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Some results of autowave modelling by cellular neural network

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Two-layer cellular neural network as a simulating model of wave processes is considered. The wave speed dependence from an intralayer connection weight is presented.

1. Introduction

Complexity of different phenomena being investigated in nonlinear dynamical systems has achieved now such a level when application of traditional mathematical models becomes rather difficult even when powerful computing systems are used. In this situation some other approaches to effective studying complex processes may be interesting. Among them a particular attention should be paid to highly parallel computing models which allow to simulate such processes with minimal time spending.

Nowadays many interesting results in simulating different complex phenomena were obtained using Cellular Neural Networks (CNN) proposed by L.O. Chua and L. Yang [1]. Being a model of parallel information processing CNNs have the following properties expected to be essential for the simulating purpose:

1. A structure of a CNN can be used as a discrete representation of different media properties.
2. Interactions between elements of the CNN are local and can have different values which make possibility of simulation of media with a wide range of properties.
3. Each cell of a CNN is a nonlinear system, so, many nonlinear properties may be simulated.

Today it is possible to select two different ways in application of CNN to different complex nonlinear processes simulation:

1. Synthesis of CNNs approximating a solution to partial differential equations [2] both for fully defined equations as its discrete representation, and in the case when only a rough knowledge of the PDE form and several values of the system output for a certain initial condition

are available [3]. Different learning algorithms are used in the latter case.

2. Synthesis of CNNs simulating complex distributed processes on the base of knowledge about local properties of the media and their variations in space and time. Here CNN is a simulating model which can be used for different computational experiments.

Up to day many interesting phenomena have been simulated by different CNN models. In this paper we concentrate on the autowave as one of the base phenomenon observed in different nonlinear distributed systems and present results of investigation of a two-layer CNN as a model of autowave processes. In the next section the general formal model of a CNN and the model being discussed are described. In Section 3 the problem of the CNN parameters choosing are discussed. Section 4 contains some simulation results.

2. Formal model

Cellular Neural Network is a regular array of cells usually of 1-, 2-, or 3-dimension. Each cell has an input u, a state x, an output y and weighted connections only with neighbor cells. The cell neighborhood is determined by a cloning template which is identical for all cells of the network.

In the case of 2-D CNN the following state equations [1] are used

$$\frac{dx_{ij}}{dt} = g(x_{ij}) + \sum_{k,l \in N_r(i,j)} a_{kl} y_{ij,kl} + \sum_{k,l \in N_r(i,j)} b_{kl} u_{ij,kl} + I_{ij} \tag{1}$$

for continuous time and

$$x_{ij}(t+1) = \sum_{k,l \in N_r(i,j)} a_{kl} y_{ij,kl}(t) + \sum_{k,l \in N_r(i,j)} b_{kl} u_{ij,kl}(t) + I_{ij} \tag{2}$$

for discrete time.

In both equations the output is a nonlinear function σ of the corresponding state:

$$y_{ij} = \sigma(x_{ij}).$$

In equations (1), (2), i and j are the coordinates of the cell in a lattice, the function g determines an influence of a state value x_{ij} on its increment (1). The coefficients a_{kl} are entries of the matrix $\boldsymbol{A}$, called a cloning template matrix. Values of these entries determine the weights of adjacent cell outputs when a cell (i,j) next state value is calculated. In the same way the entries b_{kl} of the matrix $\boldsymbol{B}$ determine weights of adjacent cell inputs. The constant member I_{ij} defines a bias of the state value. The set $N_r(i,j)$ includes pairs of coordinates of those cells which belong to the neighborhood of a cell (i, j), r being the neighborhood size. The variables $u_{ij,kl}$ and $y_{ij,kl}$ take values of neighborhood inputs and outputs respectively, including the input and output value of the cell (i, j), and constitute the matrices of inputs $\boldsymbol{U}_{ij}$ and outputs $\boldsymbol{Y}_{ij}$.

Let us enumerate all elements of the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{U}_{ij}$, and $\boldsymbol{Y}_{ij}$ and define obtained vectors as $\mathbf{A}'$, $\boldsymbol{B}'$, $\boldsymbol{U}'_{ij}$, and $\boldsymbol{Y}'_{ij}$ respectively. If we take $g(x_{ij}) = -x_{ij}$ according to [1], then equations (1), (2) may be written as follows:

$$\frac{dx_{ij}}{dt} = -x_{ij} + \boldsymbol{A}'\boldsymbol{Y}'_{ij} + \boldsymbol{B}'\boldsymbol{U}'_{ij} + I_{ij}, \tag{3}$$

$$x_{ij}(t+1) = \boldsymbol{A}'\boldsymbol{Y}'_{ij} + \boldsymbol{B}'\boldsymbol{U}'_{ij} + I_{ij}. \tag{4}$$

It is well-known that almost all autowave processes arise in two-component systems with a mathematical point model described by a system of two first-order nonlinear partial differential equations [7]. Moreover, this system of equations is autonomous and contains a sum of a nonlinear function and a diffusion member. Therefore a CNN model of autowave processes may be presented on the base of general model description (3) as a two-layer CNN with both intralayer and interlayer connections. The processes being considered are supposed to be independent from any external influence, thus each equation (3) should have no external input members, i.e., $\boldsymbol{B}' = 0$. With such assumptions it is possible to use a two-layer CNN with simplified connection structure [5], where each cell in one layer is connected to the corresponding cell in the other one only by a single weighted connection. Such two-layer CNN is a set of interconnected interlayer neuron pairs. *Each* neuron pair is described by the system of equations:

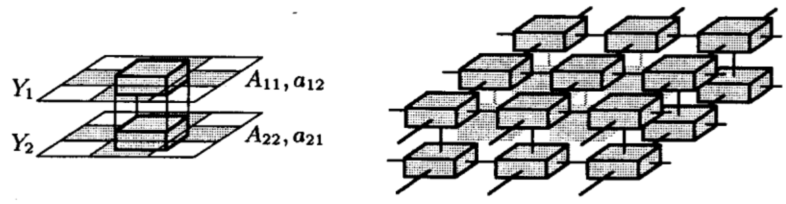
$$\begin{cases} \dfrac{dx_{1,ij}}{dt} = -x_{1,ij} + \boldsymbol{A}'_{11}\boldsymbol{Y}'_{1,ij} + a_{12,ij}y_{2,ij} + I_{1,ij}, \\ \dfrac{dx_{2,ij}}{dt} = -x_{2,ij} + \boldsymbol{A}'_{22}\boldsymbol{Y}'_{2,ij} + a_{21,ij}y_{1,ij} + I_{2,ij}, \end{cases} \tag{5}$$

and

$$y_i = \frac{1}{2}\left(|x_i + 1| - |x_i - 1|\right) \tag{6}$$

where $a_{12,ij}$ and $a_{21,ij}$ are interlayer connection weights, $y_{2,ij}$ and $y_{1,ij}$ are outputs of the pair neurons in each layer. Figure 1a shows the interconnection structure of a neuron pair according to system (5). Figure 1b shows the simple representation of the whole CNN interconnection structure.

System (5) is the formal representation of a cell with coordinates (i, j) which constitutes the two-layer CNN being investigated. Operation of the cell may be represented in a simple way by a second order differential equation when there are no interactions between neurons in each of two layers



a b

Figure 1. The two-layer CNN interconnection structure

and y_{ij} (x_{ij}) is a linear function. The cell behaviour becomes rather complex when all nonzero parameters of system (5) are used and y_{ij} (x_{ij}) is a nonlinear function. The last case is less studied and requires solving the problem of choosing parameters for system (5). Because of its complexity, this is the main problem of synthesis of the CNN-model for dynamical systems simulating.

3. CNN parameters

The formal representation of a two-layer CNN in the form of equations (5) allows to select two different factors defining the dynamics of CNN:

1. Parameters of the neuron pair with the coordinates (i, j) which are defined by a nonlinear function, by the coefficients ($a_{21,ij}$ and $a_{12,ij}$) of the pair interneuron interaction, by weight of self output value and by bias values $I_{1,ij}$, $I_{2,ij}$.
2. Weights of intralayer connections, i. e., the values $a_{11,kl}$ and $a_{22,kl}$ of the matrices $\boldsymbol{A}_{11}$ and $\boldsymbol{A}_{22}$.

Therefore, the possible approach to control the CNN behavior consists both of determining the dynamical properties of the neuron pair, namely the nonlinear function $y = \sigma(x)$, coordinates, quantity, quality of equilibrium points and trajectory form on the neuron pair phase plane, and of the diffusion parameter value defining interaction between them.

To have possibility of independent consideration of the neuron pair dynamics, we should distinguish the neuron pair equations in the form:

$$\begin{cases} \dfrac{dx_{1,ij}}{dt} = -x_{1,ij} + k_{11,ij}y_{1,ij} + a_{12,ij}y_{2,ij} + I_{1,ij}, \\ \dfrac{dx_{2,ij}}{dt} = -x_{2,ij} + k_{22,ij}y_{2,ij} + a_{21,ij}y_{1,ij} + I_{2,ij}, \end{cases} \tag{7}$$

where $k_{11,ij}$ and $k_{22,ij}$ are interlayer connection weights, $y_{1,ij}$ and $y_{2,ij}$ are defined by (6).

The neuron pair dynamical properties analysis can be done on the base of the well-known methods of the second order dynamical systems theory [6]. In particular, the phase plane of a neuron pair can be considered where phase variables take the values of neuron state variables $x_{1,ij}$ and $x_{2,ij}$. The quasi-linear function (6) devides the phase plane onto nine linear subspaces in which an analysis of linearized system (7) may be performed [5].

The following conditions for the neuron pair phase plane should be met to have a propagation of ring-like autowave with one front in the two-layer CNN:

1. There should be a stable closed orbit as a necessary and sufficient condition [4] to obtain an appropriate oscillation behaviors of the pair.
2. There should be at least one equilibrium to have the initial state of the CNN being unchanged without a wave formation offset which initiate wave formation and propagation through the CNN.

The first condition can be satisfied if we choose parameters $k_{11,ij}$, $k_{22,ij}$, $a_{12,ij}$ and $a_{21,ij}$ as follows [5]:

$$k_{11,ij} = k_{22,ij} = 1 + \mu, \quad a_{12,ij} = -a_{21,ij} = -s, \quad 0 < \mu < s.$$

In particular, we can take $s = 1.0$ and $\mu = 0.7$.

The CNN properties with bias values $I_{1,ij} = -I_{2,ij} = -0.3$ was extensively studied in [5]. Such a neuron pair has two stable equilibrium points on its phase plane. Here we also consider the neuron pair with only one stable equilibrium by having $I_{1,ij} = -0.2$ and $I_{2,ij} = 0.3$. Figure 2 shows the corresponding phase planes of neuron pairs.

It should be pointed out here that in the case of two equilibrium points the single neuron pair has no closed orbit because being started from one of them, the neuron pair always has stable equilibrium point on its way. To have a closed trajectory a neuron pair must have additional offset when passing through this point.

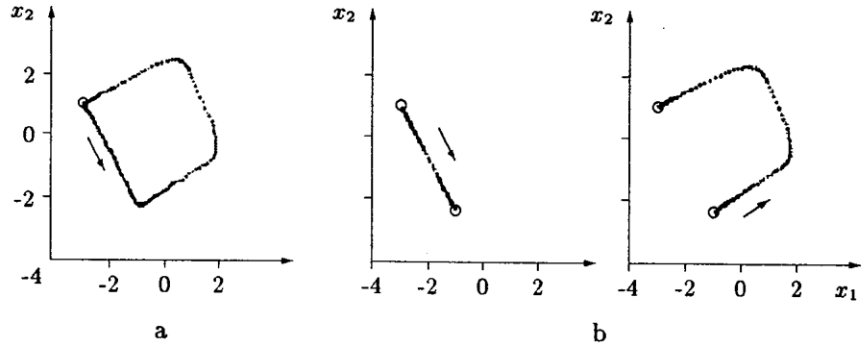


Figure 2. The phase plane of neuron pair: a – closed trajectory for one-point neuron pair, b – two parts of trajectory of two-point neuron pair

Besides of the properties of a neuron pairs, the propagation of autowave in the CNN is determined by values of intralayer communication weights, defined by elements of the matrices $\boldsymbol{A}_{11}$ and $\boldsymbol{A}_{22}$ referred to as diffusion coefficients. Let us denote them as D_1 and D_2. Then the matrices can be written as follows:

$$\boldsymbol{A}_{ii} = \begin{pmatrix} 0 & D_i & 0 \\ D_i & -4D_i + 1.7 & D_i \\ 0 & D_i & 0 \end{pmatrix} \tag{8}$$

where $i = 1, 2$.

The influence of the diffusion coefficients values ranged from 0.005 to 1.4 are shown in the experimental results section.

4. Experimental results

The experimental results presented further are based on computer simulation of a two-layer CNN with 150×150 neurons in each layer and with parameters determined in the previous section.

Two types of autowave processes have been simulated: the propagating pulse and the propagating front.

The simulating process is divided into two stages: 1) initializing of the CNN and 2) observing the evolution of cells outputs and states. In the initialization stage all cells of the CNN are settled into equilibrium condition except one (central) cell. The state and output values of this cell are shifted from the equilibrium values in such a way as to have instability propagating through the CNN. This instability initiates the autowave formation and provides its propagation through the CNN.

The required initial wave formation values of a central cell can be determined by the analysis of equilibrium points properties on the neuron pair phase plane. Each point being considered lies on the boundary between two adjacent linear subspaces and is stable in one of them and unstable in the other. Therefore to have propagating instability it is necessary to shift the cell state values in the unstable subspace. The neuron pair being considered has one equilibrium point with coordinates $(-2.9, 1.0)$ or two equilibrium points with coordinates $(-3.0, 1.0)$ and $(-1.0, -2.4)$. In the first case such shifting leads always to a propagating pulse formation while both a propagating pulse and a propagating front arise in the second one. In the experimental results presented here the central cell initial state values are put to $(-0.5, -0.9)$ for the one-equilibria neuron pair and to $(-0.5, -0.9)$ and $(-0.9, -2.0)$ for the two-equilibria one. In Figure 3 the CNN output pattern and the state values of the CNN middle cell row for each type of autowave processes are presented. In the output pattern, the state value

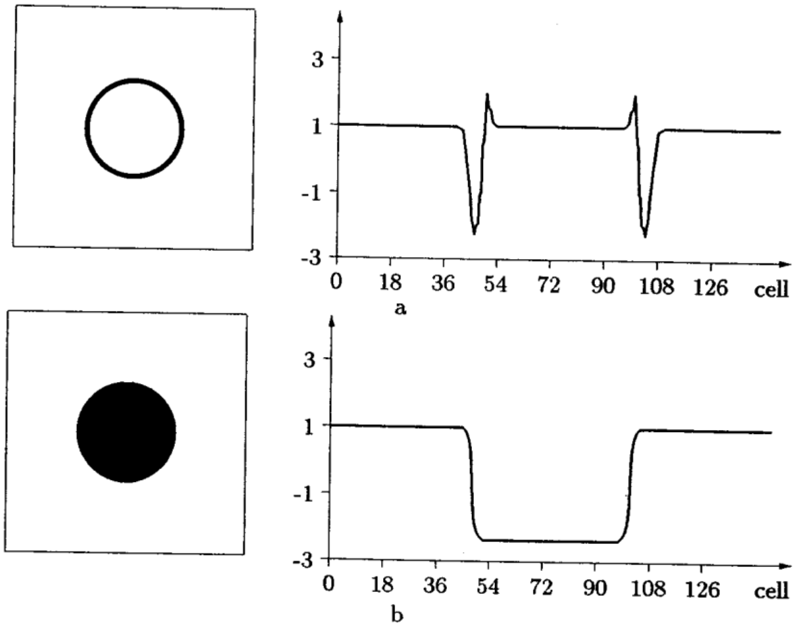


Figure 3. Outputs and cell states of the CNN for different types of autowave processes: a – the propagating pulse, b – the propagating front

equal to -1.0 is represented by black color and the state value equal to 1.0 is represented by white color.

The essential property of the model is "transparency" of the CNN boundaries, i. e., the absence of wave reflection from the boundaries of the CNN. It is achieved by using zero-flux boundary condition realized as inactivity of boundary cells of the CNN. The annihilation of two colliding autowave pulses has also been obtained as illustration of autowave properties of the processes being simulated. In Figure 4 these two properties are illustrated.

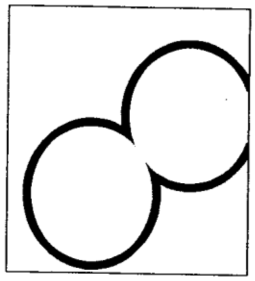


Figure 4. Annihilation of two colliding autowaves and "transparency" of the CNN boundaries

The particular attention was paid to the autowave speed dependence on the diffusion coefficient value. The results obtained for the CNN with different number of equilibrium point on the neuron pair phase plane are illustrated in Figure 5.

The speed of autowave is measured as a number of CNN cells between the central cell and some other fixed one related to the number of time steps until the selected type of the fixed neuron pair behavior has observed. There are two selected behavior types used to make the measurements, namely "start" (Figure 5a – dashed line, Figure 5b – dotted line for $x_{10} = 0.9$, $x_{20} = 2.0$

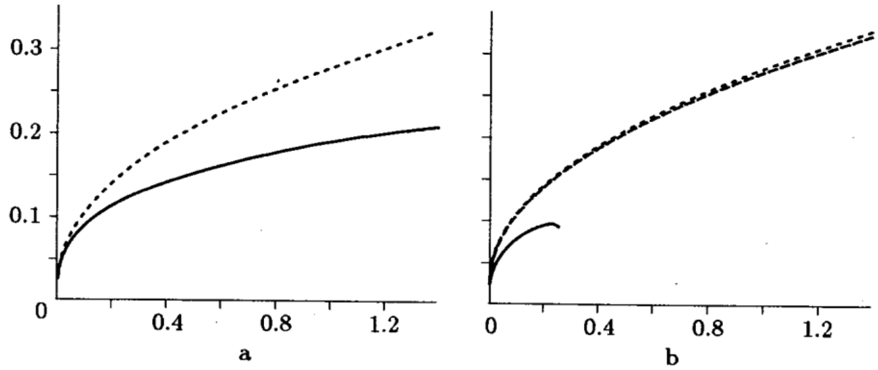


Figure 5. The diffusion dependence of the autowave speed for two types of the neuron pair phase plane: a – with one equilibrium point, b – with two equilibrium points

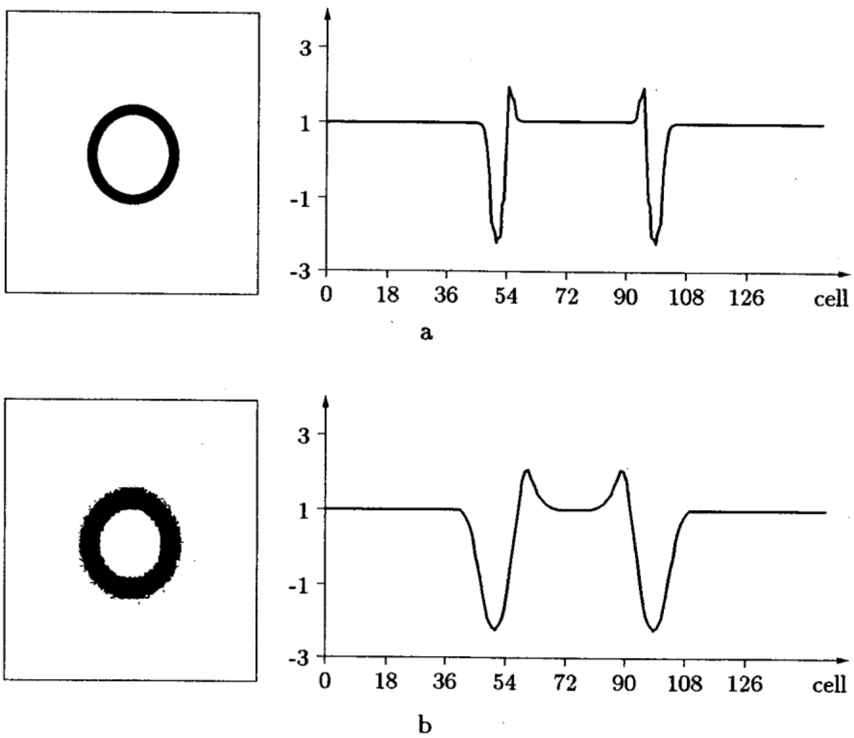


Figure 6. The influence of a diffusion value on the autowave properties: a – when $D = 0.2$, b – when $D = 1.2$

and dashed line for $x_{10} = 0.5$, $x_{20} = 0.9$), when the state values of a neuron pair begin to change from equilibrium value and "finish" (Figure 5a – solid line, Figure 5b – solid line for $x_{10} = 0.9$, $x_{20} = 2.0$ and for $x_{10} = 0.5$, $x_{20} = 0.9$) when the values return to the initial states. It is easy to see that the CNN model with the neuron pair having one stable equilibrium point on its phase plane allows to obtain propagating pulse with all values of the diffusion coefficient (see Figure 5a) while the model with two stable equilibria has only small range of these values (see Figure 5b).

In Figure 6, the influence of diffusion coefficient value on the autowave properties (the wave boundary sharpness and the wave form) is presented. The value was put to 0.2 for Figure 6a and to 1.2 for Figure 6b. As a result, an increasing of diffusion coefficient leads to the autowave boundaries being more smooth.

5. Conclusion

The two-layer cellular neural network is considered in this paper as a model which allows to simulate autowaves as a base phenomena in different complex processes. Simulation results of two different autowave types with different CNN neuron pair properties are presented. The main attention is paid to the diffusion dependence of wave propagation speed. The simplicity of a parallel realization of this model defines an effective application of the CNN model in simulating of complex processes in large distributed systems with minimal time spending.

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