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On a phase method of vibroseismic exploration*

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The possibilities of obtaining numerical estimations of the time of wave propagation and parameters to describe the form of recorded wave impulses, with the use of a sequence of sessions of the monochromatic radiation on various frequencies are investigated. The monochromatic signals are invariant in their form when propagating in the linear media of any complexity. Only two parameters – a phase displacement and amplitude vary. The frequency of signals is preset and kept with a high accuracy.

The difference in phases between the radiated and the recorded signals expresses the propagation time (delay) in a medium modulo, which is equal to its period of signal. A real delay is represented by the sum of an unknown integer of periods and the measured fractional part of a period. The integer part of the delay is calculated with the use of additional measurements on other frequencies. This principle is used in the phase radio engineering bearing and navigational systems operating in the homogeneous (one-wave) medium and differs from impulse systems of the same purpose by a very high accuracy. The objective of the present research is to extend this method to an essentially inhomogeneous seismic medium and to obtain numerical estimations of the times of propagation of separate waves without essential loss in the level of "technical" accuracy of modern equipment for vibrosounding exploration.

In experimental sounding systems of SB RAS, the phase of monochromatic signals at a sufficient duration of sessions is measured by a coherent method with errors about one degree at distances of 300 km. With the frequency of a signal 6 Hz the measurement error of a difference in phases equal to 1 degree corresponds to the time interval 0.5 ms and to the travel of a wave in solid rocks about 2 m.

The essential advantage of vibrosounding methods of exploration over the impulse methods is, that vibrational signals can be specified (synthesized) and to within fractions of milliseconds be reproduced with the help of specialized computer systems of vibrators control. The possibility to multiply radiate signals with practically constant parameters allows one to proceed to the development of sounding systems intended for the research into the

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field of geodynamics, first of all for the detection and tracing measurements of variations of the tense state of the of the earthquake-prone foci for the sake of earthquake prediction.

It is worth to note some important features of the method under consideration. The results of the exploration contain a minimum amount of data – the list of values of modules (amplitudes) and phases obtained in the course of observation of harmonic signals with various frequencies. The physical time of propagation of waves and the forms of their impulses in these data are not represented in the explicit forms. They can be obtained in the course of processing, but without application of the visual data analysis.

A decrease in the influence of microseismic noises happens only when selecting parameters of sounding signals. The parameters contain only residual random errors in the coordinates of two-dimensional vectors to describe separate harmonic signals. The errors are distributed according to the normal law and are statistically independent, as the radiation sessions are performed at various time intervals.

The major feature of the method is in that the calculation of propagation times (kinematic parameters) and the parameters defining the form of oscillations of separate waves (dynamic parameters) cannot be separately done. They are the roots of a system of nonlinear equations. Its essence is not in defining the time of propagation of separate waves, but in separation of wave impulses. In this case, these are differences of impulses in the form, which are used as indication to performing the separation. In the course of development of the method, the possibilities of separation of impulses associated with various waves and completely overlapping in time were found. In the task in question the parameter called "the time of propagation" is, in the main, for the calculation of the above-mentioned integer of periods of a harmonic signal. Actually the value of this parameter is connected with the form of a wave impulse.

Recently, in practice of decoding seismograms the propagation time is determined when constructing travel-time curves (hodographs) of waves. In phase areas (points) and/or areas of a maximum range of oscillations related to one wave are traced by a discrete set of seismic traces. They are approximated (interpolated) by a continuous curve, which is then additionally smoothed. The hodographs (travel-time curves) are constructed with allowance for a priori information obtained on the traveled part of the profile, and with well-known physical laws of seismic waves propagation in an inhomogeneous medium taken into account. The form (dynamics) of oscillations is of importance for "recognition" of types of waves in order that their separate hodograph be created. The hodograph is considered as "carrier" of the time of a wave propagation along the line, where seismic receivers are located. Using hodographs the information, necessary for the "kinematic" description of a medium, is extracted from experimental data. It is possible to say that kinematic methods are intersecting (are close to) with dynamic methods of the analysis of properties of a medium "at one point", where the hodograph crosses the seismic trace axis. In our task, these methods are intimately connected.

The substitution of real wave impulses by delta-functions corresponds to the extension of a frequency spectrum of seismograms *ad infinitum.* Such a challenging extrapolation (idealization) by no means-decreases the essence and practical value of kinematic methods, provided that the input data are sufficiently accurate. So, the task of determination of an exact moment of the wave arrival still remains to be solved. The concept of the time of the wave arrival will remain conditional, but will require a concrete definition; which point (function, functional) complex wave impulse will be the best to correspond not only to a general idea of time of propagation, but, also, to formal requirements of kinematic tasks. In this case, there is a need in a "good" system of processing of seismic signals. Such a system should use as a priori data well founded mathematical descriptions of the regularities defining the "expected" form of wave impulses. These descriptions should be "brief", contain a limited number of unknown parameters.

The above-described features of the phase method of sounding exploration correspond to general principles of experimental data processing. When there is, at least, a general idea about the processes under study and the character of the cause end effect links between processes and their manifestations, available for the direct recording, the technical observational systems, such as, system of seismic sounding are created.

The development of theoretical (model) knowledge about processes and links brings about the development of methods of observations processing. The natural objective of observations processing is the transformation of experimental data to the form convenient for the user perception. If there is need in the information about properties of a process unavailable for the direct observation, this information is calculated with the help of a mathematical model of the process by introduction to it available parameters (indirect measurements). To our systems, not the waves as they are, but the oscillations, they excite, are available at certain points of a medium.

When the data are accompanied by the intensive noise, they cannot be immediately introduced into the model of a process. Then additional models of a signal and noise are used. They contain a priori features, by which a preliminary weakening of the noise is conducted. The essence of preprocessing reduces to projected "the space of data" onto "the space of a signal", which has essentially smaller dimensionality. In our case, the data array (with noise), containing hundreds thousands of numbers, is projected onto a two-dimensional space of the harmonic signal vector. The obtained two parameters can immediately be introduced into the processing.

In a "classic" method of vibrational sounding a complex signal with a linear frequency modulation is applied. Though it is completely determined by three parameters, it is not invariant. Characteristics of a medium are expressed in changes of form. Therefore, instead of a coherent, a correlational method is used. The objective is the reduction of data to the form of an impulse seismogram. The user sees it as an array containing several thousands of numbers. In this case, no limitations are imposed on the obtained values of numbers as viewed from the model of wave propagation.

In the system in question, the monochromatic signals measurement of a complex frequency characteristic of a medium in several discrete values of frequency. The result of measurement on a separate frequency represents a vector sum of values of the frequency characteristics of separate wave impulses of a seismogram. The frequency characteristic of a seismogram has rather a complicated form due to interference. However the form of a frequency characteristic of each separate wave impulse is rather simple. The functions of the module and phase of this characteristic are smooth and can be approximated by functions with a limited number of parameters. Such a description of wave impulses is usually used in problems of investigation of dynamic properties of a medium.

The primary problem of the present method reduces to a single-valued decomposition of the two-dimensional vector S, describing a recorded signal, to components V, corresponding to separate waves delayed by the medium for various time intervals. The decomposition should satisfy the system of equations:

$$\sum_{p=1}^{m} |V_{p,k}| e^{i2\pi\{F_q t_p\}} = |S_q| e^{i\beta q}$$

Here

m is a specified number of waves,

t_p are unknown times of propagation (delays) [sec],

k is the number of parameters of the description of the dependence of V on frequency,

n is the number of sessions of sounding, ($n \geq m + k$, $q = 1 \div n$),

F_q are frequencies of monochromatic sounding [Hz],

$\{F_q t_p\}$ is a fractional part of the product $F_q t_p$.

The present system is satisfied to within errors of measurement of the signal vectors S. If the frequencies of sounding are not concentrated in a narrow band, the parametric description of the spectral characteristic of a wave packet is used. Parameters of this description are also considered to be unknowns. The result of measurements are assumed to contain a limited

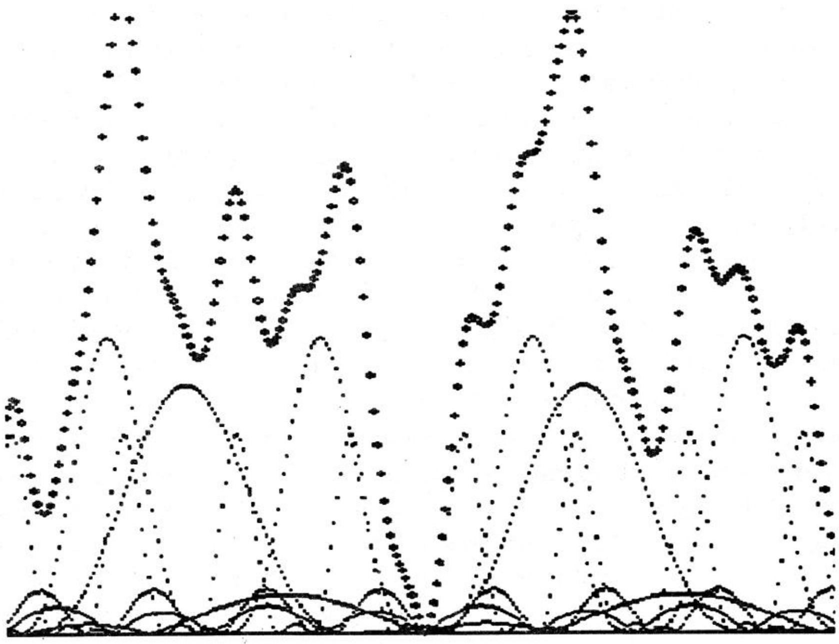

Figure 1

(according to a priori data) amount of "informative" waves, and the so-called wave code is absent. In real conditions this requirement is satisfied by a special fragmentation of sounding session with respect to time and frequencies.

The considered problem refers to a special case of decomposition of vectors to components. It is combined with the search for systems of basis vectors, which meet the requirements on convergence of the iterative procedure to one-valued solution. The solution to the system is also complicated by availability of periodic functions and multimodality of the functional to be minimized.

The character of the functional is shown in Figure 1. The cuts of the functional in six variables in arguments of the exponential function in the neighborhood of solution are shown, and the bold-fase points indicate to their sum. The acute global and a variety of local minima are distinguished. It is necessary to set the first approximation of arrival times t to within fractions of a period of sounding signals. The accuracy of results, obtained by usual methods of sounding, is insufficient for their use as first approximation. Thus there arised a necessity of the calculation (search) of the first approximation within the framework of the present task, without external data and additional sessions of sounding.

The search for the first approximations t reduces to the search for the integer parts of the desired time of propagation. This search becomes easier because it takes place on a set of integers and due to the fact that it can be done using a period of one frequencies of sounding. Each of discrete variants of selection is analytically recalculated to other frequencies. The field of the search can be decreased by a priori assignment of "approximate" values of wave arrival of the moments. However, sought for procedure is essentially complicated by the fact that it is necessary to find not one integer, but a few of them, for each of the waves separately. There are two complicating circumstances. First – the criterion of the search is convergence of rather a bulky procedure to the level of residual close to the measurement errors of signal vectors. Second it is required to find approximation to be distinguished from the exact one by the value, smaller than the quarter of a period of a sounding signal of the highest frequency.

Let us remark that the search for waves in a usual impulse seismogram also reduces to selection and search for the variants satisfying a certain feature. But in this case the search becomes easier because the seismogram contains obviously a clearly expressed indication to a wave packet – the rise of amplitude and "smooth phase variations". This indication is similarly available for visual and "computer" perception. In order that this feature be formed, the sounding signals (impulse or vibrational) with a continuous frequency spectrum are used. In our experiments there was no such an "expressed" indication. In our method of sounding, the role of such an indication is played by minimization of the functional up to the level of a random measurement error in signal vectors. The minimization is rather a bulky computing procedure, when we cannot do without computer. At earlier stages of computerization, algorithms of "manual" calculations and visual data analysis were realized in programs. The conventional scheme of creation of the source information and of experiment control was kept. In data processing with the dialogue support one can speak only about the accuracy of programmed calculations, but not about the accuracy of results to be obtained. In our case, the program does not require the dialogue with the user and accuracy of the characteristics of results of processing can be analytically expressed. We call "strong computerization" the full revision of principles of carrying out of experiments according to the possibilities of modern computing methods and tools, as well as modern representations about measuring information systems for the scientific research purpose.

Some insight into organization of the accelerated retrieval procedure using the principles of "soft calculations". For this purpose, a discrete set of orientations of vectors is introduced within the segment $[0, 2\pi]$ with a sufficiently small interval. To basis vectors, various combinations of orientations are assigned and decompositions of signal vectors to components are carried out. The appropriate polygons are constructed and the units of vector components are calculated. The "suitable" configurations of the polygons by simplified criteria are selected: the sum of modules of components should not exceed a defined threshold, and the modules of components associated with a certain wave should not considerably differ. Generally they must not essentially differ from values of the module of a priori description of the wave impulses spectrum. Such a preliminary "interpretation" decreases the area of the search from several thousands variants up to several tens. The remained combinations are checked by a rigid criterion – the condition of the iterative procedure convergence. Then the search for an integer of periods of a sounding signal is performed which is part of evaluations of the times of propagation of waves.

The program as a whole appears to be rather complicated. The applications will need the development of special versions. In particular, in the observation of variations of the tense state of a medium it is possible to exclude the retrieval procedures. The separation of overlapping wave impulses "according to the form" can be considered as independent task. When there are no models of wave impulses it is possible to use sounding signals with close frequencies and to reduce the description of the waveform to one parameter, i.e., amplitude, and "to approximate" wave impulses by delta - functions. This variant of the program is the most extensively studied. In this case some characteristic features of the algorithm have shown up.

The modified Newton method was applied to the solution of nonlinear equations. It has shown satisfactory convergence after introduction to it of some changes and additions. The simulation program was tested in the following conditions. Eight frequencies of sounding are prescribed:

5.1415	5.1425	4.950	4.954
6.010	6.018	7.000	7.011

The frequencies are selected pairwise close, thus to some extent improving the form of a functional.

Random errors of measurement of signal vectors were set by the generator of quasirandom numerical sequences with zero expectation and normal distribution.

The calculated mean square value was equal to 0.0069.

The values of "real" time of propagation of three waves (sec) are set:

110.300	112.610	112.700	with modules in conditional units
0.200	1.300	1.000	

The calculations were carried out in the mode of "statistical trials". More than 300 cycles of the calculation with various implementations of random errors of measurement were performed. This brought about errors of the results. In addition, the variability of parameters of waves was simulated. In the alternation of cycles of the calculation modifications to the delays of propagation of waves by the medium as well as changes of the waves amplitude were made:

−0.005	+0.002	0.000	for time,
+0.08	−0.10	+0.10	for the amplitude.

In this case, the initial approximations did not vary and made:

110.31	112.50	112.75
0.60	1.70	0.50

The task was put to define the relation between random errors of the obtained estimations of parameters and inessential changes occurring in the medium of propagation. The results are shown in Figure 2.

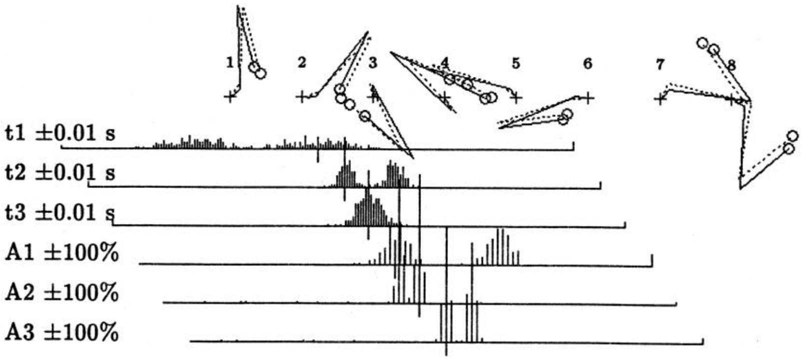

Figure 2

In the upper part, eight vector diagrams are represented. The sign '+' marks the beginning of coordinates. The circle shows the coordinate of the measured vector. Three wave vectors and a signal vector form the closed above-mentioned polygons. The dashed lines mean the introduced changes of the medium.

Based on results of many cycles of the calculation the histograms of errors of the calculated amplitudes and delays as related to "true" values were constructed. They are represented at the bottom of the figure. The histograms related to two variants of the state of a medium are not overlapped. This means, that the accuracy of results of sounding is sufficient for the detection of changes of the propagation time per units of milliseconds. The accuracy of measurement of amplitudes has appeared to be noticeably lower. This is one of the manifestations of the main advantage of phase systems of observation.

A close look at the histograms, enables us to detect on the axes separate points located far from the distribution mode. They are far beyond the scope of the scales, and their sums are represented at extreme points. The so-called threshold effect is intrinsic to all somewhat complicated systems of data processing. This is not the error of measurement, but the error of calculation due to "discharges" of random errors of measurement. Let as note that if the waves have small amplitude, the error of determination of time increases. On some frequencies, the vector to be measured modulo is less than the restored wave vectors. There are cases, when the module of the measured vector is equal to zero, but it does not bring about errors. This is only manifestation of interference. The present method uses interference, therefore the vector with zero module brings as much information, as any other. In usual practice of vibrational exploration the signal, suppressed by interference, is considered to be lost.

Let us pay an attention to the fact that the algorithm confidently separates waves (the second and the third) with close travel times.

This program was tested with an extended list of sounding frequencies, up to 20. In this case, the effect of averaging the measurement errors was exhibited, and the dispersion of errors of the result has essentially decreased. Let us remark that the residual error of the optimization process of approximation was controlled and did not exceed 1E−5, so that the calculations do not bear "eigen" additional errors. The connection between errors of measurements and results of calculations is determined by trigonometric relations and is rather transparent and available for the analytical description.

An important disadvantage of sounding of the medium by continuous monochromatic signals is in that it perceives by its parameters not only "informative" waves, but also all the consequent, including those caused in the seismic medium by sound waves. For elimination of this disadvantage we had nothing to do but to try the long-explored in the short-wave radio communication (and long-forgotten) "method of jumping frequency" for operation in ionospheric channels with multiradial propagation. In the course of transmission of telegraphic signals, the frequency "in jumps" is switched to the nearest "neighboring" value from a specific limited list and this process is repeated in cycles. The receiver synchronously does the same, but with allowance for propagation delays, selecting one or several waves. Such a fragmentation of radiation imposes special requirements on the control systems of vibrators. Until now automation of transition mode of a vibrator has not been required.

The choice of optimal frequencies of harmonic sounding is reduced to a well-known problem of the dense of stacking of multi-dimensional spheres in a multi-dimensional space. It is possible to present the arguments of trigonometric functions related to one of waves, as linear functions of the propagation time taken modulo 2π. If values of these functions are put on the orthogonal axeses and considered as coordinates of a certain point, then if the propagation time is changed, the point will fill in a multi-dimensional cube with parallel segments. The best values of frequencies correspond to the most dispersed layout of segments in the cube. Minimum distance between the neighboring segments determines a diameter of the cylindrical domain of uncertainty and the level of local minima of a functional (see Figure 1). When the error of measurements does not fall outside the limits of this area, the anomalous errors will be absent. The sizes of the adjacent spherical areas, located along a segment, express solvability. This geometric model of the algorithm and its more complicated versions allow to optimize phase sounding for obtaining a maximum accuracy and reliability with a specific level of the recording noise and to obtain estimations of its limiting information efficiency. This model enable us to calculate the law of distribution of anomalous errors and to use it for correction, as this error transfers a solution can most probably be transferred to the neighboring segment. The optimization of the present algorithm makes it closer to a theoretical limit of efficiency in the presence of noise fluctuations and by limiting radiation energy. Let us remark that the task of optimization of a system is considered to be very complicated.

A peculiar feature of the present algorithm is that the physical time "is not represented" in results of measurement of signal vectors. The time of propagation of waves is calculated as unknown parameter, the difference between conditional time moments of radiation and recording being calculated. Therefore the devices of synchronization of the radiating and of the receiving equipment of sounding should provide only the in-phase operation (matching of conditional labels of time) on the interval of a separate session. "Matching" of readings of hours to astronomical, zone and any other systems of exact time is not required.

The phase sounding method is based on the use of an interference picture of a wave field. It can be considered as extremely simplified version of the holographic system. For obtaining the table of numbers it is sufficient to record a wavefront at a limited number of points. One stationary seismic receiver is an use, and the measurement at separate points is performed by the front displacement for when the frequency of radiation varies.

This method can be used in various modes. With appropriate a priori data it is possible to reject the retrieval procedures in the mode of tracing medium variations. It is possible to program "sounding" of usual impulse seismograms and to obtain appropriate numerical estimations, needless to say if these seismograms are not strongly noised.

The efficiency of mathematical modeling in scientific researches is essentially increased, when not only qualitative, but also quantitative correspondence of mathematical models and experimental data is ensured. In this connection, the mathematical model of experimental installation should be apart of the general model of the research system. The present algorithm can be considered as an attempt to create a "transparent" version of the seismic sounding system to be available for the formal description.

We consider the proposed method as an example to show that there can be alternative approaches to the problem of vibrational sounding. In a way, this method is limiting, extreme. We see the prospects in the development of "intermediate" variants.