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Application of eddy-resolving models for penetrating turbulent convection in the atmosphere and deep lakes*

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This paper presents the results of the numerical research into the generation of the free thermal convection in the atmosphere and lakes on the basis of the mesoscale eddy-resolving models. The surface values of the heat flux into the atmosphere and the temperature of the lake surface are taken from observational data. The convective mixing processes in both natural media are analyzed.

1. Introduction

The penetrating turbulent thermal convection is an important physical mechanism providing the mass-energy transfer in the atmospheric boundary layer (ABL) and the upper mixed layer (UML) of the deep lake [1]. Due to the complexity of the phenomenon and a deficit of the natural observations data [2, 3], mathematical modeling appears an effective tool of studying both individual thermals and ensembles of coherent structures (CS) of the convective nature [4–6].

The method of large eddies simulation (LES) is applied in this paper for the direct determinate description of penetrating convection, taking place in unstable stratification of near-surface layers of air and water. Examples of the formation of such a stratification in natural conditions with simulations formation of ensembles of the penetrating turbulent convection in both natural media are processes of the night-time cooling in the diurnal cycle of the heat exchange between the lake and the atmosphere in the spring–summer period [7–9] and processing of cooling of deep lakes late in autumn and in winter [9]. In these cases, the cooling intensity of the near-surface layers of air and water is directly dependent on the horizontal transfer of the cold air masses from the coastal zone to the warm lake (land breezed and frontal advection in spring and in summer, transport of severely cooled air from the land to the lake at the time preceding freezing-over). Within the present model this process is parametrically described with the help of setting the cold air mass advection.

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2. Equations for the atmospheric model

In the convective ABL there are three main types of interaction, important in terms of energy, processes of different scales; (a) the ordered mean motions; (b) coherent structures ("large eddies") with characteristic size, comparable to the boundary layer thickness; (c) isotropic small-scale (subgrid) turbulence arising, due to the action of buoyant forces and shear of velocity.

Let us introduce the rectangular Cartesian coordinate system (x, y, z), where the axis z is directed vertically upward, and the level $z = 0$ coincides with the surface of the "water-air" interface. Let us present the sought for vector-function $\phi = (u, v, w, \theta, \pi)$, where u, v, w are components of the velocity vector along the axis x, y, z; θ is potential temperature, π is an analogy of pressure as, a sum

$$\phi = \Phi + \phi', \tag{1}$$

where the fields $\Phi(z,t) = (U, V, 0, \Theta, \Pi)$ and the fields $\phi' = (u', v', w', \theta', \pi')$ describe the processes (a) and (b), respectively, [10].

Let L_x, L_y stand for horizontal sizes of the domain, where a non-stationary penetrating convection as an ensemble of spontaneously firming thermals (CS) is generated, and let us assume the periodicity of the processes along x, y. Note, that discretization of the domain with $L_x = L_y = 10$ km on the grid point with 128×128 nodes, admits the realization of an ensemble containing up to 100 convective formations of different size and intensity. The periodically condition is associated not with an individual thermal, but with the domain as a whole and has a sense of statistical homogeneity of processes for $x > L_x$, $y > L_y$, respectively. In this case, the problem of boundary conditions with respect to x, y is simultaneously solved.

Substituting representation (1) to the equations of mesoscale atmospheric dynamics [11], let us average them in the horizontal plane, making use of Reinold's rules:

$$\overline{\phi'} = 0, \quad \overline{\phi} = \Phi, \quad \text{where} \quad \overline{f} = \frac{1}{L_x}\frac{1}{L_y}\int_0^{L_x}\int_0^{L_y} f\,dx\,dy.$$

As a result we arrive to the system of equations describing the mean current in the ABL:

$$\begin{aligned}
\frac{\partial U}{\partial t} &= l(V - V_G) + \frac{\partial}{\partial z}K\frac{\partial U}{\partial z} - \frac{\partial}{\partial z}\overline{uw},\\
\frac{\partial V}{\partial t} &= -l(U - U_G) + \frac{\partial}{\partial z}K\frac{\partial V}{\partial z} - \frac{\partial}{\partial z}\overline{vw},\\
\frac{\partial \Theta}{\partial t} + \mathrm{Adv}_\Theta &= \frac{\partial}{\partial z}K_T\frac{\partial \Theta}{\partial z} - \frac{\partial}{\partial z}\overline{\theta w},
\end{aligned} \tag{2}$$

where U_G, V_G are the geostrophic wind components caused by the baric gradient in the free atmosphere, l is the Coriolis parameter, K is the coefficient of the vertical turbulent exchange of subgrid scale, $K_T = K/\mathrm{Pr}$, Pr is the Prandtle number in the ABL, and Adv_Θ is the function describing in the parametric form the temperature advection. Here and below primes at convective deviations are omitted.

Let us simulate the mean current subgrid-scale turbulence in the ABL on the basis of equations of the semi-empirical turbulence theory:

$$\frac{\partial b}{\partial t} = \frac{\partial}{\partial z}K\frac{\partial b}{\partial z}+KJ-\epsilon, \quad \frac{\partial \epsilon}{\partial t} = \frac{1}{\sigma}\frac{\partial}{\partial z}K\frac{\partial \epsilon}{\partial z}-c_1\frac{\epsilon}{b}KJ-c_2\frac{\epsilon^2}{b}, \quad K = c_k\frac{b^2}{\epsilon}, \tag{3}$$

where b is the kinetic turbulence energy (KTE), ϵ is the dissipation rate, $J = (U_z^2 + V_z^2) - \lambda\Theta_z/\mathrm{Pr}$ is a source generating the KTE, λ is the buoyancy parameter, c_k, c_1, c_2, σ are empirical constants.

The system of equations for the description of the mesoscale convective processes (b) in the ABL is obtained by the component-by-component subtraction (2) from the respective equations of the original system:

$$\begin{aligned}
&\frac{du}{dt} + w\frac{\partial U}{\partial z} = -\frac{\partial \pi}{\partial x} + lv + \mu\Delta u + \frac{\partial}{\partial z}K\frac{\partial u}{\partial z} + \frac{\partial}{\partial z}\overline{uw},\\
&\frac{dv}{dt} + w\frac{\partial V}{\partial z} = -\frac{\partial \pi}{\partial y} - lu + \mu\Delta v + \frac{\partial}{\partial z}K\frac{\partial v}{\partial z} + \frac{\partial}{\partial z}\overline{vw},\\
&\frac{dw}{dt} = -\frac{\partial \pi}{\partial z} + \lambda\theta + \mu\Delta w + \frac{\partial}{\partial z}K\frac{\partial w}{\partial z},\\
&\frac{d\theta}{dt} + w\frac{\partial \Theta}{\partial z} = \mu\Delta\theta + \frac{\partial}{\partial z}K_T\frac{\partial \theta}{\partial z} + \frac{\partial}{\partial z}\overline{w\theta},\\
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
\end{aligned} \tag{4}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (U+u)\frac{\partial}{\partial x} + +(V+v)\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

is the operator of an individual derivative, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, and μ is the coefficient of lateral turbulence of subgrid scale.

3. Equations of the lake model

Separation of the processes in the lake as mean and conditioned CS [3] can be done in the same manner as this has been done for the ABL. In this case, the characteristic spatial scales of the hydrological CS would be, at least, two orders smaller than these scales in the ABL. Using the splitting procedure in analogy with the ABL as applied to the lake, we obtain a system of equations for the mean homogeneous current in the UML having the characteristic vertical scale $\sim$ 10 m.

$$\begin{aligned} \frac{\partial \tilde{U}}{\partial t} &= l\tilde{V} + \frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{U}}{\partial z} - \frac{\partial}{\partial z}\widehat{\tilde{u}\tilde{w}}, \\ \frac{\partial \tilde{V}}{\partial t} &= -l\tilde{U} + \frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{V}}{\partial z} - \frac{\partial}{\partial z}\widehat{\tilde{v}\tilde{w}}, \\ \frac{\partial T}{\partial t} &= \frac{\partial}{\partial z}\tilde{K}_T\frac{\partial T}{\partial z} + \beta_s\frac{\partial R_s}{\partial z} - \frac{\partial}{\partial z}\widehat{\tilde{w}\tilde{T}}, \end{aligned} \qquad (5)$$

where T is temperature of the mean current, $\tilde{T}$ are temperature deviations (here and further the values with tilde correspond to the notations of the ABL, but refer to the UML). The averaging operator $\widehat{f}$ is similar in its structure to the above-introduced operator $\overline{f}$, the sizes of the averaging domain in the UML $\tilde{L}_x, \tilde{L}_y$ are also subject to determination. The last term in the heat flux equation in (5) describes the direct solar radiation flow R_s with an absorption fraction β_s.

The equations b–ϵ of the model for the lake are of the form

$$\frac{\partial \tilde{b}}{\partial t} = \frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{b}}{\partial z} + \tilde{J}\tilde{K} - \tilde{\epsilon}, \quad \frac{\partial \tilde{\epsilon}}{\partial t} = \frac{1}{\tilde{\sigma}}\frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{\epsilon}}{\partial z} - \tilde{c}_1\frac{\tilde{\epsilon}}{\tilde{b}}\tilde{J}\tilde{K} - \tilde{c}_2\frac{\tilde{\epsilon}^2}{\tilde{b}}, \quad \tilde{K} = \tilde{c}_k\frac{\tilde{b}^2}{\tilde{\epsilon}}, \qquad (6)$$

where $\tilde{J} = (\tilde{U}_z^2 + \tilde{V}_z^2) - g\beta_T T_z/\tilde{\Pr}$, g is the acceleration due to gravity, $\beta_T(T)$ is the coefficient of the water thermal extension.

The system of equations for the description of CS in the lake has the form:

$$\begin{aligned} &\frac{\tilde{d}\tilde{u}}{dt} + \tilde{w}\frac{\partial \tilde{U}}{\partial z} = -\frac{1}{\tilde{\rho}_0}\frac{\partial \tilde{p}}{\partial x} + l\tilde{v} + \tilde{\mu}\Delta\tilde{u} + \frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{u}}{\partial z} + \frac{\partial}{\partial z}\widehat{\tilde{u}\tilde{w}}, \\ &\frac{\tilde{d}\tilde{v}}{dt} + \tilde{w}\frac{\partial \tilde{V}}{\partial z} = -\frac{1}{\tilde{\rho}_0}\frac{\partial \tilde{p}}{\partial y} - l\tilde{u} + \tilde{\mu}\Delta\tilde{v} + \frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{v}}{\partial z} + \frac{\partial}{\partial z}\widehat{\tilde{v}\tilde{w}}, \\ &\frac{\tilde{d}\tilde{w}}{dt} = -\frac{1}{\tilde{\rho}_0}\frac{\partial \tilde{p}}{\partial z} + g\beta_T\tilde{T} + \tilde{\mu}\Delta\tilde{w} + \frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{w}}{\partial z}, \\ &\frac{\tilde{d}\tilde{T}}{dt} + \tilde{w}\frac{\partial T}{\partial z} = \tilde{\mu}\Delta\tilde{T} + \frac{\partial}{\partial z}\tilde{K}\frac{\partial \tilde{T}}{\partial z} + \frac{\partial}{\partial z}\widehat{\tilde{w}\tilde{T}}, \\ &\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} = 0, \end{aligned} \qquad (7)$$

where

$$\frac{\tilde{d}}{dt} = \frac{\partial}{\partial t} + (\tilde{U} + \tilde{u})\frac{\partial}{\partial x} + (\tilde{V} + \tilde{v})\frac{\partial}{\partial y} + \tilde{w}\frac{\partial}{\partial z},$$

$\tilde{p}$ is pressure, $\tilde{\rho}_0$ is the mean value of water density.

4. Boundary and initial conditions

We define the vertical structure of the domain in the following way: H is the upper boundary of the ABL, h is thickness of near-water air layer, for which the assumption of constant turbulent fluxes is valid, $z = 0$ is the surface of the "water-air" interface, $z = \tilde{H}$ is the lower boundary of the active layer in the lake.

For the mean current equations in the ABL (2), (3) and the UML (5), (6) let us set the following conditions:

$$U = U_G, \quad V = V_G, \quad \frac{\partial \Theta}{\partial z} = \gamma_H, \quad b = 0, \quad \frac{\partial \epsilon}{\partial z} = 0 \quad \text{at } z = H; \tag{8}$$

$$K\frac{\partial U}{\partial z} = c_u|\vec{U}|U, \quad K\frac{\partial V}{\partial z} = c_u|\vec{U}|V, \quad -\rho_0 c_p K\frac{\partial \Theta}{\partial z} = Q_T,$$
$$K\frac{\partial b}{\partial z} = 0, \quad K\frac{\partial \epsilon}{\partial z} = -\kappa_0 U_* \epsilon \qquad \text{at } z = h; \tag{9}$$

$$\tilde{\rho}_0\tilde{K}\frac{\partial \tilde{U}}{\partial z} = \rho_0 K\frac{\partial U}{\partial z}, \quad \tilde{\rho}_0\tilde{K}\frac{\partial \tilde{V}}{\partial z} = \rho_0 K\frac{\partial V}{\partial z}, \quad T = T_0,$$
$$\tilde{K}\frac{\partial \tilde{b}}{\partial z} = 0, \quad \tilde{K} = \tilde{l}_0\tilde{b}^{1/2} \qquad \text{at } z = 0; \tag{10}$$

$$\tilde{U} = \tilde{V} = 0, \quad \frac{\partial T}{\partial z} = \tilde{\gamma}_H, \quad \tilde{b} = 0, \quad \frac{\partial \tilde{\epsilon}}{\partial z} = 0 \quad \text{at } z = \tilde{H}, \tag{11}$$

where γ_H is the standard stratification of the free atmosphere, c_u is the resistance coefficient, Q_T is the heat flux from the lake to the atmosphere, c_p is the specific heat of the air at constant pressure, κ_0 is the Carman constant, U_* is dynamic velocity in the near-water layer, T_0 is the water surface temperature, l_0 is the turbulence scale in the subsurface layer, connected with the wind–wave disturbance intensity, $\tilde{\gamma}_H$ is the stable temperature stratification of the lake at depth. In the given statement, we have restricted ourselves by a priori setting the values Q_T and T_0, so that the ABL- and the UML-models have appeared to be thermodynamically disconnected. Such an approach makes it possible to elucidate the possibility of occurrence of the convective instability on the basis of real values Q_T, T_0, taken from observations. A more general statement provides "sewing" of temperatures with allowance for a cold "film" in the water and formulation of the heat balance equation on the surface.

Let us formulate the boundary conditions for systems (4), (7). Formally speaking, these conditions should supplement the back ground conditions and be in agreement with them. It is possible, however, to consider the convective structures in the ABL to intensively develop above the near-water layer, i.e., at $z > h$. This fact fully corresponds to theoretical and experimental concepts on the physics of processes near to the underlying surface. Thus, we write down

$$u = v = w = 0, \quad \theta = 0 \quad \text{at } z = H; \tag{12}$$

$$u = v = w = 0, \quad \theta = \theta_0(t, x, y) \quad \text{at } z = h; \tag{13}$$

$$\tilde{u} = \tilde{v} = \tilde{w} = 0, \quad \tilde{T} = \tilde{T}_0(t, x, y), \quad \text{at } z = 0; \tag{14}$$

$$\tilde{u} = \tilde{v} = \tilde{w} = 0, \quad \tilde{T} = 0 \quad \text{at } z = \tilde{H}, \tag{15}$$

where θ_0, $\tilde{T}_0$ are random low-amplitude temperature perturbations.

The following initial conditions were set for (2) and (5)

$$\Phi = \Phi_0; \quad \tilde{\Phi} = \tilde{\Phi}_0 \quad \text{at } t = t_0, \tag{16}$$

where Φ_0, $\tilde{\Phi}_0$ are stationary solutions of systems (2), (5) in the absence of convection and $\text{Adv}_\Theta = 0$.

The formulated problem (2)–(16) was solved by an implicit finite difference method based on a version of the splitting method. Equations (4), (7) were discretized in terms of the initial variables "velocity–pressure". The numerical algorithm includes stages of the transfer and the turbulent exchange in each of the directions x, y, z and the correction stage, providing the dynamic conformity of the fields and the increase of accuracy of the scheme by the reaccount of nonlinear terms. The scheme has the second accuracy order in all the variables and is stable within the range of admissible values of physical parameters.

5. Calculation results

As source data for the model of the ABL, we look an explicit heat flux obtained from the field experiment on lake Krasnoe in the summer of 1984 [8]. The daily variation $Q_T(t)$ is shown by solid curve in Figure 1.

Assume that at the initial moment $t_0 = 18$ hours l.t., the lower part of the ABL, the near-water layer included is neutrally stratified, and above-it is stable with $\Theta_z = \gamma_H$. Let us set $U_G = V_G = 0$ according to a synoptically situation during the observations period (calm or very weak wind). The near-water layer is cooling with mean rate 1.5°C/hour on the interval from 8 p.m. to 4 a.m. This causes accumulation of the thermal energy in the lower ABL and brings about the formation of an unstably stratified near-water layer. As a result, small perturbations increase with time up to the finite amplitudes which in this case are $\theta \approx 0.3$°C, $w \approx 1.5$ m/s. The configuration and size convective elements can inferred from Figure 2, showing the isolines of the field $w(x, y)$ at $z = 200$ m obtained at $t = 2$ h.l.t. Figure 3 illustrates the vertical sections of the temperature deviations field.

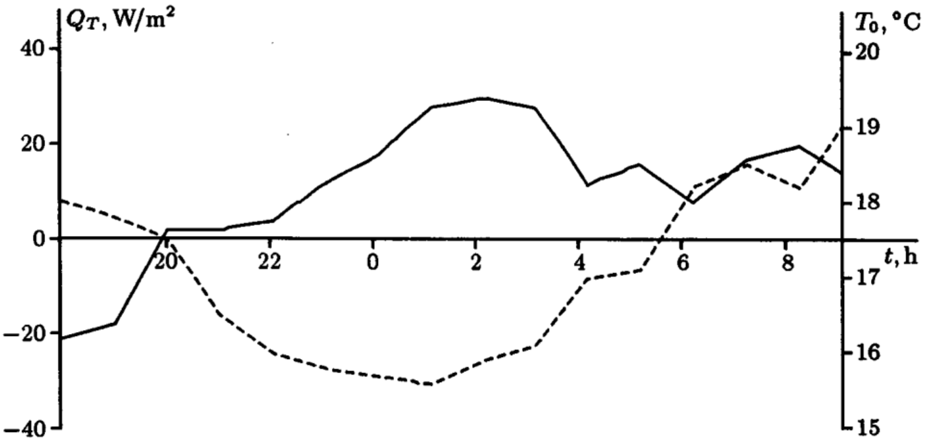

Figure 1. A fragment of the daily variation of the explicit heat flux Q_T (solid curve) and the water surface temperature T_0 (dashed curve) on evidence derived from observations [8]

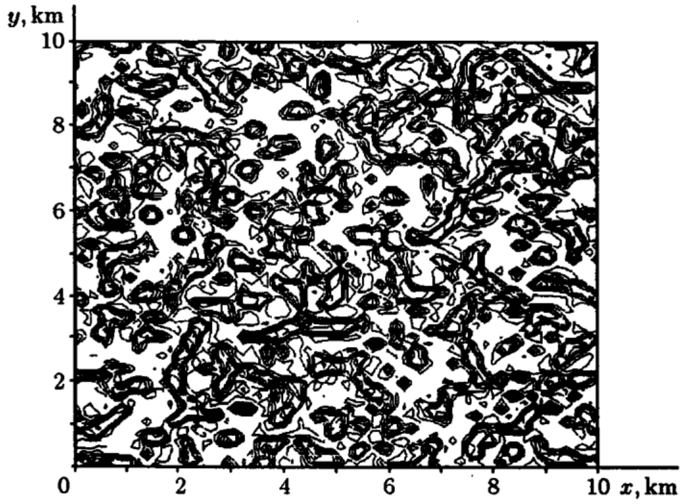

Figure 2. The field $w(x, y)$ in the convective ABL above the lake at t =2 hours, z = 200 m ($\max w$ = 1.5 m/s, $\min w$ = −0.7 m/s)

The cold "caps" are formed above the largest thermals ($z \approx$ 700 m), and the entrainment mechanism is realized at their cost. The presented structure of convective fields is similar to the structure obtained in [12], where convection is simulated in the marine boundary layer.

Figure 4a presents the vertical profile of the mean potential temperature (continuous curve 1) and the turbulent exchange coefficient (continuous curve 2). In the figure, the level z = 0 is made coincident with the upper boundary of the constant fluxes layer. The distribution of $\Theta(z)$ has

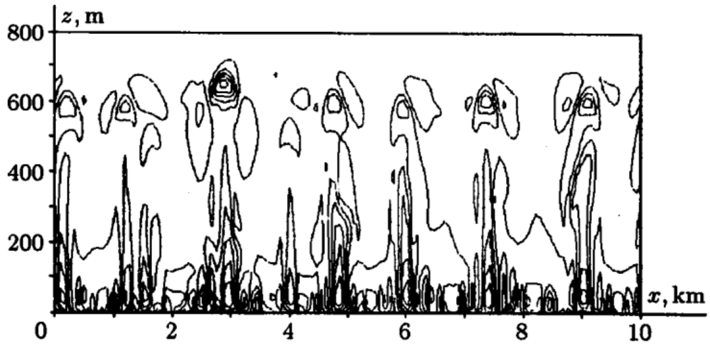

Figure 3. The vertical section of the field θ at t =2 hours

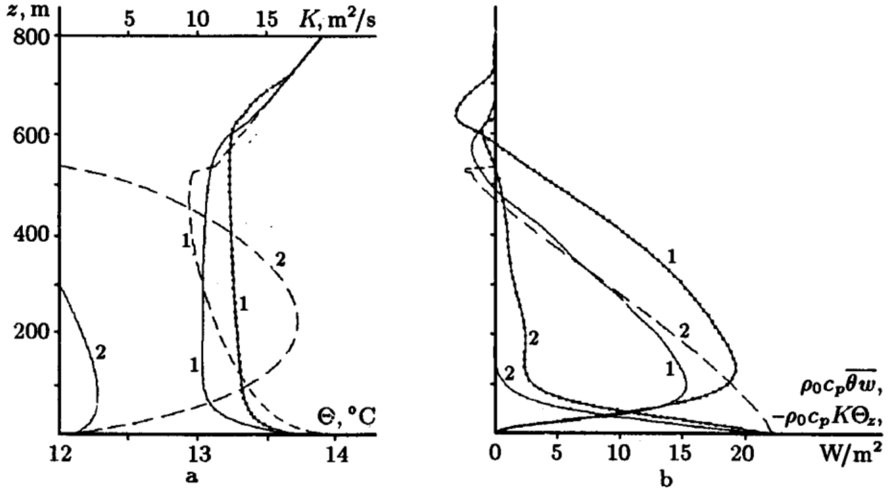

Figure 4. The vertical distribution of a) potential temperature (curves 1) and the turbulent exchange coefficient (curves 2); b) convective (curves 1) and turbulent heat fluxes (curves 2). The solid lines correspond to the complete problem with $U_G = V_G = 0$, the dotted lines correspond to $U_G = 5$ m/s and the dashed lines – to the purely gradient-diffusive problem

appeared to be intrinsic of the convective ABL, i.e., the mixing layer with weakly stable stratification up to heights $z \approx 500$ m with the characteristic reverse layer above this level; below is a thin layer (a few dozen meters) with a superadiabatic gradient Θ_z. The value of K does not exceed 5 m^2/s. The structure of the convective heat flux $\overline{\theta w}$ is shown in Figure 4b (continuous curve 1); the turbulent (subgrid) heat flux $-K\Theta_z$ is shown by continuous curve 2. The dotted curves in Figures 4a and 4b correspond to the version with $U_G = 5$ m/s (weak wind in the ABL). In the main thickness of the mixing layer $\overline{\theta w} \gg -K\Theta_z$, which is backed by the results of observations and by comparing four spatial LES-models ABL over the ocean [6]. The authors' interest to conduction the calculation with a purely diffusive K-model, i.e., when $\overline{\theta w} = 0$ is set in system (2) was quite natural. From Figure 4a (dotted line 1) it is seen that K-model unsatisfactorily reproduces characteristic features of the temperature distribution in vertical.

The stage of the maximal convection generation falls on 2–4 a.m. l.t., the mixing layer attaining the heights of $\approx$ 1000 m. As energy supply from the near-water layer at $t > 6$ hours (Figure 1, solid curve) decreases, instability of the lower part of the ABL is weakening and the convective activity gradually attenuates.

6. Conclusion

A step forward in the direction of constructing the combined ABL and UML model has been done. On the basis of such a model it appears possible to describe a very complicated mechanism of the energy-mass exchange between the lake, the adjacent land and the atmosphere. Such the model could serve a hydrodynamic basis for the development of a complex geospheric–biospheric model of ecosystems of large lakes of moderate latitudes. The constructed 3D convection model in the atmosphere and lake makes it possible to reliable describe both the mean currents fields and a thin space–time structure of the turbulent currents with CS, arising with the developed penetrating convection above a relatively warm lake at the night time*.

The development and realization of a target observational programme intended for obtaining the maximally complete data information of the parameters of the energy-mass exchange between the two most important components of nature are extremely important for construction and verification of models of the atmosphere-lake interaction.

References

[1] Zilitinkevich S.S. Penetrating turbulent convection. – Tallinn: Voltus, 1989 (in Russian).

[2] Mikhailova Z.A., Ordanovich A.E. Coherent structures in the atmosphere boundary layer (review) // Izv. AN SSSR, FAO. – 1991. – Vol. 27, № 6. – P. 593–609 (in Russian).

[3] Pushistov P.Yu. Coherent structures in deep lakes of moderate altitude (observations, evaluation of possibility of mathematical modeling) // Proceedings of the International Conference "Basic Problems of water and water resources at the thousandth anniversary". – Tomsk, 2000. – P. 341–345 (in Russian).

*Result of simulation of convection in the lake at the night cooling will be published in the near future.

[4] Deardorff J.W. Three-dimensional numerical study of turbulence in an entraining mixed layer // Boundary Layer Meteorology. – 1974. – Vol. 7. – P. 199–226.

[5] Pushistov P.Yu., Malbakhov V.M., Kononenko S.M., Vaskevich L.A. Numerical convection model with formation and evolution of cumulus clouds // Izv. AN SSSR, FAO. – 1980. – Vol. 16, № 1. – P. 3–10 (in Russian).

[6] Nieustadt F.T.M., Mason P.J., Moeng C.-H., Shumann U. Large-eddy simulation of the convective boundary layer: a comparison of four computer codes // Turbulent shear flow. – Springer-Verlag, 1991. – P. 343–367.

[7] Imberger J., Patterson J.C. Physical limnology // Advances in Applied Mechanics. – 1989. – Vol. 27. – P. 303–475.

[8] Panin G.N. Heat- and Massexchange between the Lake and the Atmosphere in Natural Conditions. – Moscow: Nauka, 1985 (in Russian).

[9] Boyarinov P.M., Petrov M.P. Processes of the Thermal Regime Formation of the Deep Freshwater Lakes. – Leningrad: Nauka, 1991 (in Russian).

[10] Pushistov P.Yu. Application of the Splitting method and the Energy Balancing Principle in setting the problems of hydrodynamic weather firecasting // Mathematical models of the atmospheric motions, part 2 / Sb. nauch. tr. VTs SO AN SSSR. – Novosibirsk, 1980. – P. 126–136 (in Russian).

[11] Pielke R.A. Mesoscale Meteorological Modelling. – Acad. Press, 1984.

[12] Walter B.A. The evolution of organized mesoscale convection in the marine planetary boundary layer during cold air outbreaks // Third Conference on Mesoscale Processes, Vancouver, B.C., Canada, 1987. – P. 202–203.