Bull. Nov. Comp. Center, Num. Anal., 4 (1993), 35–44

Special models of non-stationary random processes and non-homogeneous fields*

V.A. Ogorodnikov

In this paper some methods of statistical simulation of non-Gaussian non-stationary scalar and vectorial processes and non-homogeneous spatial fields with continuous argument on the basis of synthesis of discrete models and models on point fluxes are considered.

Introduction

The most known technique to simulation of non-stationarity and non-homogeneity reduce to the use of spectral parameteric models, in which parameters are some functions of time [1]. Covariance functions of such processes depend on two variables (for spatial fields these variables are vectorial), and in addition one of them is time increment. If analytical expressions for such functions are known, then the corresponding spectral representation is selected and this representaion determines the algorithm of simulation. Another way for simulation of non-stationary processes, for example, of non-stationary discrete sequences, is simulation of autoregressive sequences with the coefficients depending on time. In some cases one may use the methods, based on simulation of periodically correlated non-stationary processes [2].

For many applications a peculiarity of simulation is that observation data of the investigated process are the original information for a model. The estimate of correlation structure is made at fixed time points, so for the use of spectral parametric models, it is necessary to approximate the corresponding correlation matrix by some positive definite analytical functions of a proper number of variables. In practice the structure of correlation matrices may be rather complicated, so it appears difficult to find a suitable approximate function.

In this paper a simple approximate approach, which allows to avoid these difficulties and take into account in the model the change in time and

*This work was supported by Russian Fund of Fundamental Research

space not only correlation properties, but also one-dimensional probability distributions, is considered.

1. Stochastical joint of independent random vectorial sequences

This approach is based on a stochastical (randomized) joint of random processes, given on some sequence of disjoint intervals [3]. The process inside each time interval has one-dimensional distribution and correlation structure peculiar only to this interval. For simplicity, we will restrict ourselves to the joint of two random vectorial sequences of finite length. The joint of some several sequences is done by similar way.

Let us consider two independent from each other non-Gaussian vectorial sequences

$$\vec{\xi}_{(k+\nu)} = \left(\vec{\xi}_1^T, \ldots, \vec{\xi}_k^T, \vec{\xi}_{k+1}^T, \ldots, \vec{\xi}_{k+\nu}^T\right)^T,$$
$$\vec{\zeta}_{(k+m-\nu)} = \left(\vec{\zeta}_{k+\nu+1}^T, \ldots, \vec{\zeta}_{k+m}^T, \vec{\zeta}_{k+m+1}^T, \ldots, \vec{\zeta}_{2k+m}^T\right)^T,$$

with the block covariance matrices

$$K^{(1)}_{(k+\nu)} = M(\vec{\xi}_{(k+\nu)} - \vec{\mu}^{(1)}_{(k+\nu)})(\vec{\xi}_{(k+\nu)} - \vec{\mu}^{(1)}_{(k+\nu)})^T = (\sigma^{(1)}_{ij}),$$
$$i,j = 1,\ldots,k+\nu,$$
$$K^{(2)}_{(k+m-\nu)} = M(\vec{\zeta}_{(k+m-\nu)} - \vec{\mu}^{(2)}_{(k+m-\nu)})(\vec{\zeta}_{(k+m-\nu)} - \vec{\mu}^{(1)}_{(k+m-\nu)})^T = (\sigma^{(2)}_{ij}),$$
$$i,j = k+\nu+1,\ldots,2k+m,$$
$$\vec{\mu}^{(1)}_{(k+\nu)} = M\vec{\xi}_{(k+\nu)}, \quad \vec{\mu}^{(2)}_{(k+m-\nu)} = M\vec{\zeta}_{(k+m-\nu)}.$$

Here $\vec{\xi}_i$, $i = 1,\ldots,k+\nu$ and $\vec{\zeta}_j$, $j = k+\nu+1,\ldots,2k+m$ are vectors $\vec{\xi}_i = (\xi_{i,1},\ldots,\xi_{i,p})^T$, $\vec{\zeta}_j = (\zeta_{j,1},\ldots,\zeta_{j,p})^T$ with the mathematical expectation $M\vec{\xi}_i = \vec{\mu}^{(1)}$, $M\vec{\zeta}_j = \vec{\mu}^{(2)}$ and one-dimensional probability distributions

$$\vec{F}^{(1)}(x) = \left(F_1^{(1)}(x),\ldots,F_p^{(1)}(x)\right)^T, \quad \vec{F}^{(2)}(x) = \left(F_1^{(2)}(x),\ldots,F_p^{(2)}(x)\right)^T,$$

and covariance matrices

$$M(\vec{\xi}_i - \vec{\mu}^{(1)})(\vec{\xi}_i - \vec{\mu}^{(1)})^T = \sigma^{(1)}_{ii}, \quad M(\vec{\zeta}_j - \vec{\mu}^{(2)})(\vec{\zeta}_j - \vec{\mu}^{(2)})^T = \sigma^{(2)}_{jj}.$$

The sets of distributions for each component of vector $\vec{\xi}_{(k+\nu)}$ and $\vec{\zeta}_{(k+m-\nu)}$ we represent in the form of a vector for simplicity of the record. In the general case, $\sigma_{ij}^{(1)}$ and $\sigma_{ij}^{(2)}$ are square $p \times p$ block of the block-covariance matrices $K_{(k+\nu)}^{(1)}$ and $K_{(k+m-\nu)}^{(2)}$, and the matrices $\sigma_{ii}^{(1)}$ and $\sigma_{jj}^{(2)}$ are on the main diagonals of these matrices, correspondingly. Index (l) in the round brackets, for example, $(l) = (k+\nu)$, denotes that the vectorial sequence consists of l vectors and the block matrix - $l \times l$ blocks.

The procedure of stochastical joint of realizations of the sequences $\vec{\xi}_{(k+\nu)}$ and $\vec{\zeta}_{(k+m-\nu)}$ reduces to the following:

(i) relization of integer random index ν from the interval $[1, m]$ with probabilities $P(\nu = i) = p_i$, $\sum_{i=1}^{m} p_i = 1$ is choosen;

(ii) out of two independent sequences $\vec{\xi}_{(k+\nu)}$ and $\vec{\zeta}_{(k+m-\nu)}$, the sequence

$$\vec{\vec{\xi}}_{(2k+m)}=\left(\vec{\vec{\xi}}_1^{\,T},\ldots,\vec{\vec{\xi}}_{2k+m}^{\,T}\right)^T = \left(\vec{\xi}_{(k+\nu)}^{T},\vec{\zeta}_{(k+m-\nu)}^{T}\right)^T$$
$$=(\vec{\xi}_1^T,\ldots,\vec{\xi}_k^T,\vec{\xi}_{k+1}^T,\ldots,\vec{\xi}_{k+\nu}^T,\vec{\zeta}_{k+\nu+1}^T,\ldots,\vec{\zeta}_{k+m}^T,\vec{\zeta}_{k+m+1}^T,\ldots,\vec{\zeta}_{2k+m}^T)^T$$

is formed.

We choose the numbering of indices of the sequences $\vec{\xi}_{(k+\nu)}$ and $\vec{\zeta}_{(k+m-\nu)}$ such that after their joint the numbering of indices in this combined sequence $\vec{\vec{\xi}}_{(2k+m)}$ would be through, with the joint being carried out within the interval of values of indices $\nu = k+1, \ldots, k+m$.

Consider the main properties of the constructed sequence. In accordance with (i)–(ii) consider integer random index λ from the interval $[1, \ldots, 2k+m]$ with probability distribution

$$P(\lambda \le j) = \theta_j = \begin{cases} 0, & j = 1, \ldots, k, \\ \sum_{i=1}^{j-k} p_i, & j = k+1, \ldots, k+m, \\ 1, & j = k+m+1, \ldots, 2k+m. \end{cases}$$

In these notations one-dimensional distributions $\vec{F}_i(x)$ for elements of the sequence $\vec{\xi}_{(2k+m)}$ will have the form

$$\vec{F}_i(x) = (1-\theta_{i-1})\vec{F}^{(1)}(x) + \theta_{i-1}\vec{F}^{(2)}(x), \quad i = 1, \ldots, 2k+m. \qquad (1)$$

Note that two-dimensional probability distributions $F_{ij,pq}(x,y)$ of p-th component of the vector $\vec{\vec{\xi}}_i$ and q-th component of the vector $\vec{\vec{\xi}}_j$ for $i, j =$

$k+1,\dots,k+m$ are expressed by θ_i, θ_j and, also, by the given one-dimensional and two-dimensional distributions $F_i^{(1)}(x)$, $F_j^{(2)}(x)$, $F_{ij}^{(1)}(x,y)$, $F_{ij}^{(2)}(x,y)$ of the components of the sequences $\vec{\xi}_{(k+\nu)}$ and $\vec{\zeta}_{(k+m-\nu)}$. The block elements σ_{ij} of the covariance matrix

$$K_{(2k+m)} = M(\tilde{\xi}_{(2k+m)} - \vec{\tilde{\mu}}_{(2k+m)})(\vec{\tilde{\xi}}_{(2k+m)} - \tilde{\mu}_{(2k+m)})^T = (\sigma_{ij}),$$
$$i,j = 1,\dots,2k+m,$$

where $\vec{\tilde{\mu}}_{(2k+m)} = M\vec{\tilde{\xi}}_{(2k+m)}$ satisfy the following relations:

$$\begin{aligned} \sigma_{ij} = \sigma_{i,i+h} &= (1-\theta_{i+h-1})\sigma_{i,i+h}^{(1)} + \theta_{i-1}\sigma_{i,i+h}^{(2)} \\ &\quad + \theta_{i-1}(1-\theta_{i+h-1})(\vec{\mu}^{(2)} - \vec{\mu}^{(1)})(\vec{\mu}^{(2)} - \vec{\mu}^{(1)})^T, \qquad (2) \\ \sigma_{ji}^T = \sigma_{ij}, &\qquad h = 0,1,\dots,m, \quad i = 1,\dots,2k+m-h. \end{aligned}$$

The covariances within the interval $\nu = k-h,\dots,k+h$ will be called the smoothing functions. The class of these smoothing functions is determined by the corresponding elements of block covariance matrices $K_{(k+\nu)}^{(1)}$ and $K_{(k+m-\nu)}^{(2)}$, and, also, by the probabilities p_i.

In particular, if $p_i = p = 1/m$, and matrices $K_{(k+\nu)}^{(1)}$ and $K_{(k+m-\nu)}^{(2)}$ are block-stationary, then elements of block matrices as functions of i are polynomials of the second degree with respect to i. If $\vec{\mu}^{(1)} = \vec{\mu}^{(2)}$, then expression (2) essentially simplifies and within the interval $i = 1,\dots,2k+m-h$ it is a linear function with respect to i. In the block record this expression has the following form:

$$\sigma_{i,i+h} = (1-\theta_{i+h-1})\sigma_{i,i+h}^{(1)} + \theta_{i-1}\sigma_{i,i+h}^{(2)}, \qquad (3)$$

$$\theta_j = \begin{cases} 0, & j = 1,\dots,k, \\ \dfrac{j-k}{m}, & j = k+1,\dots,k+m, \\ 1, & j = k+m+1,\dots,2k+m, \end{cases}$$
$$h = 0,1,\dots,m, \quad i = 1,\dots,2k+m-h.$$

If the process is built by the joint of non-stationary sequences of finite length, then some esential simplifications are possible. Let $\vec{\xi}_{(k+\nu)}$ and $\vec{\xi}_{(k+m-\nu)}$ be two cross-independent sequences with the covariance matrices $K_{(k+\nu)}^{(1)}$ and $K_{(k+m-\nu)}^{(2)}$, $\nu = 1,\dots,m$. Since ν is an integer random quantity, then the maximal block dimension of the matrices $K^{(1)}_{(k+\nu)}$ and $K^{(2)}_{(k+m-\nu)}$ is equal to $k+m$ and minimal dimension is equal to k. It is natural in some applications to take block elements in each of these matrices, corresponding to the points i and $i+h$ from the interval $k+1,\ldots,k+m$ as equal to each other, i.e., $\sigma^{(1)}_{i,i+h} = \sigma^{(2)}_{i,i+h} = \hat{\sigma}_{i,i+h}$. For example, if non-stationary $K^{(1)}_{(k+m)}$ and $K^{(2)}_{(k+m-\nu)}$ are estimated by the data of many year observations, then for calculation these matrix elements the quantity, related to the same points of the interval $k+1,\ldots,k+m$ are used, and, hence, corresponding elements of these matrices coincide. In the case $\vec{\mu}^{(1)} = \vec{\mu}^{(2)} = \vec{\mu}$ (2) may be written down in the following form:

$$\sigma_{i,i+h} = (1 - \theta_{i+h-1} + \theta_{i-1})\hat{\sigma}_{i,i+h}, \quad h = 0,1,\ldots,m, \;\; i = k,\ldots,k+m-h.$$

Thus, the error of smoothing is determined by values of the probabilities $\theta_{i+h-1} - \theta_{i-1}$.

If the Gaussian stationary sequences with zero expectation and with some different correlation functions provided that $p_i = p$, $i = 1,\ldots,m$ are joint, then the smoothing functions are determined by the linear relation (3). For $p_i \neq p_j$ these smoothing functions are nonlinear. One-dimensional distributions are a mixture of the corresponding normal distributions. Note, also, that each realization of the constructed sequence is the joint of independent realizations of limites length sequences, therefore abrupt changes of simulated quantities are possible at the boundary of the joint.

2. Mixed models of random processes and fields

In this section, a simple modification of the methods of random processes and fields simulation based on point fluxes [4] will be considered. This modification is based on mixing these models with discrete models described in [5].

As an example, consider a simple case of such combination. Let us consider the following procedure of construction of a random process [6].

(i) In the interval $(0,T)$ the grid points $t_1 = 0$, t_2, ..., $t_n = T$ are arbitrary fixed. At these points a discrete random sequence $\xi(t_1)$, $\xi(t_2)$, ..., $\xi(t_n)$ with given one-dimenaional distributions $F_{\xi(t_i)}(x)$ and a covariance matrix $K(t_i,t_j)$, $i,j = 1,\ldots,n$ is simulated. The matrix $K(t_i,t_j)$ can be either stationary (Toeplitz's), or non-stationary.

(ii) In every interval (t_1,t_2), (t_2,t_3), ..., (t_{n-1},t_n) a random point x_i is simulated with the respect to the probability density $f_i(x)$, $x \in (t_i,t_{i+1})$. These points form a random point flux and are random boundaries of the intervals $(x_1, x_2), (x_2, x_3), \dots, (x_{n-2}, x_{n-1})$, inside which the points $t_2, \dots, t_{n-1}$ are contained. The intervals (t_1, x_1) and (x_{n-1}, t_n) are adjoining to boundaries of the interval $(0, T)$.

(iii) For every interval (x_i, x_{i+1}) we take $\xi(t) \equiv \xi(t_i)$, $i = 2, \dots, n-1$. For the intervals (t_1, x_1) and (t_{n-1}, t_n) we take $\xi(t) \equiv \xi(t_1)$ and $\xi(t) \equiv \xi(t_n)$ respectively.

The covariance function

$$K(t', t'') = M\xi(t')\xi(t'') - M\xi(t')M\xi(t'')$$

of the process $\xi(t)$, $t \in (0, T)$ for the values $t' = t_i$ and $t'' = t_j$ obviously coinsides with the given covariance matrix $K(t_i, t_j)$ for an arbitrary probability density of random values x_i in the interval (t_i, t_{i+1}), $i, j = 1, \dots, n$. For values $t \neq t_i$ a one-dimensional distribution $F_{\xi(t)}(x)$ in the general case depends on the distributions $F_{\xi(t_i)}(x)$ and density and $f_i(x)$. In particular, if $F_{\xi(t_i)}(x) \equiv F(x)$, then $F_{\xi(t)}(x) \equiv F(x)$.

The covariance function $K(t', t'')$ for $t' = t_i$, $t'' \neq t_j$ depends on two dimensional distributions of the corresponding pair of random variables x_i and x_j and also, on the covariance matrix $K(t_i, t_j)$.

As an example, consider the following process $\xi(t)$. Let

$$t_i = i, \quad F_i(x) = F(x), \quad x_i = i + \alpha, \quad i = 1, \dots, n, \tag{4}$$

where α is a random variable, uniformly distributed in the interval $(0, 1)$. Construct a realization of a process $\xi(t)$ according to the above considered procedure (i)–(iii). For simplicity we take $M\xi(t_i) = 0$. Then the covariance between values of the process $\xi(t)$ at the points $t' = i + \tau'$ and $t'' = j + \tau''$, where $\tau'' > \tau'$, $\tau', \tau'' \in (0, 1)$, $j = 1, \dots, n$, has the following form:

$$M\xi(t')\xi(t'') = K(t', t'') = (1 - t'')\sigma_i^2 + (t'' - t')K(i, j) + t'\sigma_j^2. \tag{5}$$

Here $\sigma_i^2 = M\xi_i^2$, $K(i, j)$ is a given covariance matrix of the sequence ξ_i.

If $t' = t$, $t'' = t + h$, $\sigma_i^2 = \sigma^2 = 1$, then

$$K[i + \tau, (j - 1) + t + h] = K[i, (j - 1) + h] = (1 - h) + hK(i, j).$$

In the given case, a covariance function is independent of t, hence it can be used for approximation of covariance matrices both of the stationary and non-stationary types. If α is a random quantity distributed in the interval $(0, 1)$ with the density $f(x) = 2x$, then the covariance function $K(t', t'')$ depends also on t $(t' = t,\ t'' = t + h)$

$$K(t',t'') = K[i,(j-1)+t+h] = 1-(2th+h^2)[1-K(i,j)]. \qquad (6)$$

The selection of random points x_i in the interval (t_i, t_{i+1}) can be realized by different methods, dependently or independently, with the same distribution density or with different ones, etc. All these transformations determine a family of covariance functions, approximating a given covariance matrix $K(t_i,t_j)$ and coinciding with it at the grid points.

Consider one interesting particular case [7]. Let us in (4), $t_i = i$, $x_i = i+\alpha$, $i = 1,2,\ldots$, where α be a random uniformly distributed variable in the interval $(0,1)$, and ξ_i be a stationary random discrete process with a correlation function $R(i,j) = R(|i-j|) = R_k$, $k = 0,1,\ldots$, $R_0 = 1$. According to the procedure (i)–(iii) and based on the above-mentioned conditions (4), where $i = 1,2,\ldots$, construct a random process $\xi(t)$ in the interval $(0,\infty)$ and write down the correlation function of the constructed process. Consider the interval $(i,i+1)$. The correlation coefficient between values of the process $\xi(t)$ at points t' and $t'' \in (i,i+1)$ has the form

$$R(t',t'') = (1-t'') + (t''-t')R_1 + t'.$$

Take $t'' = t'+h$, $h \in (0,1)$. Then

$$R(t',t'') = (1-h) + hR_1.$$

Since variable α is the same for all intervals (note that x_i are points of a regular point flux), then the correlation coefficient between values of the process at the points $t' \in (i,i+1)$ and $t'' \in (i+k,i+k+1)$ is equal to

$$R(t',t'') = (1-h)R_k + hR_{k+1}, \quad h \in (0,1).$$

Take $t'' = t'+\tau$, where $\tau = k+h$, $k = \mathrm{entier}(\tau)$. Then

$$R(t',t'') = R(t',t'+\tau) = (1+k-\tau)R_k + (\tau-k)R_{k+1} = R(\tau), \quad k = \mathrm{entier}(\tau). \qquad (7)$$

The constructed process $\xi(t)$ is a stationary one. The correlation function of this process in a non-negative definite, piecewise-linear function, which coincides with the given values of the correlation function R_k for $k = 0,1,2,\ldots$.

Thus, the following statement is proved:

Statement. *If a function of a discrete argument* $R(k) = R_k$, $R_0 = 1$, $k = 0,1,2,\ldots$, *is a non-negative definite function, then the function* $R(\tau) = (1+k-\tau)R_k + (\tau-k)R_{k+1}$, $k = \mathrm{entier}(\tau)$ *is a non-negative definite one.*

By analogy it is shown that this property is valid also for the processes, whose argument values are changing on bounded intervals. The considered property in some cases can be used in applications for a piecewise-linear approximation of sampling correlation functions. Note that in terms of the considered technique it is impossible to find another approximation of a discrete correlation function and in so doing not to infringe stationarity of the process, because the unit factor, determining the behavior of correlation dependence is a distribution of a random variable α in the interval $(0,1)$. It is easy to show that the sample of a variable α with a non-uniform distribution density results to non-stationary of such a process. In paper [6] it was shown that the spectral density of the process $\xi(t)$, $t \in (-\infty, \infty)$ with the correlation function (7) has the following form:

$$f(\lambda) = 2g(\lambda)\tilde{f}_1(\lambda), \tag{8}$$

where

$$g(\lambda) = \frac{1}{\pi} \cdot \frac{1 - \cos\lambda}{\lambda^2} = \frac{1}{2\pi}\left[\frac{\sin\frac{\lambda}{2}}{\frac{\lambda}{2}}\right]^2$$

is the spectral density, corresponding to the process with triangular correlation function

$$R(\tau) = \begin{cases} 1 - |\tau|, & |\tau| < 1, \\ 0, & |\tau| \geq 1, \end{cases} \tag{9}$$

and $\tilde{f}_1(\lambda)$ is a periodic continuation of the spectral density $f(\lambda)$ in the form

$$\tilde{f}_1(\lambda) = \Big(1 + 2\sum_{k=1}^{\infty} R_k \cos\lambda k\Big). \tag{10}$$

Relation (8) proves the existence of the piecewise-linear approximation of the correlation function R_k under condition of the uniform convergence of series (9), i.e., if $\sum_{k=1}^{\infty}|R_k| < \infty$. Note that in the proof of Statement 1 only existence of a discrete process is required.

In conclusion consider a generalization of this method for the simulation of non-honogeneous and non-isotropic non-Gaussian random fields $\xi(x)$, determined at an arbitrary point $x \in R^m$ of the domain $D \in R^m$, with a covariance function $K(x', x'')$ coinciding with a given covariance matrix $K(x_i, x_j)$ at fixed points of the domain. According to the method, described in [3], consider the following technique of construction of the field on a regular grid.

(j) On each coordinate axis in R^m regular grid points $r_k^{(i)}$, $i = 1, \ldots, m$, $k = 1, \ldots, p$, such that $r_{k+1}^{(i)} - r_k^{(i)} = \Delta r$, determining the regular grid with p^m points, are sampled. At these points a realization of a discrete random field $\xi(r_i)$ with a given distribution $F_\xi(x)$ and covariance matrix $K(r_q, r_l)$, $q, l = 1, \dots, p^m$, is constructed.

(jj) In each interval $(r_k^{(i)}, r_{k+1}^{(i)})$ a random $x_k^{(i)}$ is sampled. In this case $r_{k+1}^{(i)} \in (x_k^{(i)}, x_{k+1}^{(i)})$.

(jjj) In each i cuboid, with the vertex formed by adjacent points of grid with coordinates $x_i^{(k)}$, one takes

$$\xi(r) \equiv \xi(r_i), \quad r_i = (r_i^{(1)}, \dots, r_i^{(m)})^T.$$

In particular, if $m = 2$, $\Delta r = 1$, $r_i^{(1)} = i$, $r_j^{(2)} = j$, $x_i^{(1)} = i + \alpha_1$, $x_i^{(2)} = j + \alpha_2$, $x', y', x'', y'' \in (0,1)$, $i, j = 1, \dots, n$, where α_1 and α_2 are independent random values, uniformly distributed in the interval $(0,1)$, then

$$\begin{aligned}
&K(i_1 + x', j_1 + y'; i_2 + x'', j_2 + y') \\
&\quad = (1 - x'')(1 - y'')\sigma^2_{i_1 j_1} + y'(1 - x'')\sigma^2_{i_1 j_2} + (1 - y'')x'\sigma^2_{i_2 j_1} + x'y'\sigma^2_{i_2 j_2} \\
&\qquad + (1 - x'')(y'' - y')K(i_1, j_1; i_2, j_2) + (1 - y'')(x'' - x')K(i_1, j_1; i_2, j_1) \\
&\qquad + y'(x'' - x')K(i_1, j_2; i_2, j_2) + x'(y'' - y')K(i_2, j_1; i_2, j_2) \\
&\qquad + (y'' - y')(x'' - x')K(i_1, j_1; i_2, j_2).
\end{aligned}$$

If a grid is irregular, one of simple algorithms can be constructed on the basis of the well-known algorithm of simulation of homogeneous isotropic fields [4]. Consider the case for $m = 2$. Let x_i, $i = 1, \dots, q$, be the coordinates of fixed points in domain D in R^2 (any modification of method, proposed in [4], is considered).

(l) At the points $x_1, \dots, x_q$ realization of a discrete fields $\xi(x_i)$ with one-dimensional distribution $F_\xi(u)$ and a covariance matrix $K(x_i, x_j)$, $i, j = 1, \dots, q$, is simulated.

(ll) On a line z along an isotropic direction $\vec{\omega}$ the projections of the points x_i, determining the set of the intervals (z_i, z_{i+1}) are found. In each interval a random point t_i is sampled, so that $z_{i+1} \in (t_i, t_{i+1})$.

(lll) At the points t_i, $i = 1, \dots, p - 1$, the lines, perpendicular to $\vec{\omega}$ are constructed. In each subdomain D_i of the domain D, bounded by the lines passing through t_i and t_{i+1} one takes $\xi(x) \equiv \xi(x_i)$.

Unlike the method considered in the previous section, a covariance matrix of the thus constructed field depends on all elements of the matrix

$K(x_i, x_j)$. Note that considered methods are easy to generalize to a case of vectorial and spatial-time random fields. These algorithms are used for the construction, in particular, of non-homogeneous fields of daily amounts of rain precipitaion and non-homogeneous fields of the sea currents.

References

[1] Yu.I Palagin, C.V. Phedotov, A.S. Shaligin, Parametric models for statistical simulation of vectorial non-homogeneous random fields, Avtomatica. Telemechanika, No. 6, 1990, 79–89 (in Russian).

[2] Ya.P. Dragan, V.A. Rozhkov, I.N. Yavorsky, Methods of Probability Analysis Rhytmics of Oceanic Processes, Gidrometeoizdat, Leningrad, 1987 (in Russian).

[3] V.A. Ogorodnikov, L.A. Minakova, Probabilistic models of non-stationary vector sequences, Theory and Appl. Stat. Model., 1989, 17–29 (in Russian).

[4] G.A. Michailov, Minimization of Computional Costs of Non-analoque Monthe-Carlo Methods, World Scientific, Singapure, New Gersey, London, Hong Kong, 1991.

[5] V.A. Ogorodnikov, Statistical simulation of discrete random processes and fields, Soviet Journal of Numer. Anal. and Math. Modelling, Vol. 5, No. 6, 1990, 489–509.

[6] V.A. Ogorodnikov, T.P. Romanenko, One method of approximation of covariance matrices, Proceedings of VIII Russian Monte-Carlo methods Conf., Novosibirsk, Part 1, 1991, 46–49 (in Russian).

[7] V.A. Ogorodnikov, Piecewize-linear approximation of discrete correlation function, Theory and Appl. Stat. Model., Novosibirsk, 1992, 20–25 (in Russian).