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Dynamic-stochastic modeling and four-dimensional analysis of the meteorological fields*

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Abstract. On the basis of the variational principle, the method of dynamic-probabilistic numerical modeling of ensembles of independent realizations of a complex of space-time stochastic fields is proposed. The ensemble of realizations satisfies the statistical structure of real fields, and the each realization of this ensemble satisfies a numerical dynamic model. For the reconstruction of the meteorological fields, on a spatial-temporal regular grid using observation data at stations, is considered. This method is based on expansion in series of required fields in finite number of natural orthogonal basis. The basis is determined on the ensemble of the spatial-temporal realizations obtained with the help of the dynamic–stochastic method with a given real statistical structure of meteorological fields. One of the methods of fast assimilation of the given observational data is proposed. The results of numerical experiments are presented.

1. Introduction

On the basis of variational methods of optimization, one of general procedures for the dynamic-probabilistic modeling of complexes of multi-dimensional fields is proposed [1, 2]. The realizations of fields satisfy the properties given in the form of autocovariance and mutual covariance matrices, probability distributions, etc., as well as physical connections in the form of appropriate systems of differential equations. In essence, the method of stochastic modeling of multi-dimensional fields, generally non-gaussian, with an optimum adjustment of stochastic and physical properties, is proposed.

In addition, a dynamic model, given by a system of differential equations, serves as interpolator at spatial-time points of considered fields, and for their mutual adjustment and filtration of non-physical components, and the statistical model provides a given probability structure of the considered process and specifies an appropriate ensemble of independent realizations of these fields. The use of a variational method of information assimilation allows us to optimize the process of corporation of dynamic and statistical methods of numerical modeling.

As one of the possible applications of the proposed method, a method of numerical modeling of climate of the atmosphere as ensemble of possible

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realizations of spatial-time hydrometeorological fields is considered [1, 2]. A necessary analysis of the statistical structure of the climatic ensemble obtained is made, and it is shown that the model with a sufficiently high accuracy reproduces the structure of real climatic fields.

As the proposed method allows us to construct a complete set of considered hydrometeorological fields at regular spatial-time points satisfying climatic properties, i.e., to fill up numerically a lack of measurements of these characteristics, it makes it possible with sufficient reliability to carry out analysis of direct relationship and feedback coupling, the mutual influence of various processes in the atmosphere, climatic behavior of an impurity in the atmosphere, etc. Concrete applications are also discussed.

The basic essence of this method is the following. We assume that the statistical structure of the fields in question is known approximately. According to this structure, the ensemble of realizations of these fields with the use of known approximate methods of statistical modeling is constructed. The realizations of fields from this ensemble are used as input data for the problem of variational assimilation of the information with the help of a dynamic model. As a result, we obtain new ensemble, in which each realization satisfies the dynamic model, and the statistical structure of the new ensemble is close to the initial structure within the accuracy of the problem of variational assimilation.

The given approach allows us to specify the statistical structure of the fields under study from the standpoint of a dynamic model. Therefore, it is rather logical to use dynamic models of physical processes for the coordination and filtering of appropriate random fields obtained using the statistical modeling and, hence, for specification of the probability structure of real fields.

In the given paper, one of methods of the statistical modeling is considered, which consists in the following. Let R be a multi-dimensional correlation matrix. Let us present its spectral decomposition as

$$R = W \Lambda W^T, \tag{1}$$

where W is a matrix of eigenvectors of the correlation matrix R, and Λ is a diagonal matrix of appropriate eigenvalues. Let us note that representation (1) is the decomposition in the so-called primary factors, and problem (1) is the corresponding problem of definition of such primary factors. The next step consists, in our case, in definition of a square root of a matrix R as $R^{1/2} = W \Lambda^{1/2} W^T$, where $\Lambda^{1/2}$ is a diagonal matrix, on whose diagonal are the square roots of corresponding eigenvalues in the matrix Λ, and the index T defines the transposition operation.

Then one of methods of statistical modeling can be defined as

$$\vec{\xi}^{(i)}_{(n)} = D_\xi R^{1/2} \vec{\psi}^{(i)}(x_j, y_j, p_j, t_j)^T + \bar{\vec{\xi}}(x_j, y_j, p_j, t_j), \quad i = 1, 2, \dots, \tag{2}$$

where $\vec{\psi}^{(i)}(x_j, y_j, p_j, t_j)^T$, $i = 1, 2, \ldots$, is a Gaussian stochastic vector with individual variance and zero average, D_ξ is a diagonal variance matrix, $\bar{\vec{\xi}}(x_j, y_j, p_j, t_j)$ is a sampling mean of the vector simulated stochastic value $\vec{\xi}^{(i)}_{(n)}$. It is not difficult to see that the correlation matrix of the stochastic vector $\vec{\xi}^{(i)}_{(n)}$ coincides in accuracy with the matrix R.

The method of variational assimilation is based on solving the problem of minimization of the quality functional [1] with the use of the method of gradient descent. The value of the quality functional characterizes a measure of distinction between realization of a stochastic field and an appropriate field after solving the system of the dynamic equations. For definition of a gradient of the quality functional, the solutions to the basic and adjoint problems, appropriate to the considered dynamic model, are used. In fact, the average value of the quality functional, obviously, determines a measure of difference in the initial statistical structure and the statistical structure obtained by the combined modeling.

The considered method of variational assimilation of data with the help of a mathematical model is based on the Lagrangian method [3] for the search for extreme points of functionals with restrictions given as equalities, the definition of the Gato differential [4] for the construction of an appropriate gradient. In this case, the fundamental concept by G.I. Marchuk [5], of using the adjoint problems, as applied to problems of hydrothermodynamics of the atmosphere and ocean is employed.

Let us notice that for an effective use of variational assimilation, the theoretical or numerical research into the convergence of an appropriate Gato functional is necessary.

For the purposes of construction of a climatic ensemble of realizations it is necessary to consider a problem of variational assimilation on the whole time interval within the limits of predictability of an appropriate numerical model, as using the so-called consecutive step-by-step assimilation does not provide necessary smoothness of the solution and an appropriate trend for further use of the field obtained in the forecast mode.

The essence of the method of variational assimilation consists in the following. Let us consider the numerical model which is written down in the operator form

$$\frac{\partial \vec{\Phi}}{\partial t} + A(\vec{Y}, \vec{\Phi})\vec{\Phi} = 0, \tag{3}$$

where $\vec{\Phi}$ is the state vector; $\vec{Y} = \vec{\Phi}|_{t=0}$ is the vector of the parameters; $A(\vec{Y}, \vec{\Phi})$ [1] is the finite difference operator determined by a system of equations of the process under study and the appropriate boundary conditions in the domain $G_t = G \times [0, \hat{t}]$.

System of equations (3) generates a certain set of the solutions dependent on a vector of parameters $\vec{Y}$. The problem is to find the closest among this set of solutions to a concrete realization from (2) in the sense of some quality functional J_0, which we write down as

$$J_0 = \frac{1}{2}\sum_k (L\vec{\Phi}^j - \vec{\Phi}_S^k, L\vec{\Phi}^j - \vec{\Phi}_S^k)_{D_S},$$

where $(\cdot,\cdot)_{D_s}$ is the scalar product in the space of the measured data $\vec{\Phi}_S^k$, as which appropriate values of the fields $\vec{\xi}^i_{(n)}$ of dimension N_S at the points at the time t_k from ensemble of realizations (2) are used; L is an appropriate operator of interpolation, and $\vec{\Phi}^j$ is the solution of problem (3) at the moment of time t_j. Thus, it is necessary to find a minimum of the functional J_0 relative to the vector of parameters $\vec{Y}$ under restrictions (3).

For solving this problem the iterative method of gradient descent based on the Lagrangian method and the solutions of the direct and adjoint problems are used. Let us notice that, generally, a minimum of the functional J_0 is not unique, that is, determined by nonlinearity of system (3) and the degree of completeness of the assimilated data. In this case, a minimum is determined by some initial value of a vector of appropriate iterative process.

The detailed description of the combined dynamics-stochastic model and its characteristics are given in [1].

2. Modeling of climatic ensemble for a local area and research on its basis of climatic rejections in the atmosphere

Based on the above-said, the climatic ensemble of possible realizations of appropriate multi-dimensional hydrometeorological fields for a chosen interval of time and a given area [1, 2, 6–8] is considered as

$$\{\vec{\xi}^i_{(n)},\ i = 1, 2, \dots\}, \tag{4}$$

where $\vec{\xi}^i_{(n)} = (\vec{U}^i(\vec{X}_j, t_k), T^i(\vec{X}_j, t_k), H^i(\vec{X}_j, t_k), \dots)^T$ is a vector of realizations of the fields of wind speed, temperature, geopotential, etc. at the space-time points $(\vec{X}_j, t_k)$ of the considered area; n is dimension of this vector.

As compared to existing climatic numerical models based on the complete equations of hydrothermodynamics, in which the result of modeling of climate is obtained at the expense of increasing the physical complication of the model, application of the detailed space-time resolution, inclusion of various parameterizations (moisture, flows of heat, boundary layers, etc.) and integration for long time period up to some quasi-periodical conditions, in this paper we propose to directly simulate independent climatic realizations of the meteorological fields space-time with some statistical characteristics which are optimally close to simulate statistical characteristics of real fields. This approach is closest to modeling of climate with the help of stochastic models [1], thus allowing to combine properties of the deterministic numerical models of the atmosphere dynamics and stochastic models. As real data, the reanalysis data of the temperature field for 1948–2005 at ten standard levels for a winter season with time of discreteness 6 hours and 2.5×2.5 degrees in longitude and latitude, respectively, (NCEP/NCAR) were used. The sample was carried out for the given local $10 \times 10^\circ$ area of Northern hemisphere with the center at the point with coordinates 60.56° of Northern latitude and 77.7° East longitude. The problem is considered in x, y, p system of coordinates in the area, whose bottom basis is a rectangle on the tangential plane at this central point. For the grid construction 24×20 resolution with respect to x and y with steps $\Delta x = 23.85$ km and $\Delta y = 58.74$ km, respectively, was chosen.

Thus, with allowance for the external points reanalysis and two moments of time on ten isobaric surfaces, the correlation matrix of the temperature field was designed. The general qualitative character of this correlation matrix corresponds to the correlation matrix, used in experiments taken from [1, 2, 6–9].

Using formulas (1), (2), ensemble of realizations (4) is under construction.

The next step for construction of a climatic-ensemble of realizations is application of variational assimilation. To this end, for each realization from this ensemble (4), the variational assimilation problem with the help of the mathematical model of hydro-thermodynamics of the atmosphere is solved, therefore the ensemble of new realizations turns out to be as follows:

$$\{\tilde{\vec{\xi}}^{i}_{(n)},\ i = 1, 2, \ldots\}, \tag{5}$$

being different from the initial one with accuracy of the solution of the problem of variational assimilation and satisfying the properties of the mathematical model.

The designed local climatic ensemble (5) includes 2000 realizations, which has appeared quite sufficient for the corresponding statistically-valued estimations.

For the numerical analysis of dynamic processes in realizations of climatic ensemble (5), we use a problem of numerical modeling of the distribution of a passive pollution in the atmosphere, as this process characterizes, first of all properties of atmospheric movements, and by virtue of linearity of the used model of pollution distribution, the character of its space-time behavior is completely defined by appropriate fields of the wind speed from the climatic local ensemble. Therefore the properties of the obtained distributions also characterize appropriate properties of the climatic ensemble of meteorological fields as it is.

As compared to existing now numerical models of distribution of pollutions, in which as background fields of the wind speed or average climatic fields, or the fields chosen according to some characteristic scenario (dependent, as a rule, on subjective criteria) are used in the present work, the whole ensemble of realizations of space-time fields of pollutions according to the chosen ensemble of climatic realizations of the wind field are proposed. This ensures, in a certain sense, the statistical completeness of statistical estimations.

As climatic distribution of pollutions in a local area, we consider an appropriate average of the obtained ensemble of realizations of fields of pollutions.

For definition of the climatic trajectory concept, let us use a known method of indicator functions and the result of averaging the ensemble of realizations of fields of pollutions. With this purpose, consider

$$\chi(f,l) = \begin{cases} 1, & f \geq l, \\ 0, & f < l, \end{cases} \tag{6}$$

where f is a value of the tested function, and l is some given number. The values of this function, equal to unit in the considered area, from an averaged sequence of realizations from the ensemble of pollutions define the respective climatic trajectory of distribution of pollutions at a given threshold value l.

In the numerical experiments, as an initial field of a pollution determining an instant source of pollution, the simulated field of a pollution located in some grid domain of 5×5 points at a level $p = 850$ mb with a maximum equal to 1 value at the center of this domain was chosen. The value of the size l was chosen equal to 0.05.

Let us notice that the estimation of the degree of belonging of a concrete trajectory of distribution of a pollution to the climatic trajectory can be determined by various ways depending on the purpose of research. For example, the estimation of belonging as the relation of the number of points, crossing with the climatic trajectory, to the common number of points of this climatic trajectory. From the point of view of the research of emissions of a pollution it is quite justified, as in our case, with a small value of the estimation introduced to expect the increased contents of a pollution in this vicinity. Thus, if we want to have an appropriate estimation of the degree of a geometrical belonging of a concrete trajectory of distribution of a pollution to the climatic trajectory, it is necessary to consider another estimation, for example, as relation of the number of points crossing with the climatic trajectory to the common number of points of a concrete trajectory of distribution of a pollution. Further we will consider this estimation.

Let us designate this size as α. As the realizations of the obtained ensemble are a stochastic vector functions, the value of a degree of belonging to the climatic trajectory, averaged on the time interval, is also a stochastic function. This function is given in Figure 1a, where the appropriate number of realizations is plotted on the abscises axis. The average value of this function equals 0.9154, and the variance estimation is 0.123. These characteristics show that in the considered problem the processes of distribution of a pollution have some prevailing direction that allows localization of the most probable area of the given level of pollution. Figure 1b presents an appropriate bar chart of this distribution.

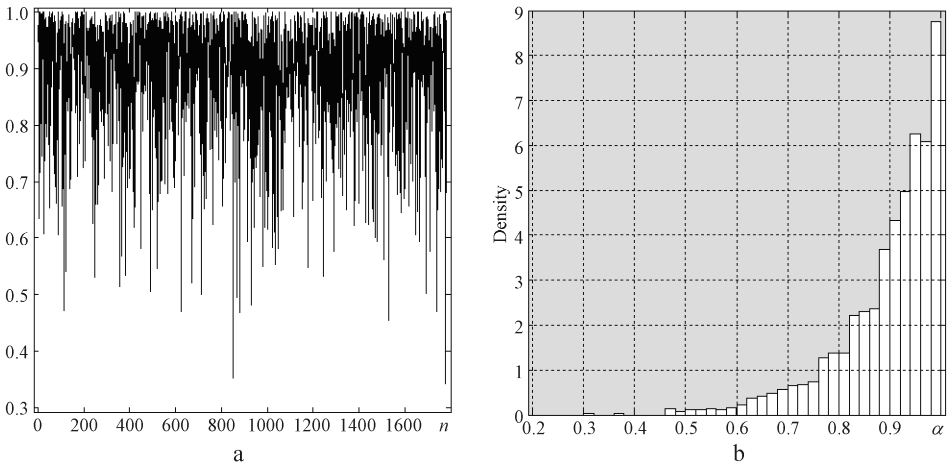

Figure 1. The diagram of the stochastic value α and the bar chart of its distribution

In Figure 2, the concrete trajectory of distribution of a pollution with a minimum degree of a belonging to the climatic trajectory is presented. The belonging of the climatic trajectory is 0.34, i.e., its larger part is outside of the climatic trajectory, and in our case, it can be considered as one of examples of ejection of a pollution. The instant source of a pollution was given in the neighborhood of a point designated in Figure 2 by letter A.

The given numerical results show that the technique proposed can be used for defining and analysis of direct and inverse climatic trajectories of dynamic processes in the atmosphere and the detailed research of rejection in the atmosphere.

In this paper, the semi-lagrangian numerical transport model was used. When realizing this numerical model the corresponding three-dimensional bicubic spline-interpolation was used. The corresponding graph of the function α calculated on the ensemble of fields of a pollution and the histogram of its distribution are presented in Figure 3. The average estimation of the function α is equal to 0.755 at a corresponding estimation of the variance

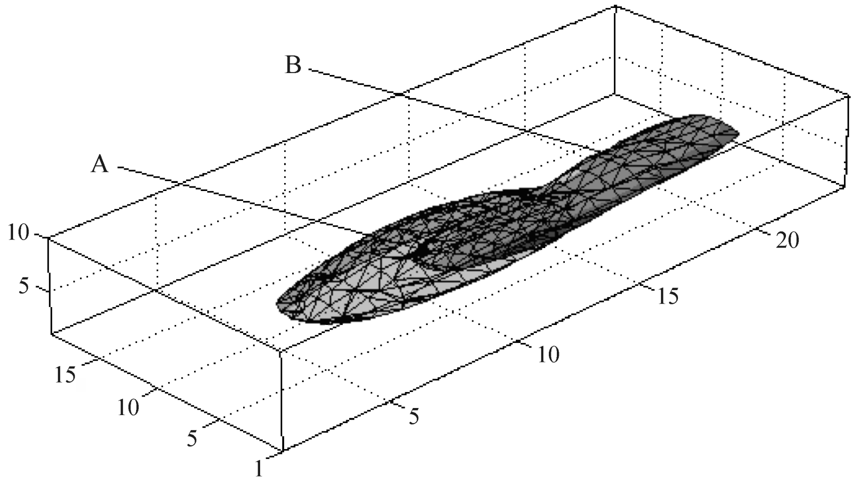

Figure 2. The combined spatial arrangement of the climatic trajectory of distribution of a pollution and the trajectory of a pollution of ejection (B). The point A corresponds to the center of the subarea of a source of pollution. Along the axes of coordinates, appropriate numbers of meshpoints are plotted

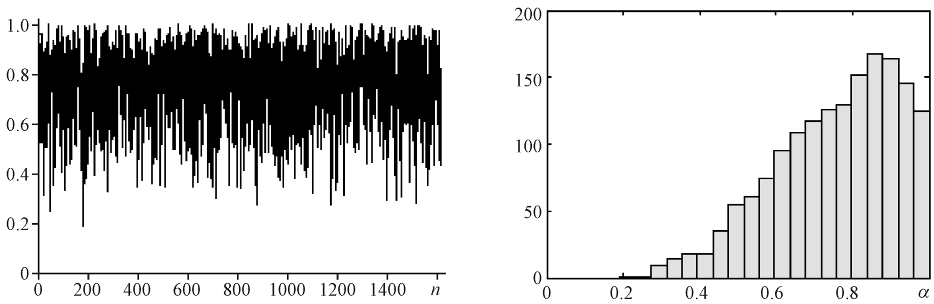

Figure 3. The graph of the stochastic function α and the histogram of its distribution (a variant of a semi-lagrangian model)

equal to 0.162. It specifies that in the considered area, the processes of pollution distribution have also some prevailing direction and allows the localization of the most probable area of the given level of pollution. The numerical probability estimation of distribution of α is presented in Figure 4. Figure 5 shows a concrete trajectory averaged in time of distribution of a pollution with minimum degree of belonging to the climatic trajectory equal to 0.193, that is, its larger part being outside of the climatic trajectory, and in our case can be considered as one of examples of ejection of a pollution. This trajectory is designated in the figure by the letter B, and the instant source of a pollution in this figure was set in the vicinity of a point, designated by the letter A. As a whole, the results, obtained in the given work, qualitatively correspond to the results from [6–8] although numerical

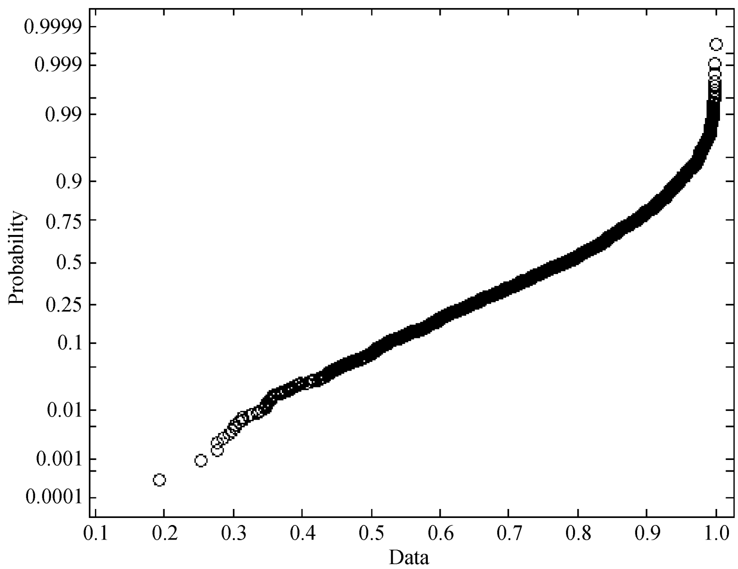

Figure 4. Estimation of probability of α-distribution (a variant of a semi-lagrangian model)

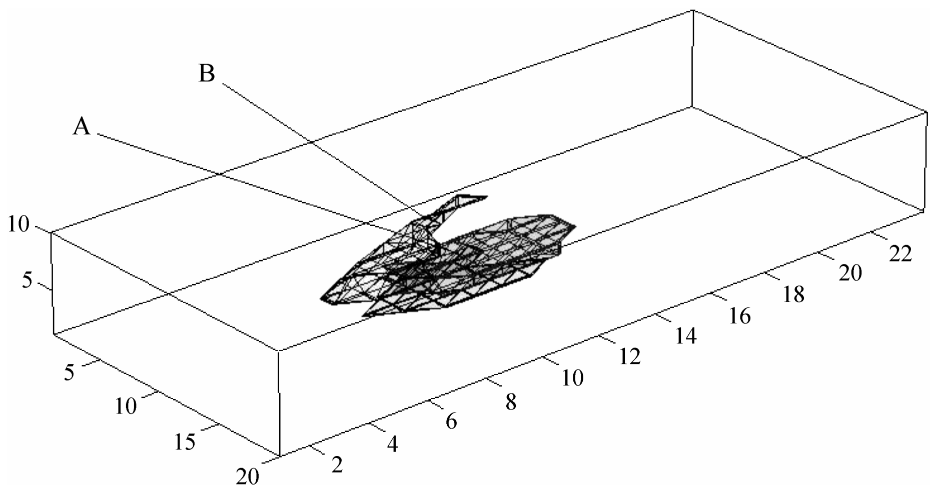

Figure 5. A combined spatial arrangement of the climatic trajectory of distribution of a pollution and the trajectory of a pollution of ejection (B). The point A corresponds to the center of the subarea of a source of pollution (a variant of a semi-lagrangian model). Along the axes of coordinates, appropriate numbers of meshpoints are plotted

estimations somewhat differ from the corresponding estimations presented there. For example, there are observed differences in the average values and in the right part of the resulted histogram, in which a strongly pronounced maximum is seen in the neighborhood of this average value.

Analysis of the numerical results shows that the offered technique can be used for definition and the analysis of direct and inverse climatic trajectories of dynamic processes in the atmosphere and researches of ejections in the atmosphere.

3. Four-dimensional analysis of the meteorological fields

One more application of the proposed method of the dynamic-stochastic modeling is the four-dimensional analysis of meteorological fields and the so-called “fast” assimilation of the hydrometeorological information. Analysis and interpretation of the real information concerns a number of major problems arising in the construction of mathematical models of physical processes and solving problems of the weather forecast, the general circulation of the atmosphere and ocean, the theory of climate, and, also, in studying and estimating the influence of human activity on the environment. One of aspects of this problem is in the development of methods of “compression” of information and its allocation in the most informative part as sum of the finite fourier series with a small number of terms.

In this paper, we propose methods of the four-dimensional analysis of the data on the basis of a climatic ensemble of possible realizations of the corresponding multivariate hydrometeorological fields for the chosen interval of time and for the region (3) for the considered grid domain of n dimensions. We use the obtained climatic ensemble (5) for solving the problem of the four-dimensional analysis of the hydrometeorological data in the atmosphere. One of algorithms of such a use offered in [10], is based on representation of the hydrometeorological field as the corresponding fourier series in the main factors of orthogonal functions designed on the real data only for a geopotential field for the winter period and on a sufficiently limited sample.

As ensemble (5) already contains statistically independent existential realizations, including a full set mutually coordinated hydrometeorological components (temperature, geopotential, wind speed), with respect to the numerical dynamic model it seems quite natural to use this technique not only for the analysis of separate hydrometeorological components, but also for the four-dimensional analysis and assimilation of the corresponding real data as a whole.

It is natural that the statistical importance of the obtained results is completely defined by ensemble (5). This approach has a number of advantages. First, the basis of the main orthogonal functions constructed on a sufficiently representative sample, have necessary properties of statistical structures of meteorological fields, that is especially important at a sparse network of stations. Second, the number of basic functions is supposed to be rather small, that allows us to construct an efficient computing algorithm for its realization. In addition, from the method of constructing a natural orthogonal basis it follows that each of its functions has the statistically coordinated components, therefore the result of restoration with the help of the method presented has the same degree of the coordination, as basic functions.

For determination of basic functions of the natural orthogonal basis (the main factors)

$$\{\vec{\varphi}_i,\ i = 1, \ldots, m\} \tag{7}$$

in ensemble (5), one of modifications of the algorithm described in [11] for the generalized covariation matrices R_a of ensemble (5) was employed. There, the initial realizations of ensemble (5) are normalized and reduced to a dimensionless form on the basis of appropriate factors of the full energy integral of system (3). This provides a combined coordination of the relative numerical model of dynamics of the atmosphere of various components (wind speed, temperature, geopotential) when constructing of corresponding correlation matrices. Logarithms of eigenvalues describing the information density of the designed basic functions, are presented in Figure 6 which shows, that 2,000 realizations in an ensemble (50 basic functions) are quite sufficient for the representation of the considered meteorological fields with good accuracy ($m = 50$).

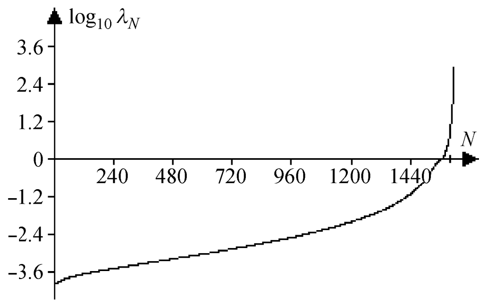

Figure 6. Logarithms of eigenvalues of a correlation matrix, designed on ensemble of realizations (1)

Thus, according to [3], we consider a vector subspace $\tilde{R}_m$ of the real vector space R_N, whose components are values of meteorological fields at points of the regular grid $\Omega^{ht} \subset \Omega$. Let vector functions (7) be the basis of the subspace $\tilde{R}_m$. Then any vector $\vec{\varphi} \in \tilde{R}_m$ can be presented as Fourier series

$$\vec{\varphi} = \Phi\vec{a}, \tag{8}$$

where Φ is a matrix of $N \times m$ dimension, made of the basic vectors $\{\varphi_i,\ i = 1, \ldots, m\}$, $\vec{a} = (a_1, \ldots, a_m)^T$ is the Fourier coefficient vector. Let in the considered area Ω, the irregular mesh grid Θ, in which the measurements data of the investigated meteorological fields are known. We consider a subspace of vectors G the Euclidean space, determined on the irregular grid Θ, and as components of the vectors we take values of the fields of one or several meteorological elements (the same, as in $\tilde{R}_m$). In this subspace, introduce the scalar product

$$(\vec{\varphi}, \vec{\psi})_M = (M\vec{\varphi}, \vec{\psi}),$$

where $\vec{\varphi}, \vec{\psi} \in G$, the symbol $(\cdot, \cdot)$ designates the scalar product in the Euclidean space, M is a positive definite symmetric matrix, whose choice is is made according to the research purposes, physical dimensions of components of vectors and a priori data about the structure of the considered fields. In this case, a scalar product is a grid analogue to the corresponding scalar product, determining the full energy integral for the hydrothermodynamics model used when solving the problem of variational assimilation.

Using the given approach, the problem of restoration of meteorological fields on a regular grid Ω^{ht} from their measured values on an irregular grid of stations is reduced to finding the vector from the factors of the formula (8) such that the interpolated values of the vector function $\vec{\varphi} \in \tilde{R}_m$ are least deviated from the corresponding measured values at points of the given irregular grid.

Let $\vec{\psi}_{\text{meas}}$ be the vector of measured values at points of the irregular grid $\Theta \subset \Omega$, and $\vec{\varphi}$ is a vector from subspace $\tilde{R}_m$ which is required to construct on the set vector $\vec{\psi}_{\text{meas}}$. Let us designate through $\vec{\psi} = L\vec{\varphi}$ an image of the vector $\vec{\varphi}$ in the subspace G, obtained with the help of the linear interpolation operator L from the regular grid to an irregular one. As a vector $\vec{\varphi} \in R_m$, as represented in (8), $\vec{\psi} = L\Phi\vec{a}$. We consider the functional, describing a measure of deviation of the vector of the measured values $\vec{\psi}_{\text{meas}}$ at points of an irregular net of stations from values of the vector functions $\vec{\varphi} \in R_m$, interpolated to the irregular grid with the help of the linear operator L:

$$J = (\vec{\psi}_{\text{meas}} - L\Phi\vec{a}, \vec{\psi}_{\text{meas}} - L\Phi\vec{a})_M. \tag{9}$$

From the condition of extremum of the functional J, for definition of factors a_i, $i = 1, \dots, m$, we obtain the linear inhomogeneous algebraic equation system

$$(L\Phi)^T M\, L\, \Phi\vec{a} = (L\Phi)^T M\vec{\psi}. \tag{10}$$

This system can be written down as

$$B\vec{a} = \vec{f}, \tag{11}$$

where $B = (L\Phi)^T M\, L\, \Phi$ is a symmetric, nonnegative definite matrix, $\vec{f} = (L\Phi)^T M\, \vec{\psi}_{\text{meas}}$ is the vector of the right-hand side of system (11).

Let us notice, that system (11) in some cases of a relative positioning of points of an irregular net of stations can appear to be ill-conditioned. Therefore, for its solution the following algorithm is used. The matrix B is presented as

$$B = W_B \Lambda W_B^T, \tag{12}$$

where Λ is a diagonal matrix of eigenvalues, W_B is the orthogonal matrix of transformation, whose matrix columns are made of eigenvectors of the matrix B.

Then, taking into account (12), the solution to system (11) is obtained from the formula: $\vec{a} = W_B \Lambda^+ W_B^T \vec{f}$, where $\Lambda^+ = diag\{\lambda_i^+\}$, $(i = \overline{1,m})$ is a diagonal matrix constructed by analogy with a pseudoinverse matrix, i.e.,

$$\lambda_i^+ = \begin{cases} 1/\lambda_i, & \lambda_i > \varepsilon, \\ 0, & \lambda_i \le \varepsilon, \end{cases}$$

ε is a sufficiently small number.

Finally, with the help of the obtained vector of the factors $\vec{a}$, it is possible to restore the vector $\vec{\varphi}$ on the regular grid Ω^{ht} from formula (8).

For an illustration of the efficiency of the above described technique the temperature fields were simulated using formulas (1), (2) for the time moments $t = 0$ and $t = 6$ hours at 10 standard levels. These data are used as input data for the problem of variational assimilation with the help of the numerical model and for the four-dimensional analysis by formulas (8)–(12). In Figure 7, the corresponding comparative results of calculations for the level of 500 mb and the moment of time $t = 0$ are presented, which show

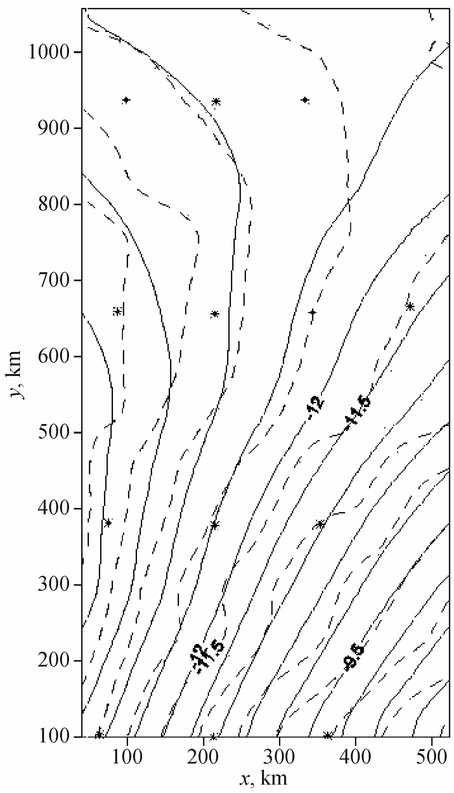

Figure 7. Isolines of temperature fields at the level of 500 mb at the moment of time $t = 0$, obtained from the variational assimilation (continuous lines) of the data set at points, designated by the symbol * and isolines of temperature fields obtained as a result of the four-dimensional analysis by the main factors (dashed lines)

a good enough qualitative agreement of corresponding isoline fields. The maximal difference between values of these fields the measurement data makes 0.93°.

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