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Modeling of seismic wave propagation during an earthquake in complex heterogeneous media

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Abstract. The direct dynamic problem of elasticity theory is solved numerically taking into account the energy dissipation caused by viscous (internal) friction, which models the formation and propagation of seismic waves from earthquakes. The problem is written in the form of dynamic equations of elasticity theory in terms of components of displacement and stress velocities for a two-dimensional Cartesian coordinate system. An effective algorithm for solving this direct dynamic problem of seismics is proposed. The numerical solution of the problem is based on the method of combining the analytical Laguerre transform and the finite-difference method. Numerical results of modeling seismic wave fields for a realistic model of the Baikal rift zone media are presented.

Keywords. Elastic medium, seismic waves, earthquakes, direct problem, Laguerre transform, difference scheme.

Introduction

The seismic waves recorded characterize not only the earthquake source but also the medium through which they propagate, so they are the main carrier of information in seismology. The most destructive waves in earthquakes are surface waves, as they have a low frequency, high amplitude and an impressive duration of action. Direct longitudinal seismic waves generated by the shift of tectonic plates of the earth's crust over large spatial areas also have a great destructive force. As a result of this type of earthquake source, an extended plane longitudinal wave with a large amplitude is generated. The amplitude of these waves is affected not only by the geological structure in the earthquake source, but also by the structure and physical properties of the overlying layers of the medium.

Mathematical methods based on the propagation of seismic waves in an acoustic or ideally elastic medium are successfully applied to various geophysical problems for the identification of geological structures [1].

In this paper, a direct dynamic problem, written as a hyperbolic system in terms of displacement velocities and stress tensor, is numerically solved for modeling the process of seismic wave propagation in an elastic medium. The problem is solved numerically by combining the analytical Laguerre transform with respect to time and the finite-difference method with respect to space. This method for solving dynamic problems of elasticity theory was first considered in [2, 3] and then developed for viscoelasticity problems [4, 5]. The proposed solution method can be considered as an analogue of the well-known spectral method based on the Fourier transform, but instead of frequency we have the parameter m, the degree of the Laguerre polynomials. However, unlike Fourier, the use of the integral Laguerre transform with respect to time allows us to reduce the original problem to solving a system of equations in which the separation parameter is present only on the right-hand side of the equations and has a recurrent dependence. Unlike the finite-difference method, the spectral-difference method, using an analytical transformation, can reduce the original problem to solving a differential system of equations in which there are derivatives only with respect to spatial coordinates. This allows the use of known stable difference schemes for the subsequent solution of such systems [6]. The works [3, 5] consider the distinctive features of this method from the accepted approaches and discuss the advantages of using the Laguerre transformation.

1. Problem statement

The propagation of seismic waves in an elastic medium in the case of energy dissipation due to viscous (internal) friction is recorded by the well-known system of first-order equations of elasticity theory [7] through the relationship between the components of the displacement velocity vector and the components of the stress tensor in a Cartesian coordinate system (x_1, x_2):

$$\frac{\partial u_i}{\partial t} + \vartheta_R u_i = \frac{1}{\rho}\frac{\partial \sigma_{ik}}{\partial x_k} + F_i f(t), \tag{1}$$

$$\frac{\partial \sigma_{ik}}{\partial t} + \vartheta_R \sigma_{ik} = \mu\Big(\frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k}\Big) + \lambda\delta_{ik}\,\mathrm{div}\,\vec{u}. \tag{2}$$

Here δ_{ik} is the Kronecker symbol, $\lambda(x_1, x_2)$, $\mu(x_1, x_2)$ are the elastic parameters of the medium, $\rho(x_1, x_2)$ is the density of the medium, ϑ_R is the coefficient of viscous friction, $\vec{u} = (u_1, u_2)$ is the displacement velocity vector, σ_{ij} components of the symmetric stress tensor, $f(t)$ is the specified time signal in the source, F_1, F_2 are the components of the force $\vec{F}(x, z) = F_1\vec{e}_x + F_2\vec{e}_z$ describing the distribution of the source localized in space. Depending on the type of the source being modeled, F_1, F_2 will have the following values [8]:

1. For a source of the "vertical force" type

$$F_1 = 0, \quad F_2 = \delta(x - x_0)\delta(z - z_0).$$

2. For a source of the "center of pressure" type

$$F_1 = \frac{\partial\delta(x - x_0)}{\partial x}\delta(z - z_0), \quad F_2 = \delta(x - x_0)\frac{\partial\delta(z - z_0)}{\partial z_2}.$$

3. For a source of the "dipole without moment" type

$$F_1 = 0, \quad F_2 = \delta(x - x_0)\frac{\partial\delta(z - z_0)}{\partial z}.$$

The problem is solved with zero initial data

$$u_i|_{t=0} = \sigma_{ij}|_{t=0} = 0 \tag{3}$$

and boundary conditions on the free surface

$$\sigma_{12}|_{x_2=0} = \sigma_{22}|_{x_2=0} = 0. \tag{4}$$

2. Solution algorithm

To solve problem (1)–(4), we use the integral Laguerre transformation in time of the form [2, 3]:

$$\vec{W}_m(x_1, x_2) = \int_0^\infty \vec{W}(x_1, x_2, t)(ht)^{-\alpha/2} l_m^\alpha(ht)\, d(ht),$$

with the inverse formula

$$\vec{W}(x, t) = (ht)^{-\alpha/2} \sum_{m=0}^{\infty} \frac{m!}{(m+\alpha)!} \vec{W}^m(x) l_m^a(ht),$$

where $l_m^\alpha(t)$ are the orthogonal Laguerre functions.

After applying the integral Laguerre transformation in time, the original problem (1)–(4) is reduced to solving a system of differential equations only in spatial coordinates (x_1, x_2):

$$\Big(\frac{h}{2} + \vartheta_R\Big) u_i^m + \frac{1}{\rho}\frac{\partial \sigma_{ik}^m}{\partial x_k} + F_i f^m - h \sum_{n=0}^{m-1} u_i^n, \tag{5}$$

$$\Big(\frac{h}{2} + \vartheta_R\Big)\sigma_{ik}^m + \mu\Big(\frac{\partial u_k^m}{\partial x_i} + \frac{\partial u_i^m}{\partial x_k}\Big) + \lambda\delta_{ik} \operatorname{div} \vec{u}^m = -h \sum_{n=0}^{m-1} \sigma_{ik}^n, \tag{6}$$

where

$$f^m = \int_0^\infty f(t)\, (ht)^{-\alpha/2}\, l_m^\alpha(ht)\, d(ht).$$

For further solution of the problem, a finite-difference approximation of derivatives on shifted grids [9] with the fourth order of accuracy is used. Let us determine the desired components of the solution vector at the following grid nodes:

$$u_1(m) \in \omega\, x_1^i \times \omega\, x_2^j, \quad u_2(m) \in \omega\, x_1^{i+1/2} \times \omega\, x_2^{j+1/2},$$
$$\sigma_{11}(m), \sigma_{22}(m) \in \omega\, x_1^{i+1/2} \times \omega\, x_2^j, \quad \sigma_{12}(m) \in \omega\, x_1^i \times \omega\, x_2^{j+1/2}.$$

As a result of finite-difference approximation of problem (5)–(6), we obtain a system of linear algebraic equations. Let us represent the desired solution vector in the following form [7]:

$$\vec{W}(m) = (\vec{V}_0(m), \vec{V}_1(m), \dots, \vec{V}_{K+N}(m))^T,$$
$$\vec{V}_{i+j}(m) = (u_1^{ij}, u_2^{i+1/2,j+1/2}, \sigma_{11}^{i+1/2,j}, \sigma_{22}^{i+1/2,j}, \sigma_{12}^{i,j+1/2})^T.$$

Then, the system of linear algebraic equations obtained as a result of transformations in vector form can be written down as:

$$\Big(A_\Delta + \Big(\frac{h}{2} + \vartheta_R\Big)E\Big)\vec{W}(m) = \vec{F}_\Delta(m-1).$$

To solve this system of linear algebraic equations, the iterative method of conjugate gradients is used. The advantage of this method is rapid convergence to the solution of the problem, provided that the matrix of the system is well conditioned. The matrix of the system obtained as a result of the Laguerre transformation has this property due to the introduced Laguerre parameter h, located on the main diagonal.

3. Numerical simulation

For numerical calculations when modeling the propagation of seismic waves generated as a result of the earthquake, a model of the medium was specified that describes the Baikal rift zone, which is characterized by increased seismic activity and has a complex geological structure. The model of the medium is shown in Figure 1. The generation of seismic waves in the epicenter of the earthquake as a result of tectonic shift of the lower layers of the earth's crust located near the Mohorovicic boundary was specified as a dipole-type source without a moment in equations (1) located at a point with coordinates $x_0 = 50$ km, $z_0 = 45$ km [10]. The coefficient of viscous friction in the equations of system (1), (2) $\vartheta_R = 0,001$. Figures 2–4 show instantaneous snapshots of the wave field for the displacement velocity component at times $t = 4$ and 8 seconds. The time signal in the sources was specified as a Puzyrev pulse:

$$f(t) = \exp\Big(-\frac{2\pi f_0(t-t_0)^2}{\gamma^2}\Big)\sin(2\pi f_0(t-t_0)),$$

where $\gamma = 4$, $f_0 = 2$ Hz, $t_0 = 0.75$ s.

For the numerical solution of the stated problem of seismic wave propagation in an elastic medium, it is necessary to introduce a limited spatial region of the medium model. The main problem in this case is the reduction to an acceptable level of the intensity of reflected waves generated by the introduced fictitious lateral boundaries. Currently, one of the methods for solving this problem is to border the computational domain with an absorbing layer with specially selected parameters, the introduction of which does not lead to the occurrence of reflections (PML — perfectly matched layer) [11–13]. This approach is effective for numerical modeling in complex elastic media, especially when using high-precision finite-difference schemes, since it does not lead to numerical instability of the solution. This approach was initially proposed for the numerical modeling of electromagnetic waves, and then used to calculate elastic wave fields. The main advantage of this approach is the fact that the attenuation of waves inside the PML layer occurs

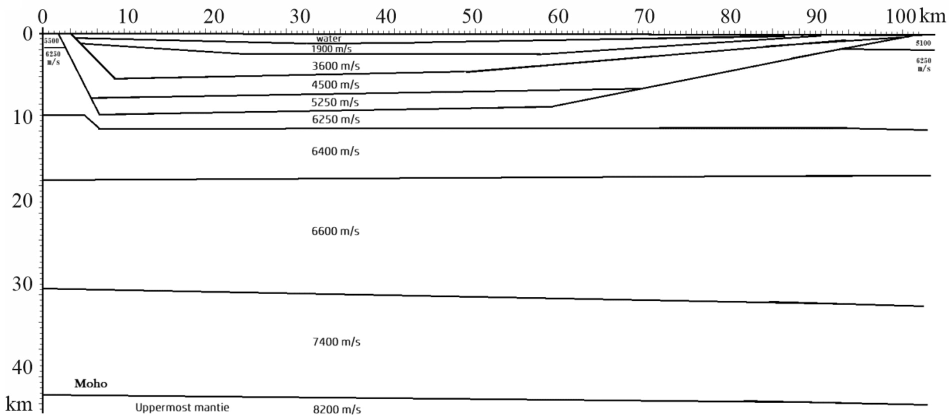

Figure 1. Model of the media

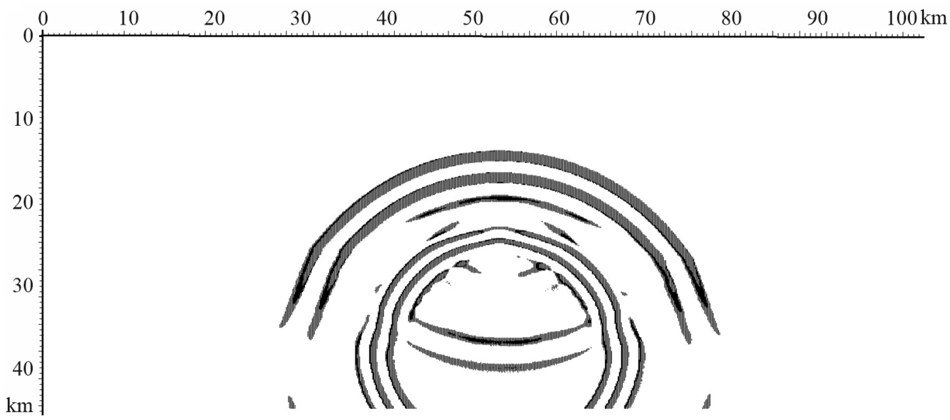

Figure 2. Snapshot of the u_2 wavefield — components for $t = 4$ seconds. Source of the “dipole without moment” type

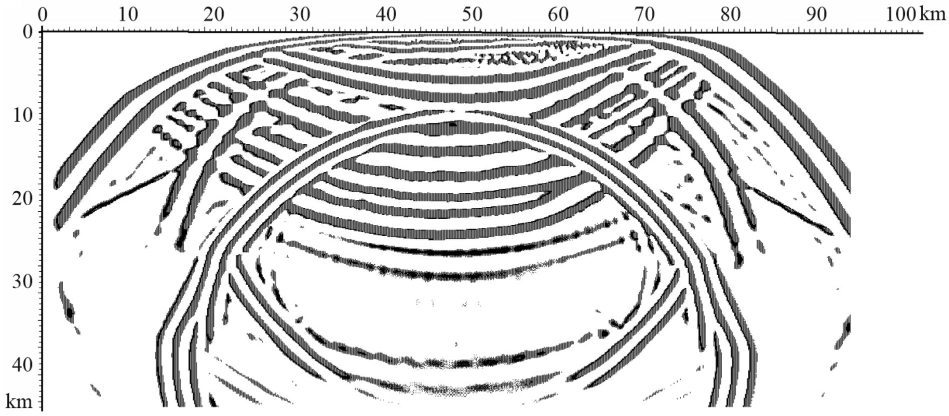

Figure 3. Snapshot of the u_2 wavefield — components for $t = 8$ seconds

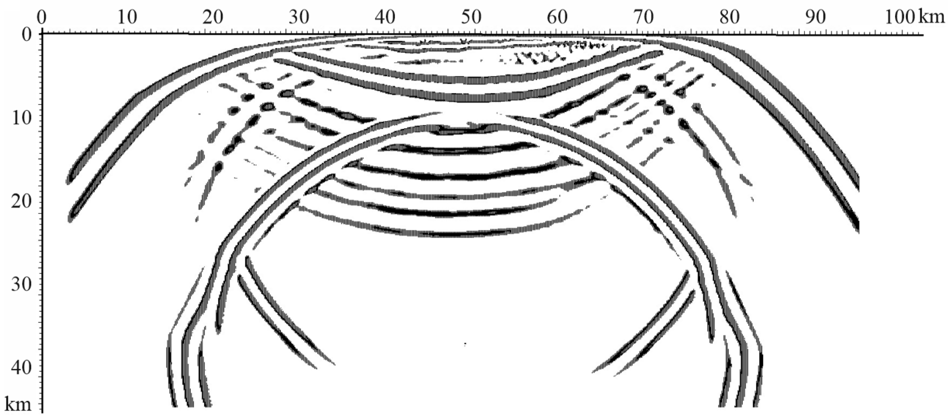

Figure 4. Snapshot of the u_2 wavefield — components with absorption at $t = 8$ seconds

regardless of the direction of their propagation. Numerical experiments using this approach have shown its high efficiency with a thickness of these layers of 2–3 wavelengths.

The snapshots of wave fields presented in Figures 3 and 4 show the formation of a complex interference pattern due to the reflection of various seismic waves from the free surface and the boundaries of the medium layers. From examining these images of the wave field, it is evident that, depending on the location of the layer boundaries, focusing of various waves on individual sections of the medium can be formed, leading to an increase in the amplitude of elastic oscillations in these places. It should be noted that the total amplitude of the waves in these focusing places decreases due to a decrease in the amplitude of the waves repeatedly reflected from the boundaries.

Conclusion

The results of numerical calculations show the efficiency of the algorithm used for solving the problem of modeling seismic wave propagation in complex heterogeneous media. Analysis of the obtained calculations of the visual picture of the wave field as a result of seismic wave propagation in such media shows the possibility of focusing the energy of seismic oscillations on certain sections of the medium in the area of the earthquake, which leads to a significant increase in the amplitude of these oscillations. This effect, as can be seen from the presented modeling results, depends on the geometry of the medium structure and the frequency of oscillations propagating in it. The results of modeling the emerging wave pattern depending on the frequency of seismic oscillations are given in [10, 14]. This fact should be taken into account when constructing technical structures on the surface, as well as inside the medium, and performing this kind of numerical modeling. In further studies, it is proposed to study the effect of the occurrence of resonance of natural oscillations in these structures and external seismic oscillations excited by earthquakes.

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