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Numerical implementation of wave mode of definition of bottom hole coordinates

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In the present paper, within the scope of the acoustic method of the wave mode of definition of the bottom hole coordinates, the procedure of estimation of drift parameters for any number of sensors ≥ 4 is discussed. Least square methods, applied in this case, with regularization and pseudo-inversion allow us to obtain confidence intervals for the coordinates (x, y, z) of a bottom hole and the mean propagation velocity of a seismic wave v.

Introduction

The definition of the bottom hole coordinates, in particular, its depth, is one of necessary components of geophysical exploration and drilling activities. The raise of accuracy of defining the coordinates of a drift allows us to improve the quality of interpretation of the logging information and to reduce the time and costs of the drilling activity [1].

The wave mode of definition of the bottom hole coordinates, that is discussed in the present paper, consists in measuring the time of wave propagation of a certain physical nature from the point of applying the action (or the measurement point) above the ground up to the bottom hole (or back). In the drilling practice, the most developed is the acoustic method based on the elastic oscillations of a medium which are passing through a pipe string and rocks [1]. When the times of wave propagation from (or up to) several points up to (or from) the drift are fixed, it is possible to apply the so-called "group" mode of definition of the coordinates of a source (or a receiver) of seismic waves, which is based on the data obtained from a group of stations.

1. Parameter estimation of a bottom hole

In paper [2], there is stated a procedure of detection and ranging of a bottom hole in the boring process for the case of five sensors. In the present paper and in [3], the procedure of parameter estimation of a bottom hole for any number of sensors ≥ 4 is stated.

Let in the Cartesian coordinate system x, y, z the axes x, y be directed along the surface of ground, and the axis z downwards to the centre of the

Earth. The mean velocity of seismic propagation wave in the neighbourhood is denoted by v. We arrange sensors, recording (or emitting) seismic signals on the ground surface or in small holes at the points with the coordinates (x_i, y_i, z_i). And let t_i be the time of propagation of a seismic signal from the source in a bottom hole (for example, chisels) up to the i-th point (or vice versa). It is required to define the coordinates (x^*, y^*, z^*) of a bottom hole and the velocity v. It is also possible to formulate the problem, in which it appears difficult to fix the time of emission of a seismic signal, and one has to include it into the number of unknowns to be defined. Then it will be necessary to define the coordinates (x^*, y^*, z^*) of a bottom hole, time in the source t^* and the velocity v. And, naturally, the minimum number of sensors will be increased up to five.

When estimating the unknowns of bottom hole parameters, let us take advantage of a nonlinear system of the so-called conditional equations [4–7]:

$$\vec{t} = \vec{\eta}(X, \vec{\theta}) + \vec{\varepsilon}, \tag{1}$$

where the following notations are used:

$\vec{t} = (t_1, t_2, \ldots, t_N)^T$ is the vector of travel times of seismic signals,

$\vec{\eta}(X, \vec{\theta})$ is N-dimensional vector of computed travel times (theoretical travel time curve) or a regression function,

$\vec{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_N)^T$ is the vector of residuals,

$\vec{\theta} = (x, y, z, v, t)^T$ is m-dimensional vector of estimated parameters,

$X = (\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N)$ is the matrix of the coordinates of sensors (or of radiation points),

N is the number of sensors (or of radiation points).

When estimating parameters, we use the information about the error distribution $\varepsilon_i = t_i(\vec{x}_i, \vec{\theta}) - \eta(\vec{x}_i, \vec{\theta})$. Further, we assume that ε_i are mutually independent random variables with distribution with zero mean and given variances: $E\varepsilon_i = 0, E\varepsilon_i\varepsilon_j = \sigma_i^2\delta_{ij}, \sigma_i = \sigma(\vec{x}_i), \delta_{ij}$ is the Kronecker delta, $i = 1, 2, \ldots, N$. In the cases with difficulties in representing variances, they are assumed to be equal and unbiased estimator of variance of observation with single weight when solving problem [6], is obtained in the manner of the present paper.

2. On methods of solution of the formulated problem

The problem of estimating the parameters $\vec{\theta}$ is a part of the so-called regression analysis, and its solution is the estimations of the least square method:

$$\vec{\theta} = \arg\min_{\vec{\theta}\in\Omega} Q(\vec{\theta}), \qquad Q(\vec{\theta}) = \sum_{i=1}^{N} \sigma_i^{-2}(t_i - \eta(\vec{x}_i, \vec{\theta}))^2. \tag{2}$$

In order to search for a minimum of the functional $Q(\vec{\theta})$, apply the iterative Gauss–Newton method or its modification based on the linear approximation of a regression function in the neighbourhood of the point $\vec{\theta}^k$:

$$J(X, \vec{\theta}^k)\Delta\vec{\theta}^k + \vec{\eta}(X, \vec{\theta}^k) - \vec{t} + \vec{\varepsilon} = 0, \tag{3}$$

where

$$J(X, \vec{\theta}) = \left(\frac{\partial\eta(\vec{x}_i, \vec{\theta})}{\partial\theta_1}, \frac{\partial\eta(\vec{x}_i, \vec{\theta})}{\partial\theta_2}, \ldots, \frac{\partial\eta(\vec{x}_i, \vec{\theta})}{\partial\theta_m}\right), \qquad i = 1, 2, \ldots, n. \tag{4}$$

Multiplying both parts of the linearized system (3) by $J^T(X, \vec{\theta}^k)$, we obtain the system of the normal equations:

$$J^T(X, \vec{\theta}^k)J(X, \vec{\theta}^k)\Delta\vec{\theta}^k = J^T(X, \vec{\theta}^k)\vec{y}(X, \vec{\theta}^k), \tag{5}$$

here $\vec{y}(X, \vec{\theta}) = (\vec{t} - \eta(X, \vec{\theta}))^T$.

The estimations $\vec{\theta}$ are obtained as a result of the realization of the iterative process ($\vec{\theta} = \lim_{k\to\infty} \vec{\theta}^k$):

$$\vec{\theta}^{k+1} = \vec{\theta}^k + \left[J^T(X, \vec{\theta}^k)J(X, \vec{\theta}^k)\right]^{-1} J^T(X, \vec{\theta}^k)\vec{y}(X, \vec{\theta}^k), \quad k = 0, 1, 2, \ldots . \tag{6}$$

The matrix

$$D(X, \vec{\theta}) = (J^T(X, \vec{\theta})J(X, \vec{\theta}))^{-1} \tag{7}$$

is called a covariance matrix (of space of parameters) and contains error estimations of the unknown parameters $\vec{\theta}$.

When implementing the iterative process (6) and its modifications, the inversion of the matrix J^TJ is not made, and, in fact, at each step of the iterative process there is a transfer from system (3) to system (5) which is solved by one of the standard methods. Therefore, process (6) can be written down as:

$$\begin{aligned} &\vec{\theta}^{k+1} = \vec{\theta}^k + \Delta\vec{\theta}^k, \\ &J^T(X, \vec{\theta}^k)J(X, \vec{\theta}^k)\Delta\vec{\theta}^k = J^T(X, \vec{\theta}^k)\vec{y}(X, \vec{\theta}^k), \qquad k = 0, 1, 2, \ldots . \end{aligned} \tag{8}$$

Two disadvantages of the discussed computational scheme, one should refer the poor conditioning of the matrix J^TJ for some cases of the location of a bottom hole and sensors. Therefore, the computational scheme of the least square method with regularization was implemented, in which the system of normal equations has the form:

$$(J^T(X,\vec{\theta}^k)J(X,\vec{\theta}^k)+\alpha^2 I)\Delta\vec{\theta}^k = J^T(X,\vec{\theta}^k)\vec{y}(X,\vec{\theta}^k), \qquad (9)$$

where α is the regularization parameter, and I is the unit matrix.

Another approach for solving the problem (1)–(4) realized by the author consists in waiving the transfer to a system of normal equations and immediate solution at each step of the iterative process of system (3). In order to solve the latter, the method of pseudo-inversion (or the generalized inversion) [8, 9], which is based on the singular expansion (the SVD-expansion), is most commonly used. In [8], for its realization a standard procedure in the language Fortran-IV has been developed. The computational scheme of the singular expansion consists in expansion of matrix (4) at each step of the iterative process in the product of three matrices.

$$J(X,\vec{\theta}^k) = U_k\Sigma_k V_k^T, \qquad (10)$$

where U_k is the orthogonal $n\times n$ matrix, V_k is the orthogonal $m\times m$ matrix, Σ_k is the diagonal $n\times m$ matrix, having the structure

$$\Sigma_k = \begin{pmatrix} S_k \\ 0 \end{pmatrix},$$

where $S_k = \mathrm{diag}(\rho_1,\rho_2,\ldots,\rho_m)$ is the diagonal matrix of singular numbers non-increasingly ranked, $\rho_i \geq \rho_{i+1}$. The method also allows for conducting the so-called singular analysis which consists in elimination of zero singular numbers and the corresponding to them columns of the matrices U and V. The iterative process in this case has the form:

$$\vec{\theta}^{k+1} = \vec{\theta}^k + V_k S_k^{-1}\vec{d}^k, \qquad k = 0,1,2,\ldots, \qquad (11)$$

where $\vec{d}^k$ is a vector consisting of the first m components of the vector $U_k^T\vec{y}(X,\vec{\theta}^k)$. When implementing this process, not only the covariance matrix of the space of parameters is readily obtained, but also the covariance matrix of the space of data, the matrix of the resolution $V_kV_k^T$, whose closeness to the unit matrix indicates to the degree of solvability of a problem, and the matrix of information denseness $U_kU_k^T$, whose closeness to the unit matrix indicates to a relative significance of separate observations [9, 10]. It is possible to show [8], that each step of the iterative process (11) simultaneously minimizes the residual norm $|J\vec{\theta}^k - \vec{y}|^2$ and the norm of the solution $|\vec{\theta}^k|^2$, and each step of process (9) is equivalent to minimization of the weighted sum of these norms $|J\vec{\theta}^k - \vec{y}|^2 + \alpha^2|\vec{\theta}^k|^2$, which ensures the uniqueness of the solution.

3. On planning the observations system

Methods (6), (11) are rather good for the solution to systems (3), (5), but, in practice, they are not very effective in the case of the poor conditioning

of matrices (4). The reason for the poor conditioning of matrix (4) is in bad organization of observations, i.e., in inappropriate arrangement of sensors relative to a bottom hole. Hence, there follows a conclusion about the necessity in planning observations, i.e., in the choice of such an arrangement of sensors within the territory, which would maximally refine the conditioning of matrix (4) and, consequently, the estimation of drift parameters. Thus, planning a system of observations, we do not try to correct poor conditioning of matrix (4) by applying various regularizations, but eliminate the reason of poor conditioning as it is.

R. Fisher, the author of design of experiments, was the first to understand that if the most effective methods of parameter estimation can yield a considerable increase in accuracy of estimated parameters by maximally several tens percent, a gain from planning an experiment (appropriate organization of measurements) can be the multiple. At present, in the theory of design of an experiment it is widely recognized that expensive experiments (for example, powerful explosions, well boring) require a preliminary skillful planning. The relevant software for planning the system of seismic observations, whose application has been stated in [4, 5], has been developed.

4. Practical implementation

Using the methods described in the present paper, which were designed as a PC program, the experimental data, obtained at 22 operating oil-wells with depths of 1000 m to 2300 m (provided by the authors of [2]), have been processed. The results of processing show the high efficiency of the developed software and good quality of the experimental material. Thus, the confidence intervals with 95 percent confidence of the drift coordinates are, basically, within 1–2 m. It is interesting to note that for the bore-holes with depths of 1000–1800 m, a drift from the aperture is separated by 60–600 m, and for the bore-holes with depths of 2000 to 2300 m – within 500–800 m.

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