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Modification of Bougeault–Andre mixing length hypothesis and its numerical verification

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This paper represents an attempt to improve the parameterization scheme, proposed by Bougeault and Andre. Modification deals with non-local feature of the turbulent mixing and proposes some kernel convolution form for the turbulent mixing length. This kernel is an influence function of the turbulence located at one point to the turbulence at some other point. The set of numerical experiments shows the advantage of proposed modification for high vertical resolution or for vanishing background turbulent coefficient.

1. Introduction

G.I. Taylor [1] was the first, who introduced the idea of the homogeneous and isotropic turbulence. Later Kolmogorov [2] developed a theory of locally homogeneous and locally isotropic turbulence. He proposed also to consider turbulent kinetic energy equation together with the Reynolds stress equations [3]. Thus the problem of the turbulence description diminishes to the description of its intensity and turbulent fluctuation scale, named hereafter as mixing length (ML). The calculation of the last one progresses from the empirically prescribed profiles to some parameterizations based on the Blackadar expression with L_∞-limit and Bougeault and Andre [8] procedure.

Proportionality of turbulent fluxes to the locally mean gradients and introduction of the turbulent viscosity coefficient means that Reynolds stresses are always of the same sign as the mean gradients and thus turbulence reduces always the kinetic energy of large scale motion. However, it does not take a place for the geostrophic fluids as reported Starr [4], Deardorff [5] and Turner [6]. The way to avoid this uncertainty is to construct from the Keller–Fridman system the higher order closures. Following this way Lykossov [7] obtained the analytical expressions for counter-gradients initially proposed by Deardorff [5].

The purpose of this paper is to build some improvement of ML hypothesis of Bougeault and Andre [8] to compare original parameterization of it with modified one.

The second chapter contains the description of statistical comparison technique. Then we give some idea of how to improve the parameterization procedure of Bougeault and Andre. The fourth chapter includes the results of numerical experiments. Finally we resume with some concluding remarks.

2. Comparison indexes

To stand out the qualitative properties of the models to reproduce adequately spatial and temporal distribution of the boundary layer characteristics let us introduce some indexes based on the statistical analysis of the data. Initially we have some correlation coefficients obtained as a result of natural data comparison with the calculated ones. As we have four characteristics of the lower atmosphere: potential temperature (θ), specific humidity (q), zone and meridian wind components (u, v) varying both in time and in space, so we can build 8 correlation coefficients. Four of them reflect the coherency of temporal variations, while the other four reflect the identity of the spatial distributions. It is quite difficult to operate with 8 coefficients. So we should build some indexes, the number of which would be less than 8, reflecting model capabilities more compactly. Each index we can build as a function of the correlation coefficients. This function is any combination of several numbers of the two-variable function (TVF). This TVF is to obey two conditions:

$$\mathcal{F}(R_1, R_2) = \mathcal{F}(R_2, R_1),$$
$$\mathcal{F}(\mathcal{F}(R_1, R_2), \mathcal{F}(R_3, R_4)) = \mathcal{F}(\mathcal{F}(R_1, R_3), \mathcal{F}(R_2, R_4)).$$

The averaging TVF, for example $\mathcal{F}(R_1, R_2) = 1/2(R_1 + R_2)$, satisfies these conditions, but according to this form, if $R_1 = 1$ and $R_2 = 0$, then $\mathcal{F} = 0.5$. It is a good value for the correlation coefficient but with $R_2 = 0$ it cannot be good. It seems to be preferable the function of geometric mean $\mathcal{F}(R_1, R_2) = \chi\sqrt{|R_1 R_2|}$, where χ is a factor equal to -1, if R_1 and/or R_2 are negative, and equal to 1 other case.

Next 5 indexes represent the following model properties:

$\boldsymbol{R_S}$ – variability of the scalar characteristics

$$\boldsymbol{R_S} = \mathcal{F}(\mathcal{F}(R_\theta^t, R_\theta^z), \mathcal{F}(R_q^t, R_q^z)),$$

where R_j^i – correlation coefficient of j-th characteristics by i-th coordinate (t – time, z – vertical coordinate);

$\boldsymbol{R_V}$ – variability of the wind vector $\boldsymbol{R_V} = \mathcal{F}(\mathcal{F}(R_u^t, R_u^z), \mathcal{F}(R_v^t, R_v^z))$;

$\boldsymbol{R_t}$ – temporal variability $\boldsymbol{R_t} = \mathcal{F}(\mathcal{F}(R_\theta^t, R_q^t), \mathcal{F}(R_u^t, R_v^t))$;

$\boldsymbol{R_z}$ – spatial variability $\boldsymbol{R_z} = \mathcal{F}(\mathcal{F}(R_\theta^z, R_q^z), \mathcal{F}(R_u^z, R_v^z))$;

$\boldsymbol{R_T}$ – total model index $\boldsymbol{R_T} = \mathcal{F}(\mathcal{F}(\boldsymbol{R_S}, \boldsymbol{R_V}), \mathcal{F}(\boldsymbol{R_t}, \boldsymbol{R_z}))$.

3. Bougeault–Andre parameterization and its modification

According to the Bougeault–Andre approach [8] let us consider the motion of a fluid parcel, initially disturbed with energy $b(z)$ equal to the turbulent kinetic energy *in situ*. The gravity and the buoyancy force this parcel to oscillate near the stationary state. For the moist air parcel we have

$$\rho_0 \frac{dw}{dt} = -\rho_0 g + \rho(z) g,$$

where $z(t)$ – vertical coordinate of the moving parcel, ρ_0 – its density, ρ – the density of the surrounding fluid. Substituting the density by the equivalent potential temperature, defined by

$$\theta_v(z) = T(z)\left(\frac{p_0}{p(z)}\right)^{\gamma}(1 + 0.61q(z)),$$

(T, p, q – temperature, pressure and specific humidity of the air) and integrating one can obtain

$$w(t) = w(t_0) - \beta \int_{z_0}^{z} (\theta_v(z) - \theta_v^*(z, z_0))dt'.$$

Here $\theta_v^*(z, z_0) = \theta_v(z_0) + \int_{z_0}^{z} \gamma_a dz'$ is the temperature of the moving parcel, changed by the adiabatic compression and phase transformation, γ_a – adiabatic gradient for the wet air, $\beta = g/\theta$ – buoyancy. The initial parcel velocity is proportional to the square root of the kinetic energy:

$$w(t_0) = \pm\mu\sqrt{b(z_0)}.$$

According to the sign of this expression one can obtain the height of the maximum ascent L_{up} and the depth of the maximum descent L_{down} of the parcel trajectory. ML is taken to be a function of L_{up} and L_{down}. Bougeault and Andre define ML as mean inverse value:

$$\frac{1}{L} = \frac{1}{2}\left(\frac{1}{L_{\text{up}}} + \frac{1}{L_{\text{down}}}\right).$$

Let us consider the homogeneous layer with the density barriers at the ends of interval $[a, b]$. In this case the above expression provides a squared dependence of L on z. ML reaches its maximum value at the center of the interval with the magnitude proportional to its length $(b - a)$. According to the previous equation the coefficient proportionality k is equal to 2 (Figure 1).

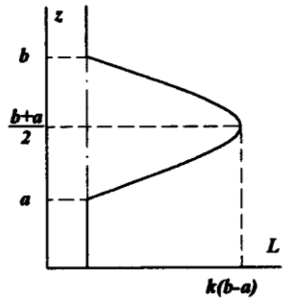

Figure 1. Vertical distribution of ML in the homogeneous layer

The advantages of this parameterization regarding the traditional parameterizations, based on Blackadar's form, are the following:

- no gross overestimation of ML near the inversion region, because for $z \to \infty$ ML vanishes;
- the clear physical interpretation of ML and its relation with the internal wave length.

Blackadar's parameterizations suppose some value L_∞ as a limit for $z \to \infty$. The reasonings of this limit are unclear, as a rule.

The procedure, proposed in [8], with the equation of motion is analogous to the turbulent energy equation, but some terms of the last equation are neglected. It is still unclear, why considered parcel only loses energy during its motion and cannot get it from the mean velocity flow shear. The interaction between the parcels is also to make some correction of the energy balance. Another question for latter discussion is the minimum ML value proposed in [8] to be equal to the numerical grid step. This proposal relates the results of numerical experiment with numerical realization of model. According to this for very fine resolution minimum value of L vanishes. It cannot be so because of the following consideration.

Let us consider the turbulent energy balance equation in the following form:

$$q\frac{\partial q}{\partial t} = \frac{\alpha_b}{3}\frac{\partial}{\partial z}L\frac{\partial(q^3)}{\partial z} - \frac{cq^3}{L} + qLG,$$

where $q = \sqrt{b}$, $G = -\beta\left(\frac{\partial \theta_v}{\partial z}\right) + \left|\frac{\partial \vec{V}}{\partial z}\right|^2$. From the analysis of this equation it is to emphasize that if at some point $q = 0$ and $G < 0$ (the stratification is stable), then L would be always equal to 0 for the definition made and hence turbulent energy would be equal to 0 too. That is turbulence never penetrates into the undisturbed regions.

This conclusion seams to be paradox and follows our suggestion that the parcel trajectory properties, starting its motion from point z_0, influence the characteristics of the turbulence at only that point. Fortunately, through the vicinity of this point move also the parcels that start from the other initial positions. Likely, they also contribute to the ML *in situ*, because they transfer the turbulent energy to the undisturbed regions. Thus ML depends not only on the z_0-parcels, but also on those separated by the distances less than L_{up} ($z < z_0$) or L_{down} ($z > z_0$).

Following this consideration, let us introduce two influence functions – upper $L^*_{\text{up}}(z, z_0)$ and lower $L^*_{\text{down}}(z, z_0)$. In case $L_{\text{up}}(z) > z_0 - z > 0$, we can let $L^*_{\text{down}}(z, z_0)$ be equal to $z_0 - z$ and the rest $L_{\text{up}}(z) - (z_0 - z)$ contribute to $L^*_{\text{up}}(z, z_0)$. We can do analogously if $L_{\text{down}}(z) > z - z_0 > 0$ takes place. Thus we have

$$L^*_{\text{down}}(z, z_0) = \begin{cases} \begin{cases} z_0 - z & \text{for } L_{\text{up}}(z) \geq z_0 - z \\ 0 & \text{for } L_{\text{up}}(z) < z_0 - z \end{cases} & \text{for } z < z_0 \\ \begin{cases} L_{\text{down}}(z) - (z - z_0) & \text{for } L_{\text{down}}(z) \geq z - z_0 \\ 0 & \text{for } L_{\text{down}}(z) < z - z_0 \end{cases} & \text{for } z \geq z_0 \end{cases}$$

$$L^*_{\text{up}}(z, z_0) = \begin{cases} \begin{cases} L_{\text{up}}(z) - (z_0 - z) & \text{for } L_{\text{up}}(z) \geq z_0 - z \\ 0 & \text{for } L_{\text{up}}(z) < z_0 - z \end{cases} & \text{for } z < z_0 \\ \begin{cases} z - z_0 & \text{for } L_{\text{down}}(z) \geq z - z_0 \\ 0 & \text{for } L_{\text{down}}(z) < z - z_0 \end{cases} & \text{for } z \geq z_0 \end{cases}$$

These expressions indicate that influence functions contain the information of the path lengths under and above z_0 of the parcels coming to that location from z. To get the net upper and lower ML one can convolute this functions at the region $\{L^{z_0}_{\text{up}} : L^*_{\text{up}}(z, z_0) > 0\}$ and at the region $\{L^{z_0}_{\text{down}} : L^*_{\text{down}}(z, z_0) > 0\}$:

$$\begin{aligned} \tilde{L}_{\text{up}}(z_0) &= \frac{1}{|L^{z_0}_{\text{up}}|} \int_{L^{z_0}_{\text{up}}} L^*_{\text{up}}(z, z_0) dz, \\ \tilde{L}_{\text{down}}(z_0) &= \frac{1}{|L^{z_0}_{\text{down}}|} \int_{L^{z_0}_{\text{down}}} L^*_{\text{down}}(z, z_0) dz, \end{aligned}$$

It is possible now to determine a new ML by the same formulae as Bougeault and Andre [8] did

$$\frac{1}{\tilde{L}} = \frac{1}{2}\left(\frac{1}{\tilde{L}_{\text{up}}} + \frac{1}{\tilde{L}_{\text{down}}}\right).$$

Proposed modification gives non zero value of ML near the turbulently disturbed region. This value depends only on the disturbance nature and do not on the numerical realization. It has also the advantages of Bougeault and Andre [8] approach.

4. Numerical experiments

First numerical experiment provides a comparison of the proposed parameterization with the original one. The whole model is analogous to those described in the previous work [9]. The initial conditions are the following:

$$\theta(z) = \theta_0 + \delta\theta \frac{z}{H}, \quad q(z) = 0, \quad u(z) = U_0 = 10 \text{ m/s}, \quad v(z) = 0, \quad q_l(z) = 0.$$

Here, θ is the potential temperature ($\theta_0 = 284^0$K, $\delta\theta = 6^0$, $H = 2$ km); q is the specific humidity; u, v are zone and meridian wind components; q_l is liquid moisture concentration.

The boundary conditions are constant except the sea surface temperature that varies as follows:

$$\theta\,|_{z=0}(t) = \theta_0 + \delta\theta_1(1 - \cos(2\pi t/\tau))$$

($\delta\theta_1 = 2^0$). The mixed layer is to develop near the surface under the above condition.

First set of figures shows the results of Bougeault and Andre model simulation. Figure 2 demonstrates potential temperature section in the plane of two coordinates – t and z. The growth of the mixed layer height takes place during the first two cycles, but on the third cycle it does not because of condensation processes at the top of it. The portion of the water vapor condenses and makes the surrounding air significantly warmer. The cloud cell forms the inversion layer at the lower height preventing the mixed layer to grow up continuously. Figure 3 indicates the distribution of liquid moisture concentration there. As one can see a small portion of liquid water arises even at the end of the second cycle, but this does not influence significantly on the temperature and turbulent characteristics' distribution.

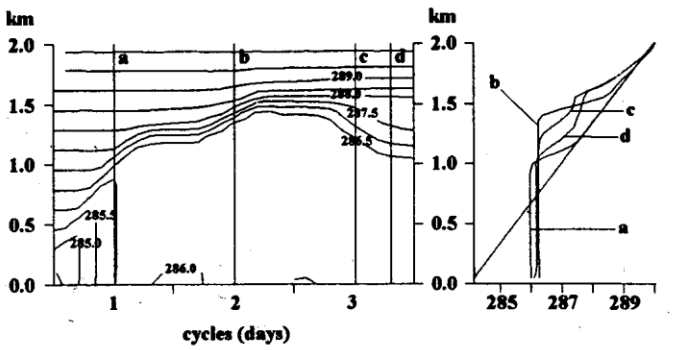

Figure 2. Potential temperature distribution with some vertical profiles

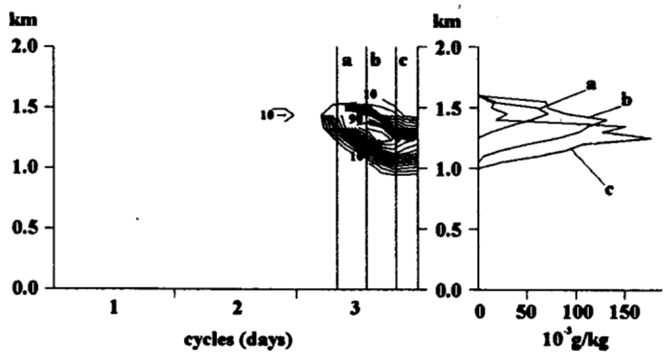

Figure 3. Liquid moisture concentration distribution with some vertical profiles

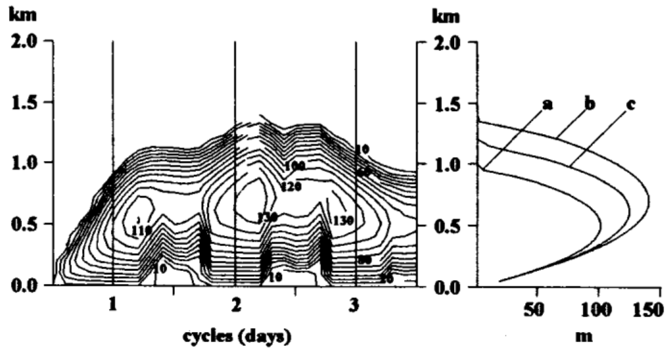

Figure 4. ML distribution with some vertical profiles

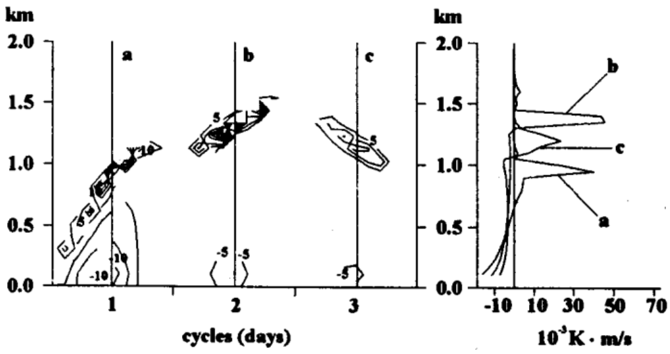

Figure 5. Turbulent potential temperature flux

The next two Figures 4 and 5 demonstrate the distributions of the ML and turbulent heat flux $(\overline{w'\theta'})$. The maximum ML value is about 150 m. The first cycle ML maximum forms at the height of about 500 m, then it decreases slightly. The second one overlaps the previous by the intensity and the vertical size. The third maximum forms under the control of the topped cloudiness, so it is less intensive and cannot reach the height of the second one, because of the energy lost against the buoyancy force. Analyzing the turbulent heat fluxes profiles we can see that these ones contain a very sharp variation at the top of the mixed layer for every cycle maximum. This is because of very narrow inversion layer. It forms provided a small turbulent penetration through it. According to the counter-gradient parameterization we have a small upward flux at the homogeneous layer.

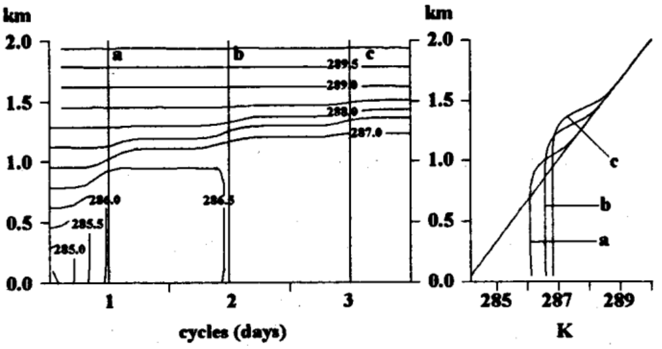

Figure 6. Potential temperature distribution with some profiles obtained by the modified version

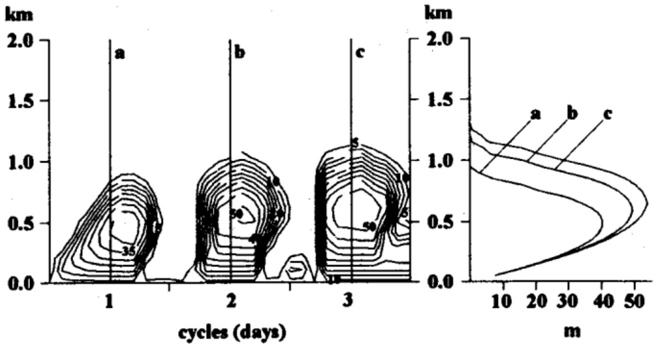

Figure 7. ML distribution with some profiles obtained by the modified version

The proposed modification of the Bougeault–Andre approach shows analogous results. The first distinction concerns the condensation processes during the third cycle. In the modified model these are not taking place, so the mixed-layer height grows continuously. Every cycle generates a new portion of turbulence that reaches the height of the previous one (Figures 6, 7, and 8). What is to say about the potential temperature flux (or heat flux) is that it becomes more smoothed and now it is in good agreement with the custom knowledge about it. The inversion layer is not so distinct as in the previous experiment, because turbulent penetration processes significantly destroy it. A large amount of turbulent energy contributes to this destruction of buoyancy barrier at the top of the mixed layer. Hence, ML has a maximum value

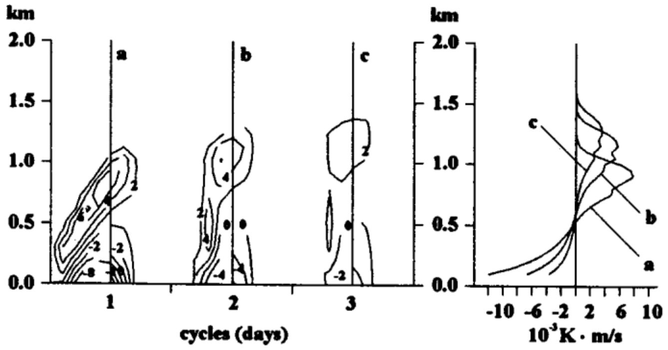

Figure 8. Turbulent potential temperature flux obtained by the modified version

that approximately 3 times less. The last is the principal difference between the original and modified the Bougeault–Andre parameterizations.

The minimum value of the ML in this experiment is allowed to be zero, while in the previous one it is equal to 1 meter (vertical resolution of the model is 50 m). Following Bougeault and Andre strictly we are to set it equal to 50 m. Meanwhile, the last experiment shows that its value is near always less than 50 m.

To examine both variants of the ML parameterization the same data set as in [9] was in use. The comparison of the model results with these data gives the correlation indexes summarized in the following table.

Parameterization types	R_S	R_V	R_t	R_z	R_T
Original Bougeault–Andre ML	78.9	42.5	54.2	61.8	57.9
Modified Bougeault–Andre ML	82.3	44.5	55.0	66.7	60.6

We can see that modification improved the model by approximately 3 %. More significant improvement takes place in vertical distribution index about 5 %. Looking at the correlation coefficients for atmosphere characteristics, presented in Figure 9, one can notice that better distribution both in time and in space arises for the specific humidity and meridian component of wind velocity.

5. Conclusion

The statistical properties of the boundary layer model using the Bougeault–Andre parameterization become more satisfactory, when model includes the proposed modification. This modification concerns the non-local feature of mixing, that is neighboring turbulent source can radiate turbulent curls capable to reach some undisturbed area. It makes also model to be more independent on the numerical space resolution.

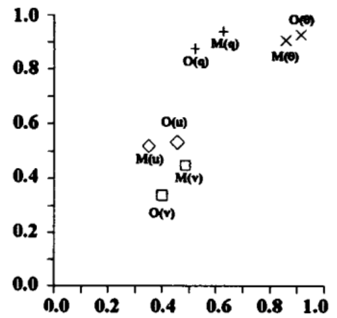

Figure 9. Spatial correlation coefficient vs. temporal one for the potential temperature, specific humidity and both components of wind velocity

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