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Numerical eddy-resolving model of nonstationary penetrative convection in the spring solar heating of ice-covered lakes*

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An eddy-resolving model of the penetrating turbulent convection in the ice-covered large lakes in moderate altitudes in the spring radiation heating has been constructed. A scheme of energy transformations in the simulated phenomenon is described. A comparison of the calculation results with natural data is done.

1. Introduction

In the winter time, in deep ice-covered lakes of moderate altitudes, the water temperature appears to be lower than the temperature of maximum density (Θ_m) and to increase with depth (stable stratification). In March–April, the temperature of the ice-covered water layer (Θ) starts to gradually rise at the expense of the volume heating of this layer by the solar radiation penetrative through ice. As a result, a layer with unstable stratification is formed. A free penetrating convection with coherent structures in the form of thermals arises [1], whose intensity increases during a few weeks. The mechanism of occurrence and the vertical thermal structures of this natural phenomenon have been well studied by experimental techniques [2–4]. The data on the spatial temporal structure of the fields of the vertical (w) and the horizontal (u, v) velocity components, temperature deviations (θ) in thermals, statistical characteristics and energy contents of coherent structures are practically absent.

The penetrative convection under the ice-layer is not subject to the wind-wave mixing. Thus, the convection under consideration is nearly an ideal object for the direct numerical eddy-resolving simulation.

This paper presents the results of constructing the LES-model (Large Eddy Simulation) of the convective boundary layer of the ice-covered lake (the CBL-ice), as well as the energy transformation scheme in the phenomenon under study. Based on comparison of the calculation results with

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natural data, a preliminary estimation of prospects of the eddy-resolving simulation methodology for studying the fine structure and the spatial-temporal evolution of hydrophysical fields of the CBL-ice of the lake has been done.

2. The LES-model of convective boundary layer of ice-covered lake

Analysis of natural data makes it possible to conclude that when simulating the ice-covered convective boundary layer it is necessary to take into account, at least, three types of processes of different space and time scales, which are important in terms of the energy transformation:

a) the time-dependent (primarily, with the daily cycle) mean state;

b) convective energy-providing large eddies, explicitly reproducible as an ensemble of coherent structures (thermals). The deterministic description of a convective ensemble is done on the basis of discrete high-resolution analogies of the Navier-Stokes equations;

c) small-scale (subgrid) turbulent motions taken into account within the Boussinesq gradient-diffusive approximation.

When constructing the LES-model of the CBL-ice, let us make use of the splitting method of the original system of equations of the thermodynamics of the lake [5] to two systems of equations, describing the deterministic interconnected processes a) and b) [6].

At the first stage of constructing the LES-model of the CBL-ice, we introduce a few assumptions to simplify things. Let us consider the mean current velocity under the ice layer to be equal to $U = V = 0$. The Coriolis force in the equations of motion is not taken into account because of the small horizontal scales of thermals. A fresh-water layer with intensive penetrating convection, is only a few tens meters, the dependence of density on pressure and mineralization not taken into account. And, finally, we will consider the 2D problem instead of the 3D problem. None of the above simplifying assumptions is strictly limiting in terms of the methodology used in construction of the LES-models of the lake.

Thus, the LES model of the CBL-ice includes:

I. The equation of the temperature (Θ) of the main state a):

$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial z}\nu\frac{\partial \Theta}{\partial z} - \frac{\partial}{\partial z}\overline{w\theta} + \frac{\partial R}{\partial z}, \tag{1}$$

where t is time, z is the vertical coordinate downwards, so that the value $z = 0$ corresponds to the lower ice edge, $R(z, t)$ is a flux of the short-wave solar radiation absorbed by the sub-ice water layer, $\overline{(\ldots)} = L^{-1} \int_0^L (\ldots)\, dz$ is the averaging operator along the horizontal axis x, L is the size of the averaging domain to be defined.

The boundary conditions for (1) are the following

$$\Theta = 0°C \quad \text{at} \quad z = 0, \qquad \frac{\partial \Theta}{\partial z} = \gamma_0 \quad \text{at} \quad z = H,$$

where H is depth of complete vanishing of convective motions, at which and below which $\gamma_0 = \text{const} > 0$, and this corresponds to the stably stratified hypolimnion.

The initial conditions were set as

$$\Theta = \Theta_* \quad \text{at} \quad t = 0,$$

where $\Theta_*(z)$ is considered given at the preconvection period.

II. Equations describing the penetrative turbulent convection (process b):

$$\frac{du}{dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\nu\frac{\partial u}{\partial z} + \frac{\partial}{\partial x}\mu\frac{\partial u}{\partial x}, \tag{2}$$

$$\frac{dw}{dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} + \lambda\theta + \frac{\partial}{\partial z}\nu\frac{\partial w}{\partial z} + \frac{\partial}{\partial x}\mu\frac{\partial w}{\partial x}, \tag{3}$$

$$\frac{d\theta}{dt} = -w\frac{\partial \Theta}{\partial z} + \frac{\partial}{\partial z}\nu\frac{\partial \theta}{\partial z} + \frac{\partial}{\partial x}\mu\frac{\partial \theta}{\partial x} + \frac{\partial}{\partial z}\overline{w\theta}, \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{5}$$

where $\frac{d}{\partial t} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z}$, ν and μ are coefficients of the vertical and horizontal exchange due to the small-scale (subgrid) gradient-diffusive turbulence (process c), $\lambda = \alpha(\Theta - \Theta_m)$. In the latter expression $\alpha = 2abg/\rho_0$, where g is acceleration due to gravity, ρ_0 is the characteristic value of density in the mixing layer, the values a, b are taken from the Kharleman–Markovsky formula [7].

The boundary conditions are the following:

$$u = w = \theta = 0 \quad \text{at} \quad z = 0, \qquad \frac{\partial u}{\partial z} = w = \frac{\partial \theta}{\partial z} = 0 \quad \text{at} \quad z = H.$$

On the left and the right boundaries the periodicity conditions are set.

The initial conditions are the following:

$$u = w = \theta = 0 \quad \text{at} \quad t = t_k,$$

where $t = t_k$ is the time of occurrence of the internal layer with the unstable stratification ($\frac{\partial \theta}{\partial z} < 0$), at which small random values $\theta(x)$ for convection generation are introduced at $t = t_k + \Delta t$ (Δt is the time step). On the interval $(0 - t_k)$, only equation (1) is integrated, in this case $\overline{w\theta} = 0$.

III. Equations for calculation of coefficients of the small-scale turbulent exchange (b–ϵ model):

$$\frac{\partial b}{\partial t} = \frac{\partial}{\partial z}\nu\frac{\partial b}{\partial z} + P_\nu - \epsilon, \qquad \nu = c_\nu\frac{b^2}{\epsilon},$$
$$\frac{\partial \epsilon}{\partial t} = \frac{1}{\sigma}\frac{\partial}{\partial z}\nu\frac{\partial b}{\partial z} + c_1\frac{\epsilon}{b}P_\nu - c^2{}_2\frac{\epsilon^2}{b},$$

where b is the kinetic turbulence energy (KTE), ϵ is the KTE dissipation rate of the small-scale turbulent eddies, P_ν is the KTE generation rate at the expense of the buoyancy forces and the shear, c_ν, c_1, c_2, σ are empirical constants. The boundary conditions:

$$b = \frac{\partial \epsilon}{\partial z} = 0 \quad \text{at} \quad z = 0, \qquad b = \frac{\partial \epsilon}{\partial z} = 0 \quad \text{at} \quad z = H,$$

For calculation of μ, the Smagorinsry relation is used:

$$\mu = \alpha_1\frac{\Delta x^2}{2}\left[\left(\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2\right]^{1/2},$$

where α_1 is the given parameter, Δx is the mesh size along x.

3. Energy cycle of processes in the CBL-ice model of the lake

To verify the correctness of the problem, formulated in Section 2, the qualitative analysis of physical processes of the simulated phenomenon and the estimation of the accuracy of the numerical solution of the problem, evolutionary equations for integral values of the available potential energy (APE) of the main state and convective perturbations for two components of the kinetic energy (KE) of convective eddies, have been obtained. These equations have the following form:

$$\frac{\partial P}{\partial t} = \{F^*, P\} + \{P, P'\} - \{P, D_\Theta\}, \tag{6}$$

where $P = \left\langle\frac{\lambda}{\gamma_0}\frac{\Theta_n^2}{2}\right\rangle$ is the APE of the main state, $\{F^*, P\} = \left\langle\frac{\lambda}{\gamma_0}\Theta_n\frac{\partial R}{\partial z}\right\rangle$ is the rate of increasing P at the expense of the volume radiative heating of the lake, $\{P, P'\} = \left\langle\frac{\lambda}{\gamma_0}\overline{w\theta}\frac{\partial\Theta_n}{\partial z}\right\rangle$, is the transfer of P to the APE of the convective structures (P'), $\{P, D_\Theta\} = \left\langle\frac{\lambda}{\gamma_0}\nu\left(\frac{\partial\Theta_n}{\partial z}\right)^2\right\rangle$ is the rate of decreasing

P at the expense of the small-scale turbulent diffusion, $\langle(\ldots)\rangle = \int_0^H(\ldots)dz$, $\Theta_n = \Theta - \Theta_c$, $\Theta_c = \gamma_0 z$ is the temperature of a standard stratified lake before the radiative heating starts when $\Theta_n = 0$;

$$\frac{\partial P'}{\partial t} = -\{P, P'\} - \{P', K_w\} - \{P', D_{pz}\} - \{P', D_{px}\} \qquad (7)$$

where $P' = \left\langle \frac{\lambda}{\gamma_0}\nu\frac{\theta^2}{2}\right\rangle$, $\{P', K_w\} = \langle\lambda(\overline{w\theta})\rangle$ is the rate of the transfer of P' to the kinetic energy of the vertical convective motions (K_w), $\{P', D_{\theta z}\} = \left\langle\overline{\frac{\lambda}{\gamma_0}\nu\left(\frac{\partial\theta}{\partial z}\right)^2}\right\rangle$ and $\{P', D_{\theta x}\} = \left\langle\overline{\frac{\lambda}{\gamma_0}\mu\left(\frac{\partial\theta}{\partial x}\right)^2}\right\rangle$ according to the diffusion rate P' at the expense of the vertical and horizontal turbulent heat exchange, $\overline{(\ldots)} = \frac{1}{L}\int_0^H\int_0^L(\ldots)\,dx\,dz$;

$$\frac{\partial K_w}{\partial t} = \{K_w, K_u\} + \{P', K_w\} - \{K_w, D_{wz}\} - \{K_w, D_{wx}\}, \qquad (8)$$

where $K_w = \left\langle\overline{\frac{w^2}{2}}\right\rangle$, $\{K_w, K_u\} = \frac{1}{\rho_0}\left\langle\overline{p\frac{\partial u}{\partial x}}\right\rangle$ is the rate of the transfer of K_w to the kinetic energy of the horizontal convective motions (K_u), $\{K_w, D_{wz}\} = \left\langle\overline{\nu\left(\frac{\partial w}{\partial z}\right)^2}\right\rangle$, $\{K_w, D_{wx}\} = \left\langle\overline{\mu\left(\frac{\partial w}{\partial x}\right)^2}\right\rangle$;

$$\frac{\partial K_u}{\partial t} = -\{K_w, K_u\} - \{K_u, D_{uz}\} - \{K_u, D_{ux}\}, \qquad (9)$$

where $K_u = \left\langle\overline{\frac{u^2}{2}}\right\rangle$, $\{K_u, D_{uz}\} = \left\langle\overline{\nu\left(\frac{\partial u}{\partial z}\right)^2}\right\rangle$, $\{K_u, D_{ux}\} = \left\langle\overline{\mu\left(\frac{\partial u}{\partial x}\right)^2}\right\rangle$.

For convenience, when analyzing of equations (6)–(9), a diagram of the energy cycle is described by the LES-model of the CBL-ice of the lake (Figure 1). Investigation of the energy of processes of the penetrative turbulent convection in the ice-covered lake is of great interest, first of all because of considerable daily variability of the source $I = \{F^*, P\}$ (in the day time $I > 0$, at night $I = 0$) combined with the variability from day to day over long periods (up to 1–1.5 month).

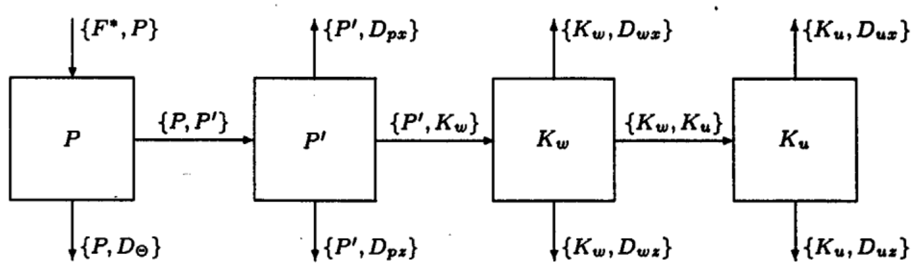

Figure 1. Diagram of the energy cycle of processes in the CBL-ice of the lake

4. Calculation results

For the numerical solution of the formulated problem, equations (2), (3), and (5) were written down in a standard manner in the "eddy-stream function". Evolution equations for the eddy of the problem and θ in (4) were approximated using the monotonic scheme by A.A. Samarsky and A.V. Gulin [8] having the accuracy $O(\Delta t, \Delta x^2, \Delta z^2)$. The boundary conditions for the eddy were calculated by relaxation formulas with supplementary iterative process. The equation for the stream function was solved by the direct method of separation of variables using the FFT (fast Fourier transform).

Taking into account the complexity of the simulated phenomenon, at the first stage of calculations we gave up applying complex models of dynamics and optics of snow and ice. We made use of a simple approximation $R(z,t) = -\alpha F_i e^{-\beta z}/(c_p \rho_0)$, F_i is the solar radiation flux at $z = 0$ (according to the Albreht formula), α is the coefficient allowing us to take into account the increase of the radiation intensity from day to day because of the ice cover melting calculated by the methods proposed in [9], $\beta = \text{const}$ is the radiation absorbing coefficient, c_p and ρ_0 are, respectively, specific heat of the water and mean water density. At the stage of test calculations, we restricted ourselves to the case of a priori setting the coefficients ν and μ.

Let us briefly describe the result of modeling of the initial stage of generation of the CBL-ice with the following values of parameters of the numerical model: $L = 120$ m ($\Delta x = 0.5$ m), $H = 40$ m, ($\Delta z = 0.5$ m), the time step 3 s, the integration time is 5 days, $\nu = \mu = 10^{-4}$ m^2/s, $\beta = 0.5$ m^{-1}.

Figure 2 shows the fields of the stream function at the time of the maximum convection evolution in the day time (15 hours l.t.) for the 1st, 3rd and 5th days. Analysis of these fields as well as of the fields w and θ allows a conclusion of the fact that the model makes it possible to reproduce a highly dynamic daily variability and the variability from day to day characteristic of turbulent currents with penetrative convection. The model describes all the stages of evolution both of individual CS (from generation in the layer of instability to dissipation in the entrainment layer) and their "integrated" influence on the variation in the structure of the main state. The latter is shown in Figure 3a.

The shown profiles Θ qualitatively correctly describe the thermal structure of the CBL-ice of the lake [4]. A comparison of natural and calculated values of the mean heating rate of a mixed layer (according to the data [4], this is a range of 0.0125 to 0.03°C days, according to the model – 0.02°C/days) and the mean deepening rate of the base mixed layer (1.5–2.0 m/days – measurements, 1.8–1.9 m/days – calculations) allow us to make a conclusion of a good potential of even a simplified LES-model according to the quantitative description of the CBL-ice structure of the lake. The possibility of applying the conventional gradient-diffusive K-models (this is

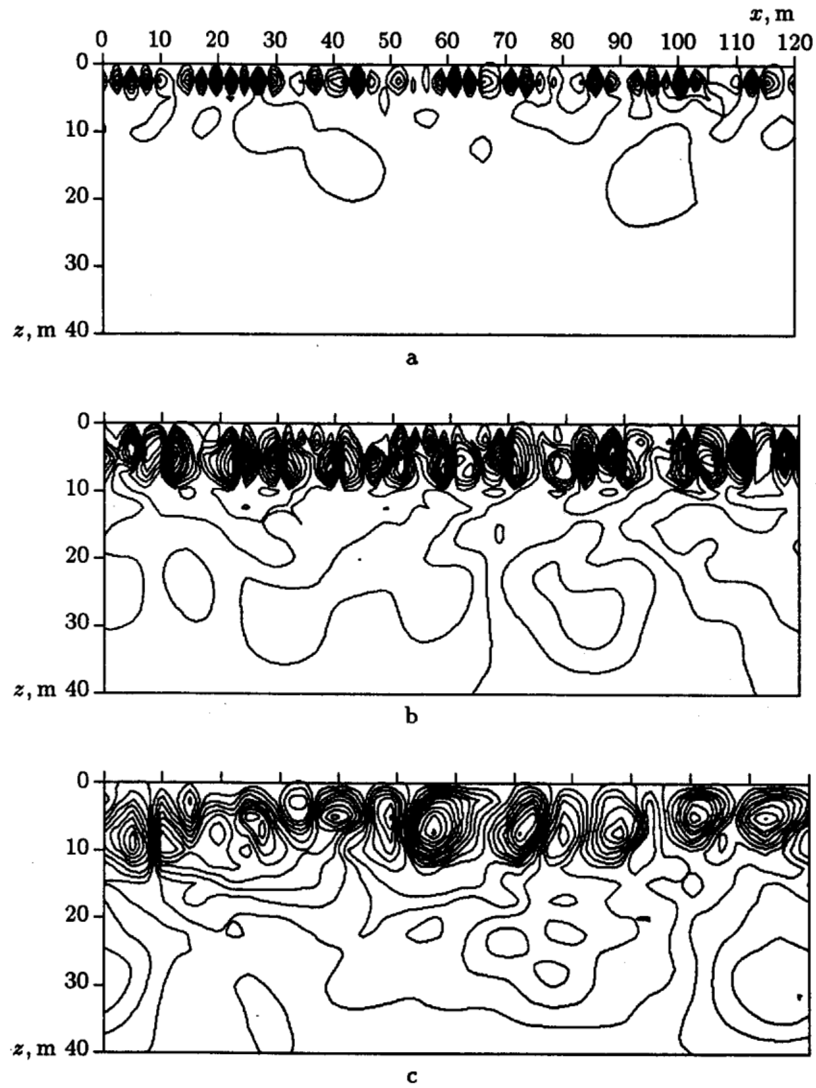

Figure 2. The stream function for the 1st, 3d and 5th days, respectively

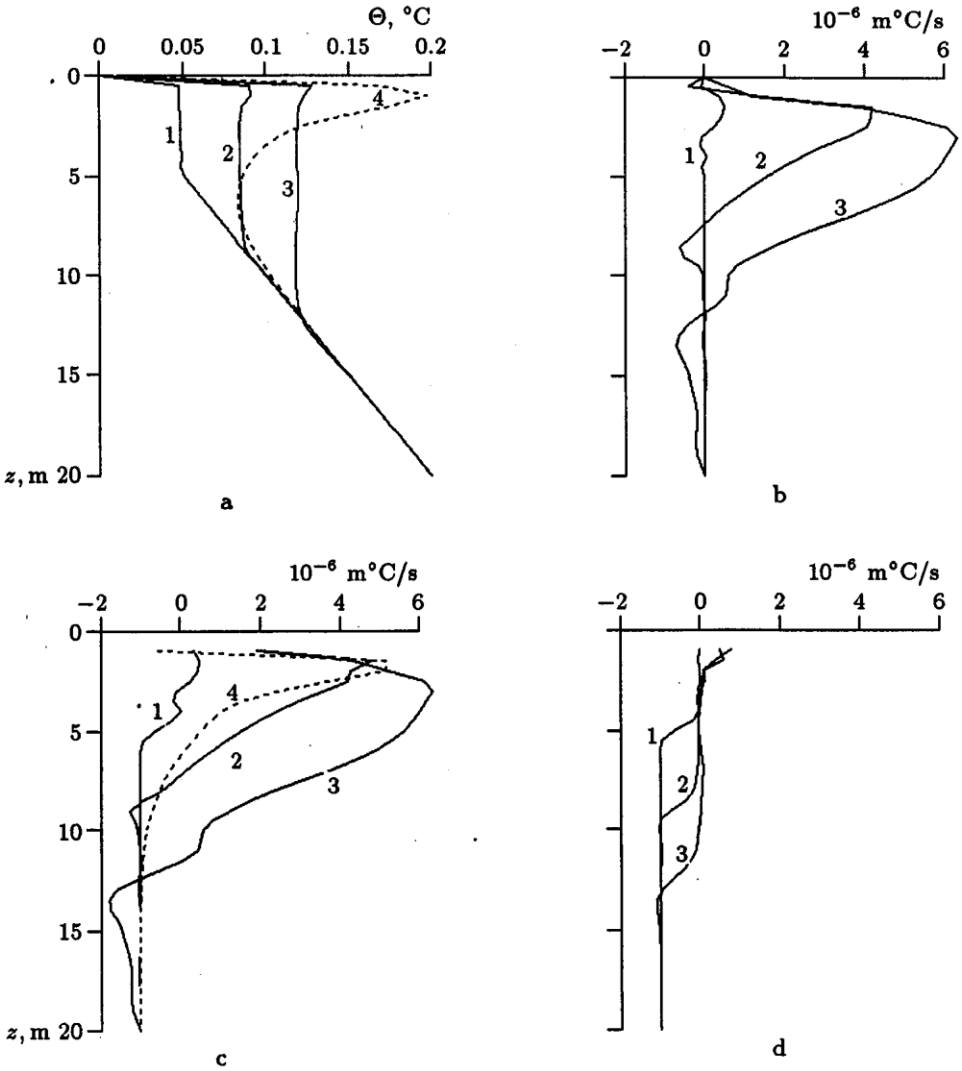

Figure 3. Vertical distribution of: a) basic temperature; b) convective, c) complete, and d) subgrid turbulent heat fluxes for the 1st, 3rd and 5th days. The complete problem – curves 1, 2, 3, respectively; the gradient-diffusive k-model – curves 4 (only the 5th day)

problem (1) at $\overline{w\theta} = 0$) for the description of the phenomenon of the sub-ice turbulent penetrative convection, is, apparently, strongly restricted, as evidenced by curve 4 in Figure 3a as well as by the calculation results shown in Figures 3b–d.

5. Conclusion

The results obtained allow us to make a conclusion about good prospects of application of the methodology of the eddy-resolving simulation for the research into the fine structure and the spatial-temporal evolution of hydrophysical fields with essentially non-stationary turbulent penetrating convection of the ice-covered lake during the spring solar heating. The LES-models can serve a good hydrodynamic basis in their combination with hydroecological models of the transport of resolved oxygen, biogenic substances, hydrosols, the dynamics of phytogenic and zoo-planktons. The authors encourage the collaboration aimed at the development of such models.

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