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Variational methods of information assimilation in the problem of probabilistic modeling of hydrometeorological fields*

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In the paper, a new method of simulation of independent climatic realizations of space-time fields of hydrometeorological elements with some set of statistical characteristics of real fields based on the variational principle of information assimilation is proposed. This method is allowed to combine some peculiarities of determinate numerical models of the atmosphere dynamic and probabilistic models.

Introduction

A method of numerical modeling of ensemble of realizations of stochastic space-time climatic fields with the use of variational methods of assimilating the observational data is considered [1, 2]. This ensemble satisfies statistical climatic characteristics in the atmosphere, and each realization of this ensemble satisfies the hydrothermodynamic numerical model. In this case, the numerical model determines the joint statistical structure of components of complexes of hydrometeorological fields. If some components of these complexes are related by linear equations, an optimization method for the correction of their joint structure is proposed. The estimations of the statistical characteristics of the ensemble of realizations constructed and their agreement with real data are presented.

1. Stochastic simulation of hydrometeorological fields

We consider the probabilistic model of hydrometeorological fields [3–6] constructed as a complex of random space-time fields of hydrometeorological elements. They, in turn, are a set of related time series at N_S observation stations in a domain G. At the instant t_k, the set of the values of

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hydrometeorological elements at the weather stations can be represented as a $N \times N_S$-dimensional vector $\vec{\xi}(t_k) = (\vec{\xi}_1^T(t_k), \ldots, \vec{\xi}_N^T(t_k))^T$. Its components $\vec{\xi}_i(t_k)$ are the hydrometeorological elements (velocity, temperature, pressure, etc.) written as deviations from the corresponding climatic values at N_S weather stations. Here N is the number of the hydrometeorological elements. In our case, the structure of the vector $\vec{\xi}(t_k)$ coincides with that of the vector $\vec{\Phi} = (u, v, T, \tau, H)$, where $\vec{U} = (u, v, \tau)$ is a velocity vector in the system of coordinates (x, y, p), T is the deviation of temperature from its standard value $\bar{T}$; H is the deviation of the geopotential from its standard value $\bar{H}$. The vectors $\vec{\xi}(t_k)$ $(k = 1, \ldots, n)$ form the following random sequence:

$$\vec{\xi}_{(n)} = (\vec{\xi}^T(t_1), \ldots, \vec{\xi}^T(t_n))^T.$$

In this paper, we assume that this sequence is Gaussian and stationary with zero expectation (the sought-for value is obtained by adding the corresponding climatic value). Then the correlation matrix $M\vec{\xi}_{(n)}\vec{\xi}_{(n)}^T = R_{(n)}$ is a block Toeplitz one. It has the form

$$R_{(n)} = \begin{bmatrix} R_0 & R_1 & \ldots & R_{n-1} \\ R_1^T & R_0 & \ldots & R_{n-2} \\ \ldots & \ldots & \ldots & \ldots \\ R_{n-1}^T & R_{n-2}^T & \ldots & R_0 \end{bmatrix}.$$

Here R_i are blocks of the dimension $N \times N_S$, for $i \neq 0$, in the general case, they are non-symmetric. The matrices $R_0, R_1, \ldots, R_{n-1}$ are, in turn, also block matrices with blocks of dimensionality N_S. The sequences of blocks, which have the same locations in all the matrices R_i, form autocorrelation and mutual matrix correlation functions of various meteorological elements. The elements of these blocks are the corresponding space-time correlations.

In order to model the sequence of vectors $\vec{\xi}(t_1), \vec{\xi}(t_2), \ldots, \vec{\xi}(t_n)$ with correlation matrix $R_{(n)}$, we employ the method of conditional expectations. Each of the vectors $\vec{\xi}(t_k)$ $(k = 1, \ldots, n)$ is calculated in the form

$$\vec{\xi}(t_k) = B^T[k-1]J_{(k-1)}\vec{\xi}_{(k-1)} + C_{(k-1)}\vec{\eta}_k, \tag{1.1}$$

with the initial value $\vec{\xi}(t_1) = C_0\vec{\eta}_1$. Here $\eta_1, \ldots, \eta_n$ are independent Gaussian vectors of dimensionality $N \times N_S$ with independent components, $J_{(k)}$ is the block matrix of reverse permutation of the vector components $\vec{\xi}_{(k)}$, $B[k] = (B_1[k], \ldots, B_k[k])^T$, and $B_i[k]$ are the matrix coefficients of the regression C_k is a lower triangular matrix such that $C_kC_k^T = S_k$, where S_k is the corresponding residual matrix [4].

The algorithm for calculating $B[k]$, S_k and, respectively, C_k in procedure (1.1), which ensures that the condition $M\vec{\xi}_{(k)}\vec{\xi}_{(k)}^T = R_{(k)}$ is satisfied at each step k, is given in [4]. It is based on the well-known Durbin–Robinson algorithm. Thus, at the n-th step we obtain the Gaussian sequence $\xi_{(n)}$ with the correlation matrix $R_{(n)}$.

When constructing the vector autoregressive process $\vec{\xi}(t_1), \vec{\xi}(t_2), \ldots$ the vector $\vec{\xi}_{(n)}$ is taken as the initial vector.

The accuracy of statistical modeling essentially depends on a priori information concerning the statistical structure of the fields studied. Therefore, it is quite reasonable to use the hydrodynamic models of atmospheric processes for adjusting and filtering the corresponding random fields obtained by statistical modeling and, hence, for refining the probabilistic structure of the actual hydrometeorological fields.

In some simplest cases, when linear connections between some components of the vector $\vec{\Phi}$ are existed, it is possible some simplifications of the matrix $R_{(n)}$ and algorithm (1.1). And what is more it is possible to correct the structure of the matrix $R_{(n)}$ with the help of optimization procedures.

Let $\vec{X} = \vec{\xi}_i(t_k)$, $\vec{Y} = \vec{\xi}_j(t_k)$ and

$$\vec{Y} = B\vec{X}. \tag{1.2}$$

Thus, the simulation process is reduced to the simplest operation (1.2) consisting in the multiplying of the matrix by a vector $\vec{X} = L_X\vec{\varphi}$, where $\vec{\varphi}$ is a random Gaussian vector with zero expectation and unity correlation matrix, and the matrix L satisfies the following equation:

$$L_X L_X^T = R_{XX}.$$

Here R_{XX} is the given covariance matrix of the vector $\vec{X}$.

Let us consider an optimization problem in which the corresponding values of variance and correlation coefficients are optimized. Let us assume that the joint distribution of the vectors $\vec{X}$ and $\vec{Y}$ is also Gaussian and given by the covariance matrices R_{XX}, R_{YY}, and R_{XY}. In addition, let us assume that the variables $\vec{X}$ and $\vec{Y}$ are related by a linear dependence of the form (1.2), where B is some $n \times m$ matrix.

Thus, we have the following mutual covariance matrix:

$$R = \begin{pmatrix} R_{XX} & R_{XY} \\ R_{YX} & R_{YY} \end{pmatrix}. \tag{1.3}$$

Here, according to (1.2), we have

$$R_{YY} = BR_{XX}B^T, \qquad R_{XY} = R_{XX}B^T, \qquad R_{YX} = R_{XY}^T. \tag{1.4}$$

Note that if the information about the linear relation (1.2) is absent, it is possible to determine uniquely the linear operator B with the help of only the covariance matrix R_{XY} from (1.3).

Consider the formulation of the optimization problem similar to those used in papers [7, 8, 2]. Thus, we assume that the covariance matrix (1.3) is given approximately in the following form:

$$\tilde{R} = \begin{pmatrix} \tilde{R}_{XX} & \tilde{R}_{XY} \\ \tilde{R}_{YX} & \tilde{R}_{YY} \end{pmatrix}.$$

It makes it possible to construct the simulation process of Gaussian joint vectors $\vec{X}^{(i)}$, $\vec{Y}^{(i)}$, where i is the realization number. We construct a new ensemble of realizations $\{\vec{\xi}_X^{(k)}, \vec{\xi}_Y^{(k)}\}$ for which equation (1.2) is satisfied. Then we adjust it to the ensemble of realizations of this process.

We consider the error functional in the form

$$J_0 = (D_X(\vec{\xi}_X^{(k)} - \vec{X}^{(i)}), \vec{\xi}_X^{(k)} - \vec{X}^{(i)}) + (D_Y(\vec{\xi}_Y^{(k)} - \vec{Y}^{(i)}), \vec{\xi}_Y^{(k)} - \vec{Y}^{(i)}), \quad (1.5)$$

where D_X, D_Y are some positive definite weight matrices. Also, we consider the problem of minimization of functional (1.5) with respect to $\{\vec{\xi}_X^{(k)}, \vec{\xi}_Y^{(k)}\}$ from this ensemble. This problem is reduced to solving the following system of linear equations:

$$(D_X^{-1} + B^T D_Y^{-1} B)\vec{\xi}_{X,\mathrm{opt}}^{(i)} = D_X^{-1}\vec{X}^{(i)} + B^T D_Y^{-1}\vec{Y}^{(i)}. \quad (1.6)$$

Due to the linearity of equation (1.6), the covariance matrix of the optimal values $\vec{\xi}_{X,\mathrm{opt}}^{(i)}$ has the form

$$\begin{aligned} \hat{R}_{XX} = {} & (D_X^{-1} + B^T D_Y^{-1} B)^{-1}(D_X \tilde{R}_{XX} D_X + B^T D_Y \tilde{R}_{YX} D_X + \\ & D_X \tilde{R}_{XY} D_Y B + B^T D_Y \tilde{R}_{YY} D_Y B)(D_X^{-1} + B^T D_Y^{-1} B)^{-1}, \end{aligned}$$

and the covariance matrix of optimal vectors $\vec{\xi}_{Y,\mathrm{opt}}^{(i)}$ is defined by the formula

$$\hat{R}_{YY} = B\hat{R}_{XX}B^T.$$

Thus, instead the original matrix $\tilde{R}$, we have the covariance matrix $\hat{R}$ that is optimal with respect to functional (1.5). In our case, we take the matrices D_X and D_Y that are equal to the real inverse matrices of the variances of vectors $\vec{X}^{(i)}$, $\vec{Y}^{(i)}$. If the matrix of real mutual correlations $\tilde{R}_{XY}$ in the calculations was absent, it was replaced by the approximate matrix obtained as a result of the following transformations taking into account that (1.2) is satisfied exactly:

$$\tilde{R}_{XY} = \frac{1}{2}\Big(B^+ \tilde{R}_{YY} + \tilde{R}_{XX} B^T\Big),$$

where B^+ is pseudoinverse matrix. Naturally, the matrix $\tilde{R}$, that was obtained in this way, may not satisfy the condition of non-negative definiteness.

Therefore, it was corrected by using spectral representation. Since we suppose that equation (1.2) is satisfied, the rank of the matrix $\tilde{R}$ must obviously be equal to the maximal rank of the matrices $\tilde{R}_{XX}$ and $\tilde{R}_{YY}$. Therefore, the corresponding minimal eigenvalues in the spectral representation were taken equal to zero.

Thus, for stochastic simulation of complexes of hydrometeorological fields taking into account their interrelations given in the form of the mutual covariance matrices, or physical relations in the form of the corresponding equations of atmospheric dynamics, it is necessary to have a procedure of their preliminary adjustment. In this paper, a general procedure of such type has been considered. Really, the presence of linear relations between individual simulated fields of meteorological elements presupposes the presence of some "basic" fields as well. They are constructed by using methods of stochastic simulation. The remaining fields are expressed in terms of them with the help of the corresponding linear operators. The problem is that the correlation structure of complexes constructed in this way must be optimally similar to the available real structure of these fields. In particular, the relation between the fields of velocity and geopotential to a first approximation may be determined with the help of geostrophic relations. Now, we consider a more general procedure of stochastic simulation of hydrometeorological fields. In this procedure, the algorithms considered above are components at the preliminary initialization step.

In this paper, we propose an approach to the realization of this problem. At the initial stage, we estimate the correlation structure of the fields at the weather stations by the corresponding processing of observational data for many years. Using this information and the above procedure, we construct an ensemble of random field realizations on the regular grid in the domain considered and the initial network of weather stations. The field realizations at these points are used as initial data for the problem of variational assimilation and adjustment. That is, the sequence of realizations of the random fields $\vec{\xi}_{(n)}$ is used as real data. These are the approximations necessary for the localization of the minimum in the assimilation problem and exact reconstruction of the climatic characteristics of the considered fields in the dynamic probabilistic model.

Thus, we consider the problem of the variation assimilation of hydrometeorological fields with the help of a general numerical model of atmospheric dynamics.

2. Variational data assimilation

We write the system of finite difference equations of atmospheric processes [2] in operator form

$$B^h \frac{\partial \vec{\Phi}^h}{\partial t} + A^h(\vec{Y}^h, \vec{\Phi}^h)\vec{\Phi}^h = 0, \tag{2.1}$$

where $B^h = \mathrm{diag}\{1,1,1,0,0\}$ is a diagonal operator;

$$\vec{\Phi}^h = (u^h, v^h, T^h, \tau^h, H^h)$$

is the state vector; $\vec{Y}^h = \vec{\Phi}^h|_{t=0}$ is the vector of the parameters; $A^h(\vec{Y}^h, \vec{\Phi}^h)$ is the nonlinear finite difference operator defined by the system of equations of hydrothermodynamic processes in the atmosphere and the corresponding boundary conditions [2] in the domain $G_t^h = [0, \hat{t}] \times G^h$, where $G^h = [0, X] \times [0, Y] \times [p_S, 0]$. System (2.1) is solved by the component-wise splitting method [2, 9]. In our case, the problem is to select such a solution from the entire set of solutions to the system of equations (2.1) that is determined by the values of the parameter vector $\vec{Y}^h$. The solution differs least from the corresponding measured values, in the sense of a quality functional at the given measurement points. We consider this problem in the finite-dimensional space of grid functions. To construct a finite-dimensional approximation for problem (2.1), the integral identities method [9] is used. Following [2, 9], in the domain G^h we introduce inner product $(\vec{\Phi}^h, \vec{\Phi}^h)_{G^h}$. This product is the finite-difference analog of the functional

$$(\vec{\Phi}, \vec{\Phi}) = \iiint\limits_G \Big(uu^* + vv^* + \frac{g}{(\gamma_a - \gamma)\bar{T}} TT^* + \alpha_\tau \tau\tau^* + \alpha_H H_S H_S^* \Big)\, dG.$$

Here $\alpha_\tau = 1\ \mathrm{m^2 mb^{-2}}$ and $\alpha_H = 1\ \mathrm{s^2 m^{-2}}$ are dimensional multipliers; γ_a is the dry-adiabatic temperature gradient; γ is the temperature gradient of the standard atmosphere $\bar{T} = \bar{T}(p)$. Here the domain G is associated with the grid domain G^h. We consider the following quality functional from [2, 9]:

$$J_0 = \frac{1}{2} \sum_k (L\vec{\Phi}^j - \vec{\Phi}_S^k, L\vec{\Phi}^j - \vec{\Phi}_S^k)_{D_S}. \tag{2.2}$$

It determines the measure of deviation of the solution obtained by model (2.1) and is interpolated at the measurement points from the actual data given by the vector $\vec{\Phi}_S^k$ of dimensionality N at the corresponding instant t_k. Here L is an interpolation operator, and the inner product is considered with a weight matrix D_S that determines the extent to which the data is reliable. Thus, we consider the problem of minimizing the quality functional J_0 in the class of vector functions $\vec{\Phi}^j$ that satisfy the system of equations (2.1). We seek for a minimum with respect to the parameter vector $\vec{Y}^h$. If we eliminate fractional steps in the component-wise splitting method for (2.1) and employ the Lagrange method of multipliers, we obtain the functional

$$J = \sum_{j=1}^{n} (\tilde{A}^h(\vec{Y}^h, \vec{\Phi}^{j-1})\vec{\Phi}^j - \vec{\Phi}^{j-1}, \vec{\Phi}^j)_{G^h} + J_0.$$

If we take the variation of this functional with respect to the parameter vector, we obtain the expression for the gradient where the vector functions $\vec{\Phi}^j$ satisfy the following adjoint system of equations:

$$\tilde{A}^{h*}(\vec{Y}^h, \vec{\Phi}^j)\vec{\Phi}^{j*} - \vec{\Phi}^{j+1*} + D_G L^*(L\vec{\Phi}^j - \vec{\Phi}^k_S) + \vec{F}^j = 0, \quad \vec{\Phi}^{n+1*} = 0.$$

Here the operator D_G is determined by the concrete form of the weight matrix and the space-time location of the measurement points. The vector function $\vec{F}^j$ is defined by the variations of the nonlinear terms in the operators $\tilde{A}^h(\vec{Y}^h, \vec{\Phi}^j)$ and is calculated explicitly. Hence, to minimize the functional J_0, we can construct the iterative process by the formula

$$\vec{\Phi}^{0\,(m+1)} = \vec{\Phi}^{0\,(m)} - \kappa^{(m)} \nabla_{\vec{Y}^h} J^{(m)}, \tag{2.3}$$

where m is the iteration number, and $\kappa^{(m)}$ is an iterative parameter. The iterative process (2.3) is repeated until it converges. We would like to point out some main properties of the above algorithm for data assimilation. Since the restrictions represented by the system of equations (2.1) of the model considered are nonlinear, the solution to the problem of minimizing functional (2.2) is not unique in the general case. This depends on the character, completeness, and location of the measurement data as well as on the concrete hydrometeorological situation, whose structure is reconstructed in the assimilation process. Since we employ the method of gradient descent (2.3) for the solution of the problem of minimizing the quality functional, it is obvious that in the general case the solution depends on the choice of the initial value of the parameter vector $\vec{Y}^h = \vec{\Phi}^h|_{t=0}$. One of the main properties of the data assimilation algorithm considered is that we obtain consistent fields of hydrometeorological factors, which are filtered relative to the numerical model, the agreement of the numerical model itself with actual data being checked simultaneously. Thus, having solved the data assimilation problem, we have a complex of space-time hydrometeorological fields on the regular grid. As a result of this procedure, we obtain realizations of the random hydrometeorological field $\vec{\xi}^c_{(n)}$, which are mutually consistent relative to the mathematical model and in which the nonphysical fluctuations are filtered, but the field structure at the observation stations is distorted slightly.

3. Numerical experiments

The results of numerical experiments on optimization of the covariance matrices of vertical profiles of the temperature and "reduced" geopotential

The results of numerical experiments on optimization of the covariance matrices of temperature and "reduced" geopotential taking into account the equation of statics

"Reduced" geopotential				Temperature			
Real correlation matrices							
1.00	0.69	0.67	0.56	1.00	0.76	0.44	−0.49
0.69	1.00	0.89	0.85	0.76	1.00	0.53	−0.56
0.67	0.89	1.00	0.90	0.44	0.53	1.00	−0.02
0.56	0.85	0.90	1.00	−0.49	−0.56	−0.02	1.00
Optimization of covariance matrices taking into account the equation of statics							
1.00	0.84	0.75	0.55	1.00	0.59	0.44	−0.37
0.84	1.00	0.91	0.66	0.59	1.00	0.53	−0.42
0.75	0.91	1.00	0.83	0.44	0.53	1.00	−0.06
0.55	0.66	0.83	1.00	−0.37	−0.42	−0.06	1.00

taking into account the equation of statics are presented in the table. The correlation matrices of the geopotential reduced to a level of 850 mb in the table correspond to standard levels of 700, 500, 300, and 200 mb. Since the mutual covariance matrices are determined by the last two equations of (1.4) together with the corresponding matrices from this table, we do not present them. One can see from the table that the initial real covariance matrices have significant changes after their mutual adjustment with respect to the finite difference analog of the equation of statics with the help of the optimization procedure, although they are in the admissible interval of values [10]. This is probably explained by the fact that the equation of statics is satisfied approximately in real atmosphere, by errors of its approximation, and by inaccurate specification of the covariance matrices. Similar correlation changes after statistical adjustment procedures are presented in [8] for the fields of temperature, pressure, and density.

In order to carry out test calculations to adjust the hydrometeorological and probabilistic models, we used the realizations of the time sequence of discrete three-dimensional random fields at nodes of the regular grid on eight isobaric surfaces, assuming that the field is distributed normally. The general correlation matrix $R_{(n)}$ of the field considered on the interval of one day with step $\Delta t = 1$ h and with space resolution over the horizontal $\Delta x = \Delta y = 300$ km with the number of nodes 20 and 24 along the coordinates x and y is given as the direct product of the correlation matrices in the corresponding directions and in time. The vertical cross-correlations at different levels p are characteristic of the North hemisphere and represented as the 8×8 correlation matrix taken from [11]. The horizontal correlation matrices are taken to be identical at each level under the assumption that the horizontal field sections are isotropic [10, 11, 2].

We calculated the spectrum of the correlation matrix $R_{(n)}$. Note that the eigenvalues decrease rapidly enough. In order to estimate the necessary number of the main components for representing any vector of the simulated random process $\vec{\xi}_{(n)}$ with a given accuracy, we calculated the ratio of the sum of the minimum eigenvalues to the sum of the maximum eigenvalues of the correlation matrix $R_{(n)}$ (see [12]). In this case, it turns out that it is sufficient to have 100 main components to achieve an accuracy of 30 per cent.

In accordance with the above structure, we constructed an ensemble of the Gaussian temperature fields by using the above procedure. In these calculations, we realize the simplified modeling procedure (1.1). With allowance for the specific character of the correlation matrix $R_{(n)}$, this procedure reduces to a successive use of similar scalar procedures. We used a sample of 166 realizations of the space-time field $\vec{\xi}_{(n)}$ in the test calculation. After the process of variational assimilation of these realizations, we performed calculations with a 10-level numerical model (2.1). The main isobaric levels considered were $p = 1000, 850, 700, 500, 400, 300, 250, 200, 150$, and 100 mb. We obtained an ensemble of new field realizations. Besides the temperature field, it also includes the corresponding fields of velocity, pressure, and ground geopotential. The mean relative assimilation error is about 30 per cent over the entire ensemble.

Using the ensemble of consistent fields, we calculated a 166×166 sampling correlation matrix for the temperature field. For the sequence of the main submatrices of this matrix we calculated the corresponding sequences of eigenvalues. They converge rapidly enough and, in the limit, they are close to the eigenvalues of the initial matrix. This indirectly points to the convergence of the sampling correlation matrices and their asymptotic closeness to the initial matrix.

We also calculated the function of the vertical temperature correlations with averaging over time and horizontal space coordinates. This function is given in Figure 1a. It is seen from this figure that after variational assimilation the character of the correlations is in good agreement with the initial correlations given in the same figure. The corresponding correlation function of the vertical geopotential profiles are presented in Figure 1b.

In Figures 2a and 2b, variances of the vertical profiles of the temperature and "reduced" geopotential are presented.

Notice that increase of the sample size up to 260 realizations does not change essentially the estimates of the sampling covariance matrices. Note also that in our case the result of optimization of the covariance matrices from Section 1 of this paper was more similar to the corresponding matrices obtained with the help of variational assimilation of the realization ensemble then to the corresponding real matrix. Besides, if we use procedure for optimization of covariance matrices described in Section 1, which is calculated

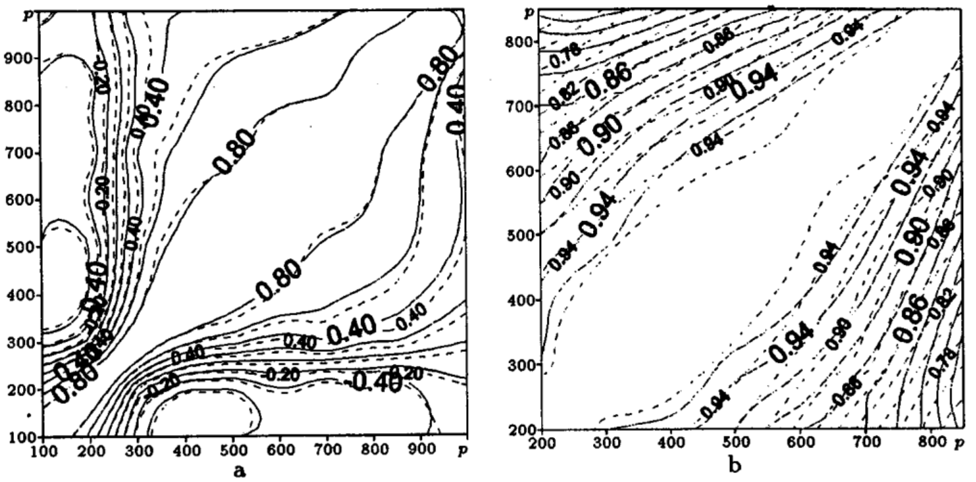

Figure 1. The correlation function of the vertical profiles for the temperature (a) and geopotential (b) fields: solid lines – the correlation function after variational assimilation; dashed lines – the real correlation function [11]

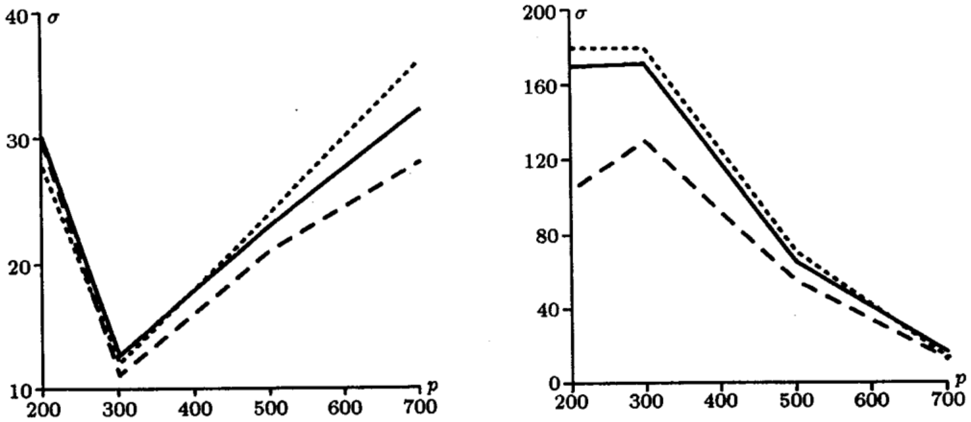

Figure 2. The variances of the vertical profiles for the temperature (a) and "reduced" geopotential (b): solid line is the real variances, dashed line is the variances after variational assimilation, and dotted line is optimization of the real variances

with the help of the variational assimilation method with the approximation of the equation of statics applied in the hydrothermodynamic model, then this procedure conserves these matrices. In this case, the operator B is reconstructed by the second formula of (1.4) with a high degree of accuracy. It follows directly from the general formulation of the variational assimilation problem.

Another important characteristic of the obtained realization ensemble is the correlation structure of the velocity fields in the horizontal section. For

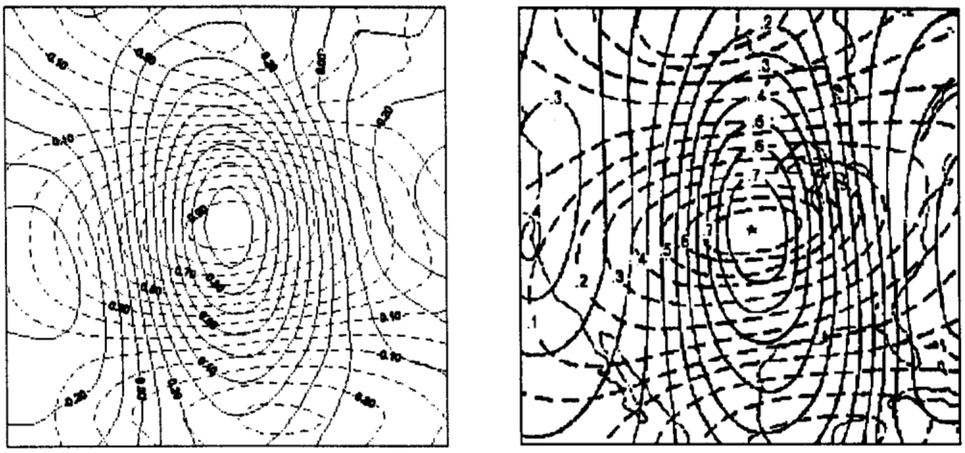

Figure 3. The correlation coefficient fields for the zonal (dashed lines) and meridional (solid lines) wind components: the left graph is the result of the stochastic modeling; the right graph was taken from [7]

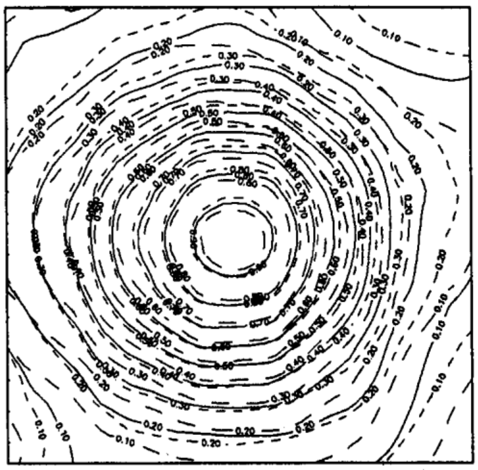

Figure 4. The correlation coefficient fields for temperature at the level $p =$ 850 mb: solid lines – the correlation coefficient field after variational assimilation for $t = 0$ hour; dashed lines with the short touches – the correlation coefficient field after variational assimilation for $t = 15$ hour; dashed lines with the long touches – the correlation coefficient field of the initial stochastic field for $t = 0$ hour

this purpose we present the contours graph of the correlation function for the horizontal wind velocity components at the level $p = 500$ mb, in comparison with the corresponding sample estimate from [7] calculated with the help of real data. It is seen from this graph that the correlation function contours calculated with the help of the model are in good agreement with the results calculated by using real data (Figure 3).

For the illustration of properties of homogeneous and isotropy the isolines of the horizontal correlation function of the temperature fields at the level 850 mb are presented (Figure 4).

Note in conclusion that we consider this paper as a preliminary step for the construction of a numerical atmospheric climatic model. We consider this model as an ensemble of possible realizations of random fields with the given covariance structure [5, 6] and satisfying the system of hydrothermodynamic equations.

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