Bull. Nov. Comp. Center, Math. Model. in Geoph., 3 (1998), 97–103
© 1998 NCC Publisher

Complex amplitude method. Applications to vibroseismic signal analysis*

R.S. Salavatov

The main aspects of the complex amplitude method were developed for analyzing the modulated oscillations in radio engineering systems. Such signals representing the oscillations of the carrier frequency ω_0 modulated by a relatively low-frequency signal $\Omega_{\max}$ are characterized by the relationship $\Omega_{\max} \ll \omega_0$. Most generally vibroseismic signals do not meet this requirement though with some stipulations the complex amplitude method applied in many cases yields correct results and makes it possible to use the tools developed for radio engineering. It is shown that the method is promising, and application results of some algorithms are presented by an example of real vibroseismic signal processing.

1. Terms and definitions

In order to prevent variant reading and useless terminological discussion, we specify definitions and terms of this paper applied to vibroseismic signals. It should be mentioned that there are some other equivalent descriptions, for example, an analytical signal representation. The complex amplitude method is attractive due to its obviousness and decent inheritance. A popular method in electrical engineering is that of writing the fixed-frequency signals in the form

$$\widehat{U} = U_0 e^{j\omega_0 t}, \tag{1}$$

where U_0 is an amplitude of the signal with the frequency ω_0. If we assume that U_0 is a slowly varying function of time and write it as $\widehat{U}_0(t) = A(t) + jB(t)$, then (1) can be generalized to a new class of signals for which, together with the complex representation formalism, all computational approaches known for the functions of complex variables are valid:

$$\widehat{U} = \widehat{U}_0(t) e^{j\omega_0 t}. \tag{2}$$

Despite the similarity of (1) and (2), they have a substantial difference. In (2), $\widehat{U}_0(t)$ is called a complex amplitude, and $s(t)$ defined by the expression

*Supported by the Russian Foundation for Basic Research under Grant 98-05-65306, 98-05-65210.

$$s(t) = A(t)\cos\omega t - B(t)\sin\omega t. \tag{3}$$

is called a narrow-band signal. It is obvious that

$$s(t) = \mathrm{Re}(\widehat{U}_0 e^{j\omega_0 t}). \tag{4}$$

The quantity $|\widehat{U}|$ related to $A(t)$ and $B(t)$ by the expression

$$|\widehat{U}_0| = \sqrt{A^2(t) + B^2(t)}. \tag{5}$$

is usually interpreted as an envelope of the signal $s(t)$, the expression

$$\varphi(t) = \arctan\left(\frac{B(t)}{A(t)}\right) + \omega_0 t, \tag{6}$$

will be a phase of the signal (3). The signal phase consists of a slow envelope describing the phase, a component, and a fast addend generated by the "carrier frequency" ω_0. As applied to vibroseismic signals, ω_0 may be chosen inside the working frequency interval. For a sweep signal, ω_0 is approximately equal to an average frequency of the sweep range. These are the necessary properties of the envelope:

1. The envelope is greater or equal to zero everywhere.
2. The envelope never crosses the signal.
3. Derivatives of the signal and the envelope are equal at the points of contact.
4. The upper frequency σ in the envelope spectrum is substantially lower than the frequency ω_0 of the quick signal component.

The last-mentioned condition should be replaced by not so rigorous one.

4°. The envelope spectrum is limited in the frequency bandwidth.

We will demonstrate that the envelope defined in (5) coincides with Hilbert's envelope definition. Write the Hilbert-conjugate signal:

$$\hat{s}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} A(\tau)\cos(\omega_0\tau)\frac{d\tau}{t-\tau} - \frac{1}{\pi}\int_{-\infty}^{\infty} B(\tau)\sin(\omega_0\tau)\frac{d\tau}{t-\tau}. \tag{7}$$

To calculate (7), apply expansion of the functions $A(t)$ and $B(t)$ into series. Taking into account that $A(t)$ and $B(t)$ are slowly varying time functions, in other words, the values of derivatives at the point $\tau = t$ are small, in the expansion of integrands into Taylor's series about the point $\tau = t$ we may neglect all terms except the first ones. Consequently,

$$\hat{s}(t) = A(t)\sin\omega t + B(t)\cos\omega t. \quad (8)$$

It can be easily seen that

$$M(t) = \sqrt{s^2(t) + \hat{s}^2(t)} = \sqrt{A^2(t) + B^2(t)}, \quad (9)$$

which coincides with the envelope defined in (5). An expression for the phase:

$$\varphi(t) = \arctan\left(\frac{\hat{s}(t)}{s(t)}\right) = \arctan\left(\frac{B(t)}{A(t)}\right) + \omega_0 t \quad (10)$$

also coincides with (6). Application of the Hilbert transform for defining the envelope and the narrow-bandwidth signal yields the same results as the complex amplitude method does. As for the assumption on the negligibly small derivatives, we should note that the perfect Hilbert transformer is not feasible, all the transformations are possible in a sufficiently narrow signal bandwidth, and this fact meets the above assumption.

2. Results of experimental data processing obtained in the field experiment of 1997

In our experiments we used centrifugal vibrators CV-100, CV-40, and a hydroresonant vibrator HRV-50 with a ground effort of 100, 40, and 50 tons as the sources of vibroseismic waves. Several multichannel systems recorded data on three components. Further we discuss data processing results obtained with a VIRS system (ICM and MG). A satellite GPS system determined the geographical coordinates and performed the time reference. The record system was placed near the settlement Savvushki, at 375 kilometers from the source. For the processing we used data recorded at night between the 10th and 11th of August, 1997. The data are characterized by a higher noise level.

The system VIRS makes it possible to store data in 15 channels divided in three groups with respect to different components X, Y, and Z of the vibroseismic field. We should note that the data are represented by integers with a limited accuracy over a rather narrow range but the processing must be carried out over a wider dynamic range. It is convenient to perform the convolution of a sweep signal and a reference signal via spectral transformations on the basis of the known statement on the correspondence between the time sequence convolution and the Fourier transform of their multiplied spectral densities. Figure 1 represents a spectral density of the recorded data belonging to Z component. The data were obtained under generation of a sweep signal by the CV-100 vibrator in the range 5.5 to 8.5 Hz. The same figure shows a spectral density of a mathematical model

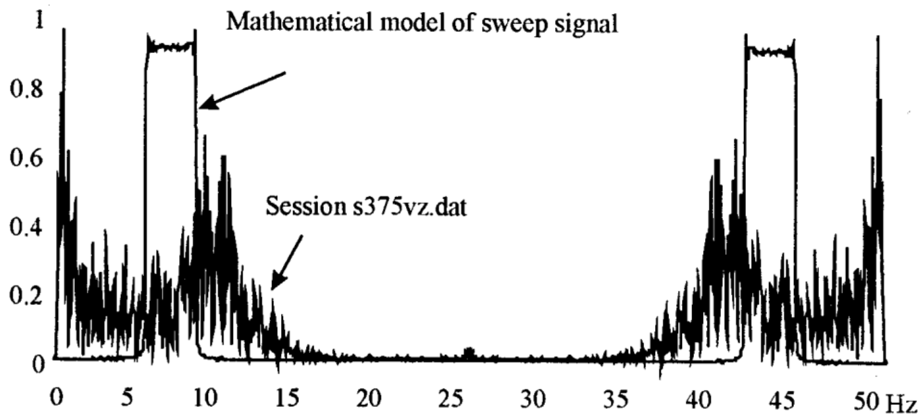

Figure 1

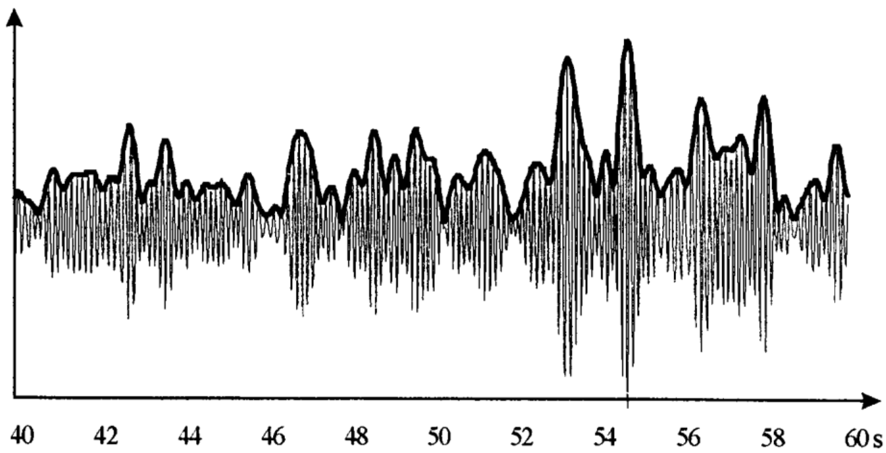

Figure 2

of the sweep signal. After inverse transformation of the spectrum of the mathematical sweep signal model multiplied by the conjugate spectrum of the signal Z-component we obtain the vibration seismogram (Figure 2). All the transformations are performed on an array of about $2.6 \cdot 10^5$ samples, where about $1.3 \cdot 10^5$ elements represent the initial data corresponding to about 2600 s of recording. To prevent the circular convolution effect, the array is padded with zeros.

A bold line in the figure shows the envelope obtained from (5). We can calculate $A(T)$ and $B(t)$ in view that

$$\begin{aligned} s(t) &= A(t)\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} - B(t)\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \\ &= \frac{1}{2}\Big[(A(t) + jB(t))e^{j\omega_0 t} + (A(t) - jB(t))e^{-j\omega_0 t}\Big], \end{aligned} \tag{11}$$

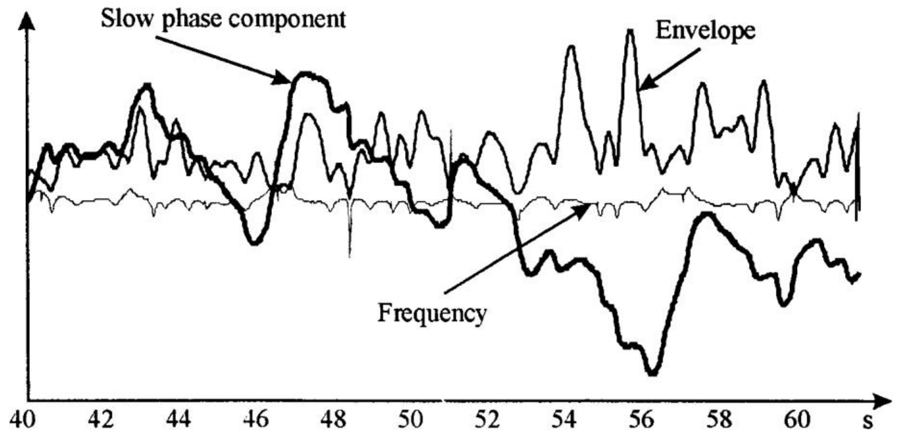

Figure 3

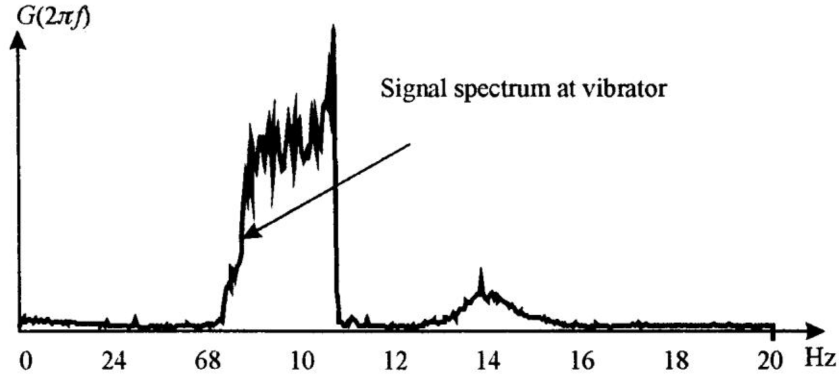

Figure 4

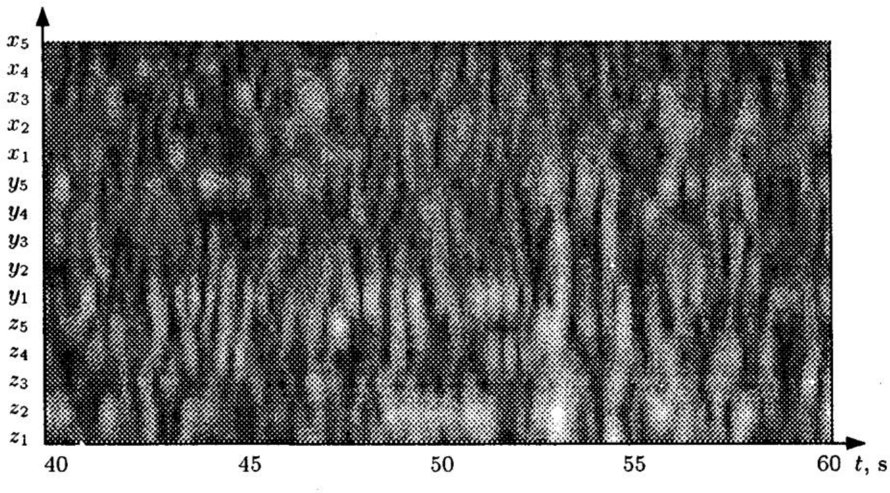

Figure 5

and, correspondingly, the spectral density

$$S(\omega) = \frac{1}{2}\left[G(\omega-\omega_0)e^{-j(\omega-\omega_0)} + G^*(-\omega_0-\omega)e^{-j(\omega+\omega_0)}\right], \tag{12}$$

where $G(\omega-\omega_0)$ is the spectral density for the envelope of the signal $s(t)$, and the complex amplitude $\widehat{U}(t) = A(t) + jB(t)$ is easily calculated as the inverse Fourier transform of the doubled $G(\omega-\omega_0)$. We merely equate to zero the complex-conjugate sequence and perform the inverse transform. The phase factor can be easily eliminated at will by applying a complex modulation to the time sequence, or it is quicker and more efficient to use the shift rule.

Figure 3 illustrates application of a signal spectrum component shift by 7.4 Hz. As a result the slow phase component in (6) has acquired the form convenient for interpretation after eliminating the term $\omega_0 t$. An average frequency of $\frac{5.5+8.5}{2} = 7$ Hz differs from that of the signal spectrum shift. This fact can be explained partly by the asymmetric form of spectral density of the vibrator radiation power. Figure 4 shows that in the low-frequency bandwidth the vibration efficiency is low. The contribution of a medium of seismic waves traveling is also evident. It is interesting to determine the weighted-mean frequency in the spectrum of a signal recorded at the vibrator:

$$f_{\text{mean}} = \frac{\int |G(2\pi f)|f\,df}{\int |G(2\pi f)|df} = 7.344 \text{ Hz}. \tag{13}$$

The weighted-mean frequency is much closer to the shift frequency correcting the quick phase component than the average frequency.

It is interesting to integrate data from the multichannel record systems in the same plot. Figure 5 shows an attempt to represent the data as a 2D plot of densities, where the intensity marks envelopes of different sensors and components.

A bright strip in the figure traces at the 53 second a wave of the first incursion. This manner of data presentation allows visual evaluation of data correlation in different channels.

3. Conclusion

Application of the complex amplitude method to vibroseismic data can sometimes make easier the interpretation of results. A real signal is usually noisy and the filling frequency ω_0 being absolutely noninformative prevents good visual perception of processing results. This property of the filling frequency is a result of the evident statement: if the signal spectrum is bounded below, then we may transfer the signal spectrum without loss in order to reduce the upper frequency. The signal will preserve the same envelope shape and slow phase component rather than the shape and full phase of the signal. Therefore, the complex amplitude described completely by the envelope and the slow phase component (5) and (6), respectively, is an invariant.

Reducing the upper frequency we can also decrease the digitization frequency. Upon shifting the spectrum by 5.5 Hz, we can digitize a real sweep signal with a bandwidth of 5.5 to 8.5 Hz by a 6 Hz frequency without loss of information and reduce the amount of data 8 times and more, complete restoration of the initial signal is always possible. Modern data storages of several Gbytes practically avoide the necessity for data compression algorithm, whereas the data transfer via limited-capacity communication lines is more effective with the use of algorithms reducing the amount of data without loss of information content.

Acknowledgements. The author would like to express gratitude to B.M. Pushnoi for fruitful discussions and V.V. Kovalevsky for kindly offered materials for processing without which this paper would lose the firm odor of geophysical problems.

References

[1] B.M. Pushnoi, *On seismogram phases*, Proc. Comp. Center SB RAS, Ser. Mathematical Modeling in Geophysics, Novosibirsk, 1994, 32–40.

[2] V.V. Kovalevsky, B.M. Pushnoi, and R.M. Salavatov, *Estimation of measurement errors of vibroseismic signals*, Problemno-orientirovannie vychislitelnye kompleksy, Novosibirsk, VC, 1996, 3–16.

[3] *Applications of Digital Signal Processing*, ed. A.V. Oppenheim, Prentice-Hall, New Jersey, 1978.

[4] L.A. Zalmanzon, *Fourier, Walsh, and Haar Transforms and their Application in Control, Communication and Other Fields*, Nauka, Moscow, 1989.

[5] W.McC. Siebert, *Circuits, Signals, and Systems*, McGrow-Hill, New York.

[6] S.L. Marple, *Digital Spectral Analysis with Applications*, Prentice Hall, New Jersey, 1987.

[7] L.R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*, Prentice Hall, New Jersey, 1975.