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Behaviour analysis of parametric time Petri nets*

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We introduce a new notion of parametric time Petri nets, an extension of time Petri nets [10] whose transitions are labelled with parametric time restrictions. Further, a time behaviour analysis algorithm for the real time branching time temporal logic TCTL [1] and a one-safe parametric time Petri net is proposed. The result of the algorithm is a set of conditions on parameter variables which is sufficient for the property expressed as a TCTL-formula being satisfied for a given parametric time Petri net. Some remarks about complexity of the algorithm are also given.

Introduction

The verification problem of real-time systems is one of the main research directions of modern programming. Traditional verification methods are suitable for checking only qualitative timing properties of the systems. However, this is unsufficient for a correctness analysis of real-time systems whose behaviour crucially depends on quantitative properties. Several temporal logics for quantitative analysis of systems [3] have been proposed.

Concurrent real-time systems are often modelled by using time automata with a finite set of clocks [2], timed transition systems [4] and time process algebras (see, for example, [8]). However, all these formalisms are based on interleaving semantics and therefore information about concurrency is lost. On the other hand, time Petri nets [6] have been proposed as a suitable formalism for modelling concurrent real-time systems.

Model-checking is an effective verification tool. In [9], deductive and algorithmic verification methods based on temporal language CTL were proposed. In [5], a temporal logic was introduced for analysis of 'fairness' conditions of labelled Petri nets. However, model-checking algorithms usually require redundantly detailed specifications of systems, which can leave a user in repetitive trial-and-error cycles to select a parameter valuation. It should be useful to have a tool for a less-detailed specification of real-time systems and their properties.

In this paper we introduce a behaviour analysis algorithm of real-time systems represented as parametric time Petri nets. This kind of nets is an extension of time Petri nets [10] by means of parametric time constraints

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with character strings which represent unspecified timing constants as parameter variables. We assume that the value of each parameter variable of a parametric time Petri net does not exceed some fixed bound. Further we restrict ourselves to one-safe parametric time Petri nets. The result of the algorithm is a set of conditions on parameter variables which is sufficient for the property expressed as a TCTL-formula true for the given parametric time Petri net.

The rest of the paper is organized as follows. The basic definitions concerning parametric time Petri nets are presented in Section 1. Section 2 recalls the syntax and semantics of TCTL. In Section 3 we define a notion of a region for a time Petri net and construct the region graph in order to get a finite representation of the net. A time behaviour analysis algorithm is provided in Section 4. Some remarks about the complexity of the algorithm are finally given.

1. Parametric time Petri nets

Let $\mathbf{N}$ be the set of natural numbers and $\mathbf{R}^+$ be the set of nonnegative real numbers. Let Var be a set of parameter variables which are character strings representing unspecified timing constants and A be an arbitrary set.

The syntax of a *time predicate* η over Var and A is defined as follows:

$$\eta ::= x \sim \theta \mid \eta_1 \wedge \eta_2,$$

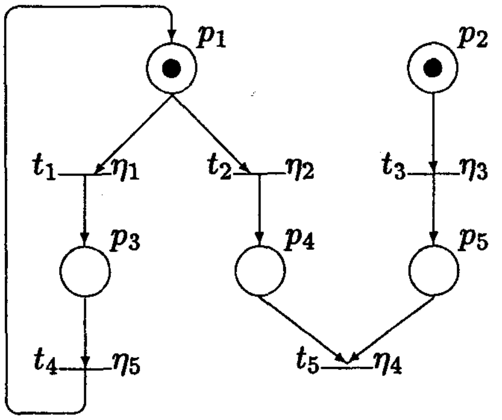
where $x \in A \cup \mathbf{N}$, $\theta \in Var \cup \mathbf{N}$, η_1 and η_2 are time predicates over Var and A, and $\sim$ stands for one of the binary relations $\{<, \leq, =, \geq, >\}$. Let B^A_{Var} be the set of all time predicates over Var and A. For a time predicate $\eta \in B^A_{Var}$, the notation $[\eta]^{y_1,\cdots,y_l}_{a_1,\cdots,a_l}$ is the result of substitution of values $a_1, \cdots, a_l$ instead of every occurence of $y_1, \cdots, y_l$ in η.

Definition 1.1. A *parametric time Petri net* is a tuple $\mathcal{N} = (P, T, F, Var, \tau, m_0)$, where

- $P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places;
- $T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions ($P \cap T = \emptyset$);
- $F \subseteq (P \times T) \cup (T \times P)$ is a flow relation;
- $Var = \{\theta_1, \theta_2, \ldots, \theta_k\}$ is a finite set of parameter variables;
- $\tau : T \to B^T_{Var}$ is a function that associates a time predicate from B^T_{Var} with each transition from T;
- $m_0 \subseteq P$ is the initial marking.

For $t \in T$, ${}^\bullet t = \{p \in P \mid (p,t) \in F\}$ and $t^\bullet = \{p \in P \mid (t,p) \in F\}$ denote the *preset* and *postset* of t, respectively. To simplify the presentation, we

$\mathcal{N}_1$:



η_1	$t_1 = \theta_1$
η_2	$t_2 \geq \theta_2^1 \wedge t_2 \leq \theta_2^2 \wedge \theta_2^2 \leq 2$
η_3	$t_3 = 1$
η_4	$t_4 = 0$
η_5	$t_5 = 0$

Figure 1. The parametric time Petri net $\mathcal{N}_1$

assume that ${}^\bullet t \cap t^\bullet = \emptyset$ for every transition t. For the sake of convenience, we fix a parametric time Petri net $\mathcal{N} = (P, T, F, Var, \tau, m_0)$ and work with it in what follows.

The parametric time Petri net $\mathcal{N}_1$ is shown in Figure 1, where the table contains the time predicates η_i $(1 \leq i \leq 5)$ representing time constraints on transitions t_i.

A *marking* m of $\mathcal{N}$ is any subset of P. A transition t is *enabled* in a marking m if ${}^\bullet t \subseteq m$ (all its input places have tokens in m), otherwise it is *disabled.* Let *enable*(m) be the set of transitions enabled in m.

Let $\mathcal{V} = [T \to \mathbf{R}^+ \cup \{\#\}]$ be the set of *time assignments* for transitions from T where $\#$ is a special symbol for labelling disabled transitions such that $\# \sim \theta = true$ for each $\theta \in Var \cup \mathbf{N}$ and $\sim \in \{<, \leq, =, \geq, >\}$. Assume that $\nu \in \mathcal{V}$ and $\delta \in \mathbf{R}^+$. Then

$$(\nu + \delta)(t) = \begin{cases} \nu(t) + \delta, & \text{if } \nu(t) \neq \#, \\ \nu(t), & \text{otherwise.} \end{cases}$$

A *state* q of $\mathcal{N}$ is a pair $\langle m, \nu \rangle$, where m is a marking and $\nu \in \mathcal{V}$. The *initial state* in $\mathcal{N}$ is a pair $q_0 = \langle m_0, \nu_0 \rangle$, where

- m_0 is the initial marking in $\mathcal{N}$;
- $\forall t \in T.\ \nu_0(t) = \begin{cases} 0, & \text{if } t \in enable(m_0), \\ \#, & \text{otherwise.} \end{cases}$

Let S denote the set of states of $\mathcal{N}$.

A *parameter valuation* χ of $\mathcal{N}$ is a mapping from Var into $\mathbf{N}$. Given a time assignment $\nu \in \mathcal{V}$ and a transition $t \in T$, we use the following notations:

$$\tau^\chi(t) = [\tau(t)]^{\theta_1, \ldots, \theta_k}_{\chi(\theta_1), \ldots, \chi(\theta_k)}; \quad \tau^{\nu,\chi}(t) = [\tau(t)]^{t_1, \ldots, t_n, \theta_1, \ldots, \theta_k}_{\nu(t_1), \ldots, \nu(t_n), \chi(\theta_1), \ldots, \chi(\theta_k)};$$

$$\mathcal{N}^\chi = (P, T, F, Var, \tau^\chi, m_0)$$

There are two causes of state changes in $\mathcal{N}$, i.e. (1) firing of transition, and (2) time passage.

In a state $q = \langle m, \nu \rangle$ of $\mathcal{N}$, a transition $t \in T$ is *fireable*, if $t \in enable(m)$ and there exists a parameter valuation χ such that $\tau^{\nu,\chi}(t) = true$. In this case, the state $q' = \langle m', \nu' \rangle$ of $\mathcal{N}$ is *obtained from q by firing t* (written $q \overset{0}{\Rightarrow} q'$), if

- $m' = (m \setminus {}^\bullet t) \cup t^\bullet$, and
- $\forall t' \in T.\ \nu'(t') = \begin{cases} 0, & \text{if } t' \in enable(m') \setminus enable(m), \\ \#, & \text{if } t' \in enable(m) \setminus enable(m'), \\ \nu(t'), & \text{otherwise.} \end{cases}$

In a state $q = \langle m, \nu \rangle$ of $\mathcal{N}$, time $\delta \in \mathbf{R}^+$ *can pass*, if for all $t \in enable(m)$ there exist $\delta' \geq \delta$ and a parameter valuation χ such that $\tau^{\nu+\delta',\chi}(t) = true$. In this case, the state $q' = \langle m', \nu' \rangle$ of $\mathcal{N}$ is *obtained from q by passing δ* (written $q \overset{\delta}{\Rightarrow} q'$), if

- $m' = m$, and
- $\nu' = \nu + \delta$.

A *q-run* r of $\mathcal{N}$ is an infinite sequence of states $q_i \in S$ and time values $\delta_i \in \mathbf{R}^+$ of the form:

$$q = q_1 \overset{\delta_1}{\Rightarrow} q_2 \overset{\delta_2}{\Rightarrow} \ldots \Rightarrow q_n \overset{\delta_n}{\Rightarrow} \ldots,$$

satisfying the *progress condition:* for each $w \in \mathbf{R}^+$ there is $n \in \mathbf{N}$ such that $\sum_{1 \leq i < n} \delta_i \geq w$. We denote the expression $\sum_{1 \leq i < n} \delta_i$ by $time(r, n)$.

A state q is *reachable* if it belongs to some q_0-run. Let $RS(\mathcal{N})$ denote the set of all reachable states of $\mathcal{N}$.

$\mathcal{N}$ is *one-safe*, if for every $\langle m, \nu \rangle \in RS(\mathcal{N})$ and for every $t \in enable(m)$ it holds that $t^\bullet \cap m = \emptyset$.

A transition $t \in T$ is *time bounded* if for each parameter valuation χ and for each parameter variable θ from $\tau(t)$ there exists a constant c such that $\chi(\theta) < c$. $\mathcal{N}$ is *time bounded* if each its transition is time bounded. The set of all such constants c for the given $\mathcal{N}$ is denoted by $C(\mathcal{N})$.

Further $\mathcal{N}$ will always denote a one-safe time bounded parametric time Petri net.

2. TCTL: syntax and semantics

Timed Computation Tree Logic (TCTL) was introduced by R. Alur, C. Courcoubetis, D. Dill [1] as a specification language for real time systems. We now review the syntax and semantics of TCTL.

Let AP be a set of atomic propositions. For our purpose, it is convenient to take $AP = P$.

Definition 2.1. The *formula* ϕ of TCTL is inductively defined as follows:

$$\phi ::= p \mid \neg\phi_1 \mid \phi_1 \ \wedge \ \phi_2 \mid \forall\phi_1\mathcal{U}_{\sim c}\phi_2 \mid \exists\phi_1\mathcal{U}_{\sim c}\phi_2,$$

where $p \in AP$, $c \in \mathbf{N}$, ϕ_1 and ϕ_2 are formulas of TCTL, $\sim$ stands for one of the binary relations $\{<, \leq, =, \geq, >\}$.

Informally, $\exists\phi_1\mathcal{U}_{<c}\phi_2$ means that for some computation path there exists an initial prefix of time length less than c such that ϕ_2 holds in the last state of the prefix, and ϕ_1 holds in all its intermediate states.

We define the derived connectives of the propositional calculus, such as $\vee$ and $\rightarrow$, in terms of $\neg$ and $\wedge$ in the usual way. In addition, some of the commonly used abbreviations are:

- $\forall\Diamond_{\sim c}\phi \equiv \forall \mathit{true}\ \mathcal{U}_{\sim c}\phi$,
- $\exists\Diamond_{\sim c}\phi \equiv \exists \mathit{true}\ \mathcal{U}_{\sim c}\phi$,
- $\forall\Box_{\sim c}\phi \equiv \neg\exists\Diamond_{\sim c}\neg\phi$,
- $\exists\Box_{\sim c}\phi \equiv \neg\forall\Diamond_{\sim c}\neg\phi$.

The unrestricted temporal operators correspond to TCTL-operators subscripted by '≥ 0'.

Definition 2.2. Given a parameter valuation χ, a TCTL-formula ϕ and a state $q = \langle m, \nu\rangle \in RS(\mathcal{N}^\chi)$, we define the satisfaction relation $q \models \phi$ inductively as follows:

$$\begin{array}{lcl}
q \models p & \Leftrightarrow & p \in m; \\
q \models \neg\phi_1 & \Leftrightarrow & q \not\models \phi_1; \\
q \models \phi_1 \rightarrow \phi_2 & \Leftrightarrow & q \models \phi_1 \text{ and } q \models \phi_2; \\
q \models \exists\phi_1\mathcal{U}_{\sim c}\phi_2 & \Leftrightarrow & \text{for some } q\text{-run } r \text{ of } \mathcal{N}^\chi,\ r \models \phi_1\mathcal{U}_{\sim c}\phi_2; \\
q \models \forall\phi_1\mathcal{U}_{\sim c}\phi_2 & \Leftrightarrow & \text{for every } q\text{-run } r \text{ of } \mathcal{N}^\chi,\ r \models \phi_1\mathcal{U}_{\sim c}\phi_2.
\end{array}$$

For a q-run $r = (q = q_1 \overset{\delta_1}{\Rightarrow} q_2 \overset{\delta_2}{\Rightarrow} \ldots)$ in $\mathcal{N}^\chi$ the relation $r \models \phi_1\mathcal{U}_{\sim c}\phi_2$ holds iff there exist k and $\delta \leq \delta_k$ such that:

1. $(\delta + \mathit{time}(r,k)) \sim c$;
2. $\langle m_k, \nu_k + \delta\rangle \models \phi_2$;
3. $\forall\, 1 \leq i < k\ \forall\, 0 \leq \delta' < \delta_i\,.\ \langle m_i, \nu_i + \delta'\rangle \models \phi_1$;
4. $\forall\ 0 \leq \delta' < \delta\,.\ \langle m_k, \nu_k + \delta'\rangle \models \phi_1$.

$\mathcal{N}^\chi$ satisfies a TCTL-formula ϕ (written $\mathcal{N}^\chi \models \phi$) iff $q_0 \models \phi$. A TCTL-formula ϕ is satisfiable iff there is $\mathcal{N}^\chi$ such that $\mathcal{N}^\chi \models \phi$.

Theorem 2.1. The satisfiability question for TCTL is Σ_1^1-hard.

Proof. Follows from Definition 2.2 and the corresponding theorem from [1]. □

A TCTL-formula containing $\mathcal{U}$-operator is said to be an *U-formula.*
Further we fix a TCTL-formula ϕ. χ is a *satisfiable valuation of $\mathcal{N}$ for ϕ* if $\mathcal{N}^\chi \models \phi$. *To analyze $\mathcal{N}$ for ϕ* means to find the set of satisfiable valuations of $\mathcal{N}$ for ϕ.

3. Region graph

Since a parametric time Petri net constitutes a dense time model, the number of its states is infinite. In order to get a finite representation of the behaviour of a parametric time Petri net, we define a notion of a region [1]. Two states of a parametric time Petri net are in the same region iff their markings coincide and the corresponding time assignment values agree on the integral parts and on the ordering of the fractional parts.

We extend a parametric time Petri net with a clock tick indicator κ which is conceptually a time assignment of a transition t^κ that can not be reset and whose actual value is of no concern. But we can test whether it is an integer or not at any moment.

Let c_ϕ be the maximal constant appearing in ϕ.

Then $K_{\mathcal{N}:\phi} = \max(\{c_\phi\} \cup \{c \mid \theta \in C(\mathcal{N})\})$. For any $\delta \in \mathbf{R}^+ \cup \{\#\}$, $fract(\delta)$ denotes the fractional part of δ, and $\lfloor \delta \rfloor$ denotes the integral part of δ. Assume that $\lfloor \# \rfloor = \#$, $fract(\#) = 0$.

Definition 3.1. Given $\nu, \nu' \in \mathcal{V}$, $\nu \simeq_\phi \nu'$ iff the following conditions are met:

- for each $x \in T$ if either $\nu(x) \leq K_{\mathcal{N}:\phi}$ or $\nu'(x) \leq K_{\mathcal{N}:\phi}$, then $\lfloor \nu(x) \rfloor = \lfloor \nu'(x) \rfloor$;
- for each $x, x' \in T \cup \{t^\kappa\}$
  - $fract(\nu(x)) \leq fract(\nu(x'))$ iff $fract(\nu'(x)) \leq fract(\nu'(x'))$;
  - $fract(\nu(x)) = 0$ iff $fract(\nu'(x)) = 0$.

Given $\nu \in \mathcal{V}$, let us denote $[\nu]_\phi = \{\nu' \mid \nu' \simeq_\phi \nu\}$. A *region* of $\mathcal{N}$ with respect to ϕ is a pair $[q]_\phi = \langle m, [\nu]_\phi \rangle$, where $\langle m, \nu \rangle \in RS(\mathcal{N})$. We denote $m(\langle m, [\nu]_\phi \rangle) = m$.

Figure 2 shows the set of regions of the parametric time Petri net $\mathcal{N}_1$.

Lemma 3.1. Let $\langle m, \nu \rangle, \langle m, \nu' \rangle \in RS(\mathcal{N})$ with $\nu \simeq_\phi \nu'$. For every TCTL-formula ψ such that $K_{\mathcal{N}:\phi} \geq K_{\mathcal{N}:\psi}$ and a parameter valuation χ such that $\langle m, \nu \rangle \in RS(\mathcal{N}^\chi)$ the following holds: $\langle m, \nu' \rangle \in RS(\mathcal{N}^\chi)$ and $\langle m, \nu \rangle \models \psi \Leftrightarrow \langle m, \nu' \rangle \models \psi$.

v	m	$[\nu]$				
		t_1	t_2	t_3	t_4	t_5
v_0	11000	[0,0]	[0,0]	[0,0]	#	#
v_1	01010	#	#	[0,0]	#	#
v_2	01010	#	#	(0,1)	#	#
v_3	01010	#	#	[1,1]	#	#
v_4	00011	#	#	#	#	[0,0]
v_5	00000	#	#	#	#	#
v_6	11000	(0,1)	(0,1)	(0,1)	#	#
v_7	11000	[1,1]	[1,1]	[1,1]	#	#
v_8	10001	[1,1]	#	#	#	#
v_9	00101	#	#	#	[0,0]	#
v_{10}	10001	[0,0]	#	#	#	#
v_{11}	10001	(0,1)	#	#	#	#
v_{12}	10001	[1,1]	#	#	#	#
v_{13}	10001	(1,2)	#	#	#	#
v_{14}	10001	[2,2]	#	#	#	#
v_{15}	01100	#	#	[0,0]	[0,0]	#
v_{16}	01100	#	#	(0,1)	[0,0]	#
v_{17}	01100	#	#	[1,1]	[0,0]	#

Figure 2. The regions of $\mathcal{N}_1$

Proof. Before proving the lemma, we need to show that there exist $\langle m_0, \nu_0\rangle$-paths r and r' in $\mathcal{N}$ containing $\langle m, \nu\rangle$ and $\langle m, \nu'\rangle$, respectively, which are similar in following sense. Let us consider a $\langle m_0, \nu_0\rangle$-path r: $\langle m_0, \nu_0\rangle \overset{\delta_0}{\Rightarrow} \langle m_1, \nu_1\rangle \overset{\delta_1}{\Rightarrow} \ldots \langle m_k, \nu_k\rangle = \langle m, \nu\rangle \overset{\delta_k}{\Rightarrow} \langle m_{k+1}, \nu_{k+1}\rangle \ldots$ We show that there exists a $\langle m_0, \nu_0\rangle$-path r' : $\langle m_0, \nu'_0\rangle \overset{\delta'_0}{\Rightarrow} \langle m_1, \nu'_1\rangle \overset{\delta'_1}{\Rightarrow} \ldots \langle m_k, \nu'_k\rangle = \langle m, \nu'\rangle \overset{\delta'_k}{\Rightarrow} \langle m_{k+1}, \nu'_{k+1}\rangle \ldots$ with the same sequence of markings as r taken at "almost" the same time moments. Formally, for every $i \geq 1$ the following correspondences between $(\nu_i, time(r,i))$ and $(\nu'_i, time(r',i))$ are met (written $(\nu_i, time(r,i)) \simeq (\nu'_i, time(r',i))$):

- the time assignments ν_i and ν'_i are equivalent: $\nu_i \simeq \nu'_i$;
- the time values $time(r,i)$ and $time(r',i)$ agree on integer parts:
  - $\lfloor time(r,i)\rfloor = \lfloor time(r',i)\rfloor$,
  - $fract(time(r,i)) = 0 \Leftrightarrow fract(time(r',i)) = 0$;
- the time values $time(r,i)$ and $time(r',i)$ are in the same order with time assignments $\nu(t)$ and $\nu'(t)$, correspondingly: for each transition $t \in T$ it holds:

- $fract(\nu_i(t)) < fract(time(r,i)) \Leftrightarrow$
  $fract(\nu_i'(t)) < fract(time(r',i))$,
- $fract(\nu_i(t)) > fract(time(r,i)) \Leftrightarrow$
  $fract(\nu_i'(t)) > fract(time(r',i))$.

Given r, we construct the desired path r' in two steps:

1. First, we construct a prefix ρ' (ending in $\langle m, \nu' \rangle$) of r' corresponding to the prefix ρ of r: $\langle m_0, \nu_0 \rangle \overset{\delta_0}{\Rightarrow} \langle m_1, \nu_1 \rangle \overset{\delta_1}{\Rightarrow} \ldots \langle m_k, \nu_k \rangle = \langle m_0, \nu_0 \rangle$. Let us construct ρ' step by step from $\langle m, \nu' \rangle$ to $\langle m, \nu \rangle$. Suppose that $(k-i)$ states of ρ' have already been constructed. Given δ_i, we try to find δ_i' so that we can extend ρ' with $\langle m_{i-1}, \nu_{i-1} \rangle$ so that the equivalence is maintained at the $(k-i+1)$th step.

Let Δ be the set $\{time(r,i)\} \cup \{\nu_i(t) \mid t \in T\}$. The set Δ' is defined analogously. Because of the equivalence at the $(k-i)$th step, the nessesary and sufficient requiment of the desired δ_i' is such that each element of Δ' should cross the same number of integer boundaries (due to the addition of δ_i') as the corresponding element of Δ does (due to the addition of δ_i). The reader can convince himself that the existence of δ_i' meeting the constraints depends only on the ordering of the fractional parts of the elements of Δ. Hence, the existence of δ_i guarantees the existence of δ_i'. Let $\langle m, \nu \rangle \in RS(\mathcal{N}^{\mathcal{X}})$. For each i the constraint η on the time of reaching a state $\langle m_{i+1}, \nu_{i+1} \rangle$ from the state $\langle m_i, \nu_i \rangle$ depends only on ν_i and contains only integer constants. Using $\nu_i \simeq \nu_i'$ we get $\lfloor \nu_i \rfloor = \lfloor \nu_i' \rfloor$. So, the constraint η' on the time of obtaining a state $\langle m_{i+1}, \nu_{i+1}' \rangle$ from the state $\langle m_i, \nu_i' \rangle$ is satisfied iff η is satisfied. Then ρ' is a prefix in $\mathcal{N}^{\mathcal{X}}$ and, hence, $\langle m, \nu' \rangle \in RS(\mathcal{N}^{\mathcal{X}})$.

2. Secondly, we consider the path r' as a concatenation of the prefix ρ' and some $\langle m, \nu' \rangle$-path constructed step by step, starting from the state $\langle m, \nu' \rangle$ for a given $\langle m, \nu \rangle$-path. The construction of the $\langle m, \nu' \rangle$-path is similar to the construction of ρ'.

Now we show that the paths r and r' are similar. The base case follows since $\nu \simeq \nu'$ and $time(r,1) = time(r',1) = 0$. Suppose that we have constructed r' correctly up to i steps. It is easy to see that the choice of δ_i' depends only on an ordering of the fractional parts of elements on Δ' which is the same as in Δ. Hence, the existence of δ_i guarantees the existence of δ_i' satisfying the above conditions.

We shall proceed by induction on the structure of ψ. The base case and the cases corresponding to the logical connectives follow immediately. We will consider the case $\psi = \exists \phi_1 \mathcal{U}_{\sim c} \phi_2$. Suppose, $\langle m, \nu \rangle \models \psi$. There exists an $\langle m, \nu \rangle$-path r such that $r \models \phi_1 \mathcal{U}_{\sim c} \phi_2$, i.e. there exist k and $\delta \leq \delta_k$ such that:

1. $(\delta + time(r,k)) \sim c$;
2. $\langle m_k, \nu_k + \delta \rangle \models \phi_2$;

3. $\forall 1 \leq i < k \ \forall \delta'.(\delta' = 0 \vee 0 < \delta' < \delta_i) \ \langle m_i, \nu_i + \delta' \rangle \models \phi_1$;
4. $\forall\, 0 \leq \delta' < \delta\, .\ \langle m_k, \nu_k + \delta' \rangle \models \phi_1$.

Let us construct an $\langle m, \nu' \rangle$-path r' as above. We have to show that $r' \models \phi_1 \mathcal{U}_{\sim c} \phi_2$.

1. We know that $(\nu_{k-1}, time(r, k-1)) \simeq (\nu'_{k-1}, time(r', k-1))$. Because of the construction of r' for a given δ we can find δ' such that $(\nu_k, time(r, k)) \simeq (\nu'_k, time(r', k))$. Using the fact that $(\delta + time(r, k)) \sim c$, we get $(\delta' + time(r', k)) \sim c$.
2. We have $\langle m_k, \nu_k + \delta \rangle \models \phi_2$ and, by induction hypothesis, $\langle m_k, \nu'_k + \delta' \rangle \models \phi_2$.
3. Let $i < k$ and $0 \leq \delta' < \delta'_i$. Now using the equivalence of $(\nu_i, time(r, i))$ and $(\nu'_i, time(r', i))$, we can find $0 \leq \delta \leq \delta_i$ such that $(\nu_{i+1}, time(r, i+1)) \simeq (\nu'_{i+1}, time(r', i+1))$. Since $\langle m_i, \nu_i + \delta \rangle \models \phi_1$, by induction hypothesis, we get $\langle m_k, \nu'_k + \delta' \rangle \models \phi_1$;
4. Similar to item 3.

Thus, $r' \models \phi_1 \mathcal{U}_{\sim c} \phi_2$ and $\langle m, \nu' \rangle \models \psi$. □

Lemma 3.2. The number of equivalence classes of $\mathcal{V}$ induced by $\simeq_\phi$ is bounded by $\mid T \mid ! \cdot 2^{2|T|} \cdot (K_{\mathcal{N}:\phi} + 1)^{|T|}$.

Proof. We can represent an equivalence class $[\nu]_\phi$ of $\mathcal{V}$ induced by $\simeq_\phi$ by a triple of arrays $\langle \alpha, \beta, \gamma \rangle$ as follows:

The array α is a T-indexed array associating one of the intervals from $[0,0]$, $(0,1)$, $[1,1]$, $\cdots$, $[K_{\mathcal{N}:\phi}, K_{\mathcal{N}:\phi}]$, $(K_{\mathcal{N}:\phi}, \infty)$ with each transition $t \in T$. The array α represents a time assignment ν iff $\nu(t) \in \alpha(t)$ for each $t \in T$.

Let T_α be the set of transitions t such that $\alpha(t)$ is not of the form $[i, i]$ for some $i \leq K_{\mathcal{N}:\phi}$. Thus T_α is a set of transitions for which an order of fractional parts of time assignments is essential.

The array $\beta : T_\alpha \to \{1 \cdots |T_\alpha|\}$ is a permutation of T_α. It gives the ordering of fractional parts of time assignments corresponding to the transition from T_α with respect to $\leq$. The array β represents a time assignment ν iff for each pair $t_1, t_2 \in T_\alpha$ we have: if $\beta(t_1) \leq \beta(t_2)$, then $fract(\nu(t_1)) \leq fract(\nu(t_2))$.

The array γ is a boolean T_α-indexed array, and is used to specify which transition in T_α has the same fractional parts of corresponding time assignments. For each transition $t \in T_\alpha$, $\gamma(t)$ tells whether or not the fractional part of $\nu(t)$ equals to the fractional part of its β-predecessor. The array γ represents a time assignment ν iff for each $t_1 \in T_\alpha$ it holds that $\gamma(t_1) = 0$ iff there is a transition $t_2 \in T_\alpha$ such that $\beta(t_2) = \beta(t_1) + 1$ and $fract(\nu(t_1)) = fract(\nu(t_2))$.

It is easy to see that the number of equivalence classes of $\mathcal{V}$ induced by $\simeq_\phi$ is bounded by the number of triples $\langle \alpha, \beta, \gamma \rangle$ of the desired form. The number of ways to choose α is $(K_{\mathcal{N}:\phi}+1)^{|T|}$. For a given α, the number of ways to choose β is bounded by the number of permutations over T_α which is bounded by $|T|!$, and the number of ways to choose γ is bounded by the number of boolean arrays over T_α which is bounded by $2^{|T|}$. □

Definition 3.2. The *region graph* of $\mathcal{N}$ and ϕ is defined to be a labelled graph $G(\mathcal{N}, \phi) = (V, E, l)$. The set of vertices V is the set of regions of $\mathcal{N}$ with respect to ϕ. The set of arcs $E \subseteq V \times V$ consists of two types of arcs:

- The arc $([q], [q'])$ may represent firing of a transition in $\mathcal{N}$: $q \overset{0}{\Rightarrow} q'$.
- The arc $([q], [q'])$ may represent the passage of time:
  - $q' = q + \delta$ for some $\delta \in \mathbf{R}^+$;
  - there are no $\bar{q}$ and $\bar{\delta} \in \mathbf{R}^+$, $0 < \bar{\delta} < \delta$ such that $[\bar{q}]_\phi \neq [q]_\phi$, $[\bar{q}]_\phi \neq [q']_\phi$, $q + \bar{\delta} = \bar{q}$, and $\bar{q} + \delta - \bar{\delta} = q'$.

The function l labels an arc either with a transition $t \in T$ (if the arc represents firing of t) or with the sign $'\delta'$ (if the arc represents the passage of time). For the sake of convenience, we fix a region graph $G(\mathcal{N}, \phi) = (V, E, l)$ and work with it in what follows.

Figure 3 shows the region graph of the parametric time Petri net $\mathcal{N}_1$ (see Figure 1).

Lemma 3.3. The number of regions of $\mathcal{N}$ for ϕ is bounded by

$$| T |! \cdot 2^{|P|+2|T|} \cdot (K_{\mathcal{N}:\phi}+1)^{|T|}.$$

Proof. Assume $\langle m, [\nu]_\phi \rangle$ to be a region of $\mathcal{N}$. Since m is an array of the length $| P |$ consisting of '0' and '1', the number of different markings of $\mathcal{N}$ is $2^{|P|}$. Moreover, the number of equivalence classes of time assignments is bounded by $| T |! \cdot 2^{2|T|} \cdot (K_{\mathcal{N}:\phi}+1)^{|T|}$, due to Lemma 3.2. Hence, the number of regions of $\mathcal{N}$ for ϕ is $| T |! \cdot 2^{|P|+2|T|} \cdot (K_{\mathcal{N}:\phi}+1)^{|T|}$, by Lemma 3.2. □

For each arc $([q]_\phi, [q']_\phi)$ from E, we let $\varepsilon([q]_\phi, [q']_\phi) =\uparrow$ if going from q to q' the value of κ increases from a noninteger to an integer; $\varepsilon([q]_\phi, [q']_\phi) =\downarrow$ if going from q to q' the value of κ increases from an integer to a noninteger; otherwise $\varepsilon([q]_\phi, [q']_\phi) = 0$.

Given a vertex v in V, a *path starting from* v (written Γ^v) is a sequence of vertices $\langle v_1 v_2 \ldots \rangle$ from V, where $v_1 = v$ and for every $i \geq 1$ $(v_i, v_{i+1}) \in E$ if v_{i+1} exists. We denote by $l(\Gamma^v)$ a sequence of labels corresponding to Γ^v. Then $tr(\Gamma^v)$ is a restriction of $l(\Gamma^v)$ to T. Given a transition $t \in tr(\Gamma^v)$ and $v', v'' \in \Gamma^v$ such that $l(v', v'') = t$, we denote $in(t, \Gamma^v) = v'$.

Given v, v' from V, we use $\Gamma^v_{v'}$ to denote the finite path starting from v and ending in v'. We define the *time* of $\Gamma^v_{v'} = \langle v = v_1 v_2 \ldots v_k = v' \rangle$ (written

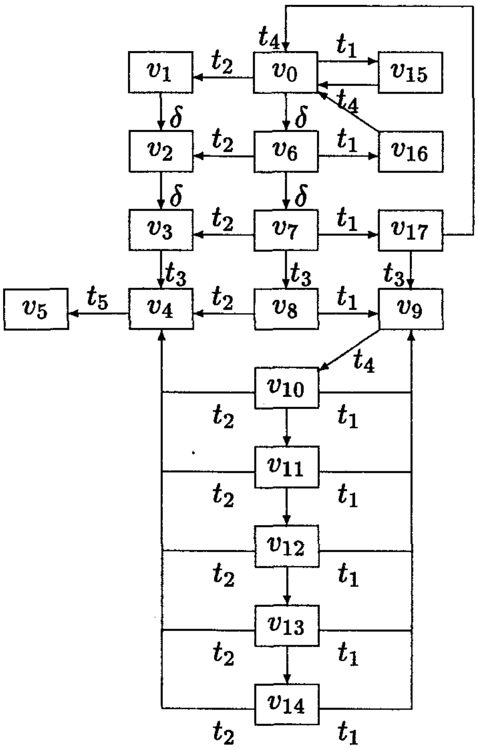


Figure 3. The region graph of the time Petri net $\mathcal{N}_1$

$time(\Gamma^v_{v'}))$ as the number of arcs (v_i, v_{i+1}) such that $\varepsilon(v_i, v_{i+1}) =\uparrow$ for $1 \leq i < k-1$. Let *Simple* be the set of all simple paths in $G(\mathcal{N}, \phi) = (V, E, l)$.

A *cycle* is a finite path $\langle v_1 \dots v_m \rangle$ such that $m \geq 2$ and $v_1 = v_m$. Γ^v is a *short path* if each simple cycle is traversed at most once along Γ^v. Γ^v is a *slim path* if each cycle of zero time is traversed at most once along Γ^v.

Lemma 3.4. Given two vertices v, $v' \in V$ and $d \in \mathbf{N}$, there is a path from v to v' of time d iff there is a slim path from v to v' of time d.

Proof follows from the fact that a path which is not slim can be reduced to a slim one by deleting duplicate zero cycles. □

Given a simple path $\Gamma^{v_1}_{v_k} = \langle v_1 v_2 \dots v_k \rangle$ and a finite set H of simple cycles in $G(\mathcal{N}, \phi)$, we call $(\Gamma^{v_1}_{v_k}, H)$ a *cactus structure* iff for each $\Omega \in H$ there is a finite sequence $\Omega_1, \cdots \Omega_m$ ($\Omega_i = \langle v^i_1 v^i_2 \dots v^i_{k_i} \rangle$, $1 \leq i \leq m$) of simple cycles in H such that

- $\Omega_1 = \Omega$, and
- for each $1 \leq i < m$ there is $1 \leq j \leq k_{i+1}$ such that $v_j^{i+1} = v_1^i$, and
- for some $1 \leq i \leq k$ $v_i = v_1^m$ holds.

Given a set of nonnegative integers $r_1, \dots, r_m$, let $gcd(r_1, \dots, r_m)$ and $lcm(r_1, \dots, r_m)$ be, respectively, the *greatest common divisor* and the *least common multiple* of the nonzero elements in $r_1, \dots, r_m$.

Given a cactus structure $(\Gamma_{v'}^v, \{\Omega_1, \dots, \Omega_m\})$. Let us denote $r_i = time(\Omega_i)$ for each $1 \leq i \leq m$. (Υ, Ψ) is an *offset-period pair* of $\Gamma_{v'}^v$, where

$$\Upsilon = time(\Gamma_{v'}^v) + \sum_{1 \leq i \leq m} r_i + m \cdot lcm(r_1, \dots, r_m), \Psi = gcd(r_1, \dots, r_m).$$

Let $Path(\Gamma_{v'}^v, \{\Omega_1, \dots, \Omega_m\})$ be the set of paths induced by the cactus structure $(\Gamma_{v'}^v, \{\Omega_1, \dots, \Omega_m\})$.

Lemma 3.5. Given v, $v' \in V$ and $d \in \mathbf{N}$ such that

$$d \geq \max_{\Gamma_{v'}^v \in Simple} \{\Upsilon \mid (\Upsilon, \Psi) \text{ is an offset-period pair of } \Gamma_{v'}^v\}.$$

Proof follows from the Definition of cactus structure and Lemma 3 [11]. □

There exists a path $\Gamma_{v'}^v$ of time d iff there is $i \geq 0$ and offset-period pair (Υ, Ψ) of $\Gamma_{v'}^v$ such that $d = \Upsilon + i \cdot \Psi$.

Given a cactus structure $(\Gamma_v^{v'}, H)$ with the offset-period pair (Υ, Ψ). A short path $\Gamma_{v'}^v$ *corresponds to the constraint* '$\sim c$' if either there exists a slim path $\bar{\Gamma}^v \in Path(\Gamma_{v'}^v, H)$ such that $time(\bar{\Gamma}^v) < \Upsilon$ and $time(\bar{\Gamma}^v) \sim c$, or there exists $i > 0$ such that $\Upsilon + i \cdot \Psi \sim c$.

4. Labelling algorithm

To decide a time behaviour analysis problem, we need to label a pair consisted of a vertex of $G(\mathcal{N}, \phi)$ and a TCTL-formula by a first-order-logic formula (called *a condition*) with parameter variables as free variables.

Informally, a condition is a constraint on parameter variables on which the given TCTL-formula is true on the states corresponding to the given vertex.

Suppose, we want to analyze $\mathcal{N}$ with respect to ϕ, i.e. to label the initial vertex v_0 with a condition $\mathrm{L}_{v_0}^{\phi}$. We label the vertices of $G(\mathcal{N}, \phi)$ with subformulas of ϕ or its negation starting from the subformulas of length 1, then of length 2, and so on; then we construct conditions L_v^{ψ} for all subformulas ψ of ϕ and for all vertices of $G(\mathcal{N}, \phi)$.

Given a vertex $v = \langle m, [\nu]_\phi \rangle$ of $G(\mathcal{N}, \phi)$ and a time predicate $x \sim \theta$, the notation $[\nu]_\phi(x) \sim \theta$ means $\tilde{\nu}(x) \sim \theta$ for all $\tilde{\nu} \in [\nu]_\phi$. Then $\tau^v = [\tau]^{t_1,\ldots,t_n}_{[\nu]_\phi(t_1),\ldots,[\nu]_\phi(t_n)}$.

A function $before : B^T_{Var} \to B^T_{Var}$ is inductively defined as follows:

- Case $\eta = x \sim \theta$. Then $before(\eta) = (x \leq \theta \vee x \sim \theta)$;
- Case $\eta = \eta_1 \wedge \eta_2$. Then $before(\eta) = before(\eta_1) \wedge before(\eta_2)$.

Given a TCTL-formula ψ, we define a proposition a^ψ which is true on the given vertex v iff v is labelled with ψ.

Let ψ be a subformula of ϕ. Assume that the vertices are already labelled with each subformula ψ' of ψ and the condition $\mathrm{Ł}^{\psi'}_{v'}$ is already constructed for each vertex $v' \in V$. We shall label a vertex v with a formula ψ or its negation and construct a condition $\mathrm{Ł}^{\psi}_{v}$. Let us consider the structure of ψ.

Case $\psi \in P$. If $\psi \in m$, then label v with ψ, else with $\neg\psi$. $\mathrm{Ł}^{\psi}_{v} = true$.

Case $\psi = \neg\psi_1$. If v is labeled with ψ_1, then label it with $\neg\psi$, else with ψ. If ψ is a U-formula, then $\mathrm{Ł}^{\psi}_{v} = \neg \mathrm{Ł}^{\phi}_{v}$, else $\mathrm{Ł}^{\psi}_{v} = true$.

Case $\psi = \psi_1 \wedge \psi_2$. If v is labelled with $\neg\psi_1$ and with ψ_2, then label it with ψ, else with $\neg\psi$.
If ψ_1 or ψ_2 is a U-formula, then $\mathrm{Ł}^{\psi}_{v} = \mathrm{Ł}^{\psi_1}_{v} \wedge \mathrm{Ł}^{\psi_2}_{v}$, else $\mathrm{Ł}^{\psi}_{v} = true$.

Case $\psi = Q\psi_1 \mathcal{U}_{\sim c}\psi_2$, where Q is either an existential or universal quantifer.

A *ψ-path* is a short path Γ^v for which there exists $i \geq 1$ such that for all $j < i$, v_j is labelled with ψ_1, v_i is labelled with ψ_2 and $\Gamma^v_{v_i}$ corresponds to the constraint '$\sim c$'. The set of such v_i for the given Γ^v is called $R(\Gamma^v)$. The set of short paths of $G(\mathcal{N}, \phi)$ starting from v is denoted by $\mathcal{F}^v(\psi)$. A vertex v should be labelled with ψ if there exists a ψ-path Γ^v of $G(\mathcal{N}, \phi)$ (all Γ^v are ψ-paths of $G(\mathcal{N}, \phi)$ depending upon Q).

Given ψ-path Γ^v and a vertex $\bar{v}$ from Γ^v,

$$\mathrm{Ł}^{\psi}_{v}(\bar{v}, \Gamma^v) = \mathrm{Ł}^{\psi_2}_{\bar{v}} \wedge \bigwedge_{v' \in \Gamma^v_{v'}} \mathrm{Ł}^{\psi_1}_{v'} \wedge \bigwedge_{t \in enable(m(\bar{v}))} \tau^{\bar{v}}(t) \wedge \bigwedge_{t \in tr(\Gamma^v_{v'})} (\tau^{in(t,\Gamma^v_{v'})}(t) \wedge$$

$$\wedge \bigwedge_{t' \in enable(m(\in(t,\Gamma^v_{v'})))} before(\tau^{in(t,\Gamma^v_{v'})}(t')));$$

$$\mathrm{Ł}^{\psi}_{v} = \bigvee_{\Gamma^v \in \mathcal{F}^v(\psi)} (\bigvee_{\bar{v} \in R(\Gamma^v)} \mathrm{Ł}^{\psi}_{v}(\bar{v}, \Gamma^v) \to a^\psi) \ \text{ if } Q = \exists;$$

$$\mathrm{Ł}^{\psi}_{v} = \bigwedge_{\Gamma^v \in \mathcal{F}^v(\psi)} (\bigvee_{\bar{v} \in R(\Gamma^v)} \mathrm{Ł}^{\psi}_{v}(\bar{v}, \Gamma^v) \to a^\psi) \ \text{ if } Q = \forall.$$

As an example, we consider an application of the above algorithm to the parametric time Petri net $\mathcal{N}_1$ (see Figure 1) and a TCTL-formula $\phi = \exists true \mathcal{U}_{\geq 2}(p_4 \wedge p_5)$, which unformally means that the transition t_5 fires not earlier than in time 2. Using the algorithm, we obtain the condition on the parameter variables $(\theta_1 = \theta_2^1 = \theta_2^2 = 1) \vee (0 \leq \theta_2^1 \leq \theta_2^2 \leq 2 \wedge \theta_1 = 2)$.

Theorem 4.1. Given a TCTL-formula ψ with $K_{\mathcal{N}:\phi} \geq K_{\mathcal{N}:\psi}$, a vertex $v = \langle m, [\nu]_\phi \rangle$ of $G(\mathcal{N}, \phi)$ and a parameter valuation χ such that $\langle m, \nu \rangle \in RS(\mathcal{N}^\chi)$. χ satisfies the condition L_v^ψ constructed by the above labelling algorithm iff $\langle m, \nu \rangle \models \psi$.

Proof. The proof is conducted by induction on the structure of ψ. The base case and the cases corresponding to the logical connectives follow immediately. We prove that χ satisfies to L_v^ψ iff $\langle m, \nu \rangle \models \psi$, where $\psi = \exists \psi_1 \mathcal{U}_{\sim c} \psi_2$. The other case $\psi = \forall \psi_1 \mathcal{U}_{\sim c} \psi_2$ is similar.

($\Leftarrow$) Suppose, $\langle m, \nu \rangle \models \psi$. This means that there is an $\langle m, \nu \rangle$-path r in $\mathcal{N}^\chi$: $\langle m, \nu \rangle = \langle m_1, \nu_1 \rangle \stackrel{\delta_1}{\Rightarrow} \langle m_2, \nu_2 \rangle \stackrel{\delta_2}{\Rightarrow} \ldots$ for which $r \models \phi_1 \mathcal{U}_{\sim c} \phi_2$. This implies that there exist k and $\delta \leq \delta_k$ such that:

1. $(\delta + time(r, k)) \sim c$;
2. $\langle m_k, \nu_k + \delta \rangle \models \phi_2$;
3. $\forall 1 \leq i < k \ \forall \delta'.(\delta' = 0 \vee 0 < \delta' < \delta_i) \ \langle m_i, \nu_i + \delta' \rangle \models \phi_1$;
4. $\forall \ 0 \leq \delta' < \delta \, . \ \langle m_k, \nu_k + \delta' \rangle \models \phi_1$.

For a given r in $\mathcal{N}^\chi$ we shall construct the corresponding path in $G(\mathcal{N}, \phi)$. For each $i \geq 1$ we can find a finite path $\Gamma^{\langle m_i, [\nu_i]_\phi \rangle}_{\langle m_{i+1}, [\nu_{i+1}]_\phi \rangle}$ in $G(\mathcal{N}, \phi)$ in the following way. If $\langle m_{i+1}, \nu_{i+1} \rangle$ is obtained from $\langle m_i, \nu_i \rangle$ by passing δ_i, then $\Gamma^{\langle m_i, [\nu_i]_\phi \rangle}_{\langle m_{i+1}, [\nu_{i+1}]_\phi \rangle} = \{\langle m_i, [\nu_i]_\phi \rangle = \langle m_i, [\nu_i^0]_\phi \rangle, \langle m_i, [\nu_i^1]_\phi \rangle, \ldots, \langle m_i, [\nu_i^{j_i}]_\phi \rangle = \langle m_{i+1}, [\nu_{i+1}]_\phi \rangle\}$ for some j_i (where $\langle m_i, [\nu_i^{l+1}]_\phi \rangle = succ_\delta(\langle m_i, [\nu_i^l]_\phi \rangle)$ for each $0 \leq l < j_i$). Otherwise $\Gamma^{\langle m_i, [\nu_i]_\phi \rangle}_{\langle m_{i+1}, [\nu_{i+1}]_\phi \rangle} = \{\langle m_i, [\nu_i]_\phi \rangle, \langle m_{i+1}, [\nu_{i+1}]_\phi \rangle\}$ (where $\langle m_{i+1}, [\nu_{i+1}]_\phi \rangle = succ_t(\langle m_i, [\nu_i]_\phi \rangle)$ for some $t \in fireable(\langle m_i, [\nu_i]_\phi \rangle)$). Note that for all $0 \leq \delta' \leq \delta_i$ there exists $0 \leq l \leq j_i$ such that $\nu_i + \delta' \simeq_\phi \nu_i^l$. Let $\Gamma^{\langle m, [\nu]_\phi \rangle}$ be a path in $G(\mathcal{N}, \phi)$ obtained as a concatenation of paths $\Gamma^{\langle m_i, [\nu_i]_\phi \rangle}_{\langle m_{i+1}, [\nu_{i+1}]_\phi \rangle}$.

Now we show that $\Gamma^{\langle m, [\nu]_\phi \rangle}$ satisfies the formula $\phi_1 \mathcal{U}(\phi_2 \wedge p_{\sim c})$, i.e. there are k and $l < j_k$ such that: a) $\Gamma^{\langle m, [\nu]_\phi \rangle}_{\langle m_k, [\nu_k^l]_\phi \rangle}$ corresponds to the constraint '$\sim c$', b) $\langle m_k, [\nu_k^l]_\phi \rangle$ is labelled with ϕ_2 and c) $\langle m_{k'}, [\nu_{k'}^{l'}]_\phi \rangle$ appearing in $\Gamma^{\langle m, [\nu]_\phi \rangle}$ before $\langle m_k, [\nu_k^l]_\phi \rangle$ is labelled with ϕ_1. Using the construction of $\Gamma^{\langle m_k, [\nu_k]_\phi \rangle}_{\langle m_{k+1}, [\nu_{k+1}]_\phi \rangle}$, we obtain that there is $l \leq j_k$ such that $\nu_k + \delta \simeq_\phi \nu_k^l$.

a) Since $\lfloor time(r,k)\rfloor = time(\Gamma^{\langle m,[\nu]_\phi\rangle}_{\langle m_k,[\nu_k]_\phi\rangle})$, we have $\delta + (\Gamma^{\langle m,[\nu]_\phi\rangle}_{\langle m_k,[\nu_k]_\phi\rangle}) \sim c$. Hence $time(\Gamma^{\langle m,[\nu]_\phi\rangle}_{\langle m_k,[\nu^l_k]_\phi\rangle}) \sim c$. For a cactus structure $(\Gamma^{\langle m,[\nu]_\phi\rangle}_{\langle m_k,[\nu^l_k]_\phi\rangle}, H)$, either $time(\Gamma^{\langle m,[\nu]_\phi\rangle}_{\langle m_k,[\nu^l_k]_\phi\rangle}) < \Upsilon$, or by Lemma 3.5 there exists $i > 0$ such that $\Upsilon + i\Psi \sim c$. Thus $\Gamma^{\langle m,[\nu]_\phi\rangle}_{\langle m_k,[\nu^l_k]_\phi\rangle}$ corresponds to the constraint '$\sim c$'.

b) Using the fact that $\langle m_k, \nu_k + \delta\rangle \models \phi_2$, we have $\langle m_k, \nu^l_k\rangle \models \phi_2$. Then by induction hypothesis $\langle m_k, [\nu^l_k]_\phi\rangle$ is labelled with ϕ_2.

c) Let us consider $\langle m_{k'}, [\nu^{l'}_{k'}]_\phi\rangle$ appearing in S before $\langle m_k, [\nu^l_k]_\phi\rangle$ (i.e. either $k' < k$, or ($k' = k$ and $l' < l$)). By the construction of $\Gamma^{\langle m_{k'},[\nu_{k'}]_\phi\rangle}_{\langle m_{k'+1},[\nu_{k'+1}]_\phi\rangle}$, there is $\delta' \leq \delta_{k'}$ such that $\nu_{k'} + \delta' \simeq_\phi \nu^{l'}_{k'}$. Then $\langle m_{k'}, \nu_{k'} + \delta'\rangle \models \phi_1$ implies $\langle m_{k'}, \nu^{l'}_{k'}\rangle \models \phi_1$ by Lemma 3.1. Hence, by induction hypothesis $\langle m_{k'}, [\nu^{l'}_{k'}]_\phi\rangle$ is labelled with ϕ_1.

Note that $\Gamma^{\langle m,[\nu]_\phi\rangle}$ is a slim path down to the progress condition of r. Hence, $\Gamma^{\langle m,[\nu]_\phi\rangle}$ is a ψ-path. We show that χ satisfies the condition $\mathrm{L}^\psi_{\langle m,[\nu]_\phi\rangle}(\langle m_k,[\nu^l_k]_\phi\rangle, \Gamma^{\langle m,[\nu]_\phi\rangle})$. Really, by induction hypothesis, χ satisfies the conditions $\mathrm{L}^{\psi_2}_{\langle m_k,[\nu^l_k]_\phi\rangle}$ and $\mathrm{L}^{\psi_1}_{\langle\tilde m,[\tilde\nu]_\phi\rangle}$ for all $\langle\tilde m,[\tilde\nu]_\phi\rangle$ from $\Gamma^{\langle m,[\nu]_\phi\rangle}$. And using the firing rules in $\mathcal{N}^\chi$, we obtain that χ satisfies the condition

$$\bigwedge_{t\in enable(m(\bar v))} \tau^{\bar v}(t) \wedge \bigwedge_{t\in tr(\Gamma^v_{v'})} (\tau^{in(t,\Gamma^v_{v'})}(t) \wedge \bigwedge_{t'\in enable(m(\in(t,\Gamma^v_{v'})))} before(\tau^{in(t,\Gamma^v_{v'})}(t'))).$$

Thus χ satisfies $\mathrm{L}^{\exists\psi_1\mathcal{U}_{\sim c}\psi_2}_v$.

($\Rightarrow$) Suppose that χ satisfies $\mathrm{L}^{\exists\phi_1\mathcal{U}_{\sim c}\phi_2}_v$. Then, according to the above algorithm, there is a ψ-path $\Gamma^{\langle m,[\nu]_\phi\rangle}$: $\{\langle m,[\nu]_\phi\rangle = \langle m_1,[\nu_1]_\phi\rangle, \langle m_2,[\nu_2]_\phi\rangle \ldots\}$ in $G(\mathcal{N},\phi)$, i.e. there is $n \in \mathbf{N}$ such that $\langle m_i,[\nu_i]_\phi\rangle$ is labelled with ϕ_1 for each $1 \leq i < n$, $\langle m_n,[\nu_n]_\phi\rangle$ is labelled with ϕ_2 and $\Gamma^{\langle m,[\nu]_\phi\rangle}_{\langle m_k,[\nu_k]_\phi\rangle}$ corresponds to the constraint '$\sim c$'. We shall construct the corresponding $\langle m,\nu\rangle$-path r in $\mathcal{N}$: $\langle m,\nu\rangle = \langle m_1,\nu_1\rangle \overset{\delta_1}{\Rightarrow} \langle m_2,\nu_2\rangle \overset{\delta_2}{\Rightarrow} \ldots$ such that for each $i \geq 1$ there is $\delta_i \in \mathbf{R}^+$ for which $\langle m_i,\nu_i\rangle \overset{\delta_i}{\Rightarrow} \langle m_{i+1},\nu_i+\delta_i\rangle$ and $\nu_i + \delta_i \in [\nu_{i+1}]_\phi$. Note that, by the construction of $\mathrm{L}^{\exists\psi_1\mathcal{U}_{\sim c}\psi_2}_v$, r is a path in $\mathcal{N}^\chi$. Then, similar to the first part of the proof, we can show that $\langle m,\nu\rangle \models \psi$. □

Theorem 4.2. Given a TCTL-formula ϕ, there is a procedure of analysis of $\mathcal{N}$ with respect to ϕ bounded by:

$$O[|\phi| \cdot 2^{|T|!\cdot 2^{|P|+2|T|}\cdot(K_{\mathcal{N}:\phi}+1)^{|T|}}].$$

Proof. From the definition of the region graph and from Lemma 3.3, it follows that $|V| = O[|T|! \cdot 2^{|P|+2|T|} \cdot (K_{\mathcal{N}:\phi}+1)^{|T|}]$. For a vertex

v of $G(\mathcal{N}, \phi)$ there are at most $\mid T \mid$ output edges representing the firing of transitions and an edge representing time passage. Hence $\mid E \mid = O[\mid T \mid! \cdot 2^{|P|+2|T|} \cdot (K_{\mathcal{N}:\phi} + 1)^{|T|}]$. The region graph $G(\mathcal{N}, \phi)$ can be constructed in time $O[\mid V \mid + \mid E \mid]$ (written $\mid G \mid$). When we construct a condition for a given formula ϕ and a vertex v of $G(\mathcal{N}, \phi)$, we need to consider all short paths in $G(\mathcal{N}, \phi)$ begining from this vertex. The number of simple cycles is $\mid G \mid !$. So, the number of short paths is $(\mid G \mid !)^2$. The time of considering a short path is $\mid G \mid + (\mid G \mid + 1)!$. Then, to construct a condition for $\mathbf{L}^{\phi}_{v}$, we need the time $O[\mid \phi \mid \cdot (\mid G \mid !)^3]$. The complexity follows from the bounds on the size of V and E. □

Conclusion

In this paper we introduce a notion of a parametric time Petri net and propose a TCTL-based algorithm for behaviour analysis of it, which provides a technique for adjustment of timing limitations with respect to the system properties.

In future we suppose to optimize the proposed algorithm and to construct an algorithm for behaviour analysis of parametric time Petri nets in terms of parametric TCTL [11], an extension of TCTL obtained by using parameter variables in $\mathcal{U}$-operators.

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