamma_1 b(v) \right) \rho(v) \operatorname{MinCT}(v) + \sum_{r \in R} \left(c(r) + \sum_{v \in F(r)} \gamma_2 d_v(r) \right) \rho(r),$$
(1)

where $v \in V' \subseteq V$ if $\exists r \in R$ such that $v \in F(r)$. Let $v \in V'$ and $v \in F(r_i)$, $r_i \in R, i = 1, ..., h$, then $\operatorname{MinCT}(v) = \operatorname{MinCT}(t(r_1), ..., t(r_h))$.

4. The algorithm

The basic idea of our algorithm is to find an initial estimate of HN and its total cost (1) and to improve it. The initial estimate is found by the Floyd algorithm or the Greedy algorithm.

The cost of $v \in V$ is $(a(v) + \gamma_1 b(v) + \gamma_2 c(r) + d_v(r))\rho(v)$ (for simplicity $\gamma_2 c(r) + d_v(r) = \text{const for all } r$) for the Floyd and the Dijkstra algorithms on the primary network PN.

Below we outline the steps of the Greedy algorithm:

Divide a set of types into incompatible subsets.

repeat

Choose a subset of the types T' with a maximum number of edges (return the values a(v), b(v) for all $v \in F(r)$ in the graph PN).

repeat

Find all the shortest paths (x_i, x_j) $i, j = \overline{1, n}, i \neq j$ in the graph PN by the Floyd algorithm $(t(x_i, x_j) \in T')$.

Choose a minimum value from a set of the shortest paths, where $(x_i, x_j) \in R$ in the graph WN (the path (x_i, x_j) is not assigned for any $r \in R$).

Assign the edge $r \in R$ to the shortest path (x_i, x_j) in the graph PN. State a(v) := 0, b(v) := 0 for all $v \in F(r)$ in the graph PN

(the costs of branches are zero for assigning a path).

until $\forall r \in R$ in the graph WN, there is an assigned path (x_i, x_j) in the graph PN.

until $T' \neq \emptyset$.

The Greedy algorithm is approximate, therefore the solutions obtained are not always optimal. We can improve a solution if for an edge $r \in R$ we reassign new paths F(r). For example, a new path F(r) is cheaper if it includes a greater number of branches $v \in F(r)$ with zero cost (branches that have already been used for other edges $r \in R$).

We can order edges by a certain technique. Earlier we have considered a few techniques [16], for example, the ordering by the inclusion of the most rarely used branches. However, on the average, the results obtained did not strongly depend on the ordering method used. We can use some of such iterations for finding better results.

5. Results of numerical experiments

For numerical experiments we have chosen the 10×10 grid as a primary network PN. The number of edges in WN for the laying in the PN varied from 10 up to 4000 (the abscissae axis). For a chosen number of edges n, we randomly place n sources and n consumers in the grid. As a result, a grid node can contain more than one object: source(s) and consumer(s). A random placement is carried out 10 times for a given value of n. As a laying cost for n we take the average among 10 values found. We assume that there are three different types of edges.

In Figure 1, the normalized costs are shown (i.e. we find a minimum cost and divide all the obtained values by it). The results show that the Greedy algorithm is more convenient for a small number of edges; otherwise the Floyd algorithm gives a better solution.



Figure 1. Numerical results: the normalized costs for 10×10 grid, the number of edges from 10 up to 800



Figure 2. Numerical results: the normalized costs for 10×10 grid, the number of edges from 10 up to 1000

In Figure 2, we show the results of the previous numerical experiments for the case of compatibility of all resources types for the laying in the same track [16]. The Performing Improvement stage with different orders of branches leads us to the techniques Improve1 and Improve2. The results are fairly similar with the numerical experiment in Figure 1.

6. The utility network laying with reliability constraint

In this section, we improve the algorithm proposed in order to obtain not only the cheapest, but also a reliable enough solution. Most of the results presented coincide with those from our previous publication [15].

Let us assume that branches (edges of the primary network) are subject to random failures that occur statistically independently with known probabilities p_i , $1 \le i \le g$.

We define the reliability of an edge $r \in R$ as

$$R_r(\mathrm{HN}) = \prod_{v \in F(r)} p(v).$$
(2)

If for an edge $r \in R$ the path F(r) has the endpoints a and b, we use the notation $R_{ab}(\text{HN})$ instead $R_r(\text{HN})$. If for the nodes a and b there are more

than one of such edges, we choose one with a maximum reliability value.

We consider the following reliability measure R(HN), taking into account the fact that failures occur in the primary network, and all consumers should be connected with the corresponding sources:

$$R(HN) = \min\{R_{ab}(HN), \ a \in Y_{\text{source}}, \ b \in Y_{\text{consumer}}\}.$$

It is assumed that we are given a reliability threshold $0 < R_0 \leq 1$. The objective is to find a sufficiently reliable solution of problem (1). In other words, we find a solution HN of problem (1) such that $R(\text{HN}) \geq R_0$. In the case of the reliability constraint, we use the Ant Colony Algorithm instead of the Floyd Algorithm in the two-stage algorithm.

Below in Figure 3 we present the numerical results of the algorithm proposed for $p_i = 0.99$, $1 \le i \le g$, $R_0 = 0.9$. For the primary network PN we consider 10×10 grid (|X| = 100).

The number of edges was from 10 up to 100 in WN (the abscissae axis). The axis of ordinate is the cost Q(HN). The first and the second algorithms are the Floyd and the Greedy respectively, without constraint (2). Therefore, the found value Q(HN) is less than the result of FloydProb and AntColony (FloydProb is the Floyd algorithm with constraint (2)). Figure 3 shows that the AntColony results are better than the FloydProb results only for a small number of edges.



Figure 3. Numerical results: costs for 10×10 grid, the number of edges from 10 up to 100

7. Conclusion

The new method for the tracks laying of a utility network is proposed, which, unlike the conventional methods of optimization, considers naturaltechnical system of a land plot and a utility network within the framework of the unified mathematical model.

This method, which is based on the hypernet approach, gives an appropriate solution with allowance for the compatibility of different types of resources for placing them in the same track.

The numerical experiments were conducted to calculate the cost of a laying a secondary network in a primary network with allowance for the network reliability and the compatibility of different types of resources for placing them in the same track.

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