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Impact of analog-to-digital transformation on precision of vibroseimic signal measurments*

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The present work deals with the problems connected with precision of measurements of vibroseismic signals and techniques of setting-up an amplification factor of the seismometric channel.

1. Introduction

An important problem in the research of the Earth by vibroseismic methods is the setting-up techniques an amplification factor of the seismometric channel.

Many experimenters conduct the set-up procedure of an amplification factor on the basis of such representation that the maximum relative precision of measurement is attained with highly efficient use of the dynamic range of an analog-to-digitial converter (ADC). Thus, in pursuit of "precision" the short-term ADC overloads (overfilling of the register) are acceptable, considering their contribution insignificant as they are seldom, while the attainable precision being essential.

In addition, based on the given representation it is widely believed that the more digits the ADC has, the higher is a relative precision of the measurement of a vibroseismic signal.

Such an incorrect representation brings about the overstated engineering requirements to ADC on the one hand and incorrect techniques of setting-up an amplification factor on the other, which in turn results in undesirable nonlinear distortions such as intermodulations making the field data defective. Moreover, such intermodulation distortions can result in false anomalies, which can be taken for the real ones, especially when studying dynamic anomalous manifestations of the tensely-deformed state of the Earth's interior.

Until the present time this problem has not been given much attention. The work [1], which is relative in subject, deals with the qualitative level with determination of the requirement to ADC in terms of the possibility of distinguishing weak signals against the noise background.

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The present work deals with the problems connected with precision of measurement of vibroseismic signals and the setting-up techniques an amplification factor of the seismometric channel. The inaccuracy of the above-mentioned representations leading to undesirable nonlinear distortions of a vibroseismic signal and to the high technical requirements to ADC is proved.

2. Error of analog-to-digital transformation

With the help of the limited amount of ADC digits it is possible to present only a discrete set of values. In this connection there arises an error of transformation representing a difference between an actual value of a signal and the closest discrete value. With such a mechanism of rounding-off the given error represents a continuous random value ξ with values within the interval $[-\Delta x/2; \Delta x/2]$, where Δx is an ADC quantum (the difference between the two closest discretization levels).

The given random value is assumed to be uniformly distributed on this interval as all the values it takes are physically equivalent (homogeneous). Hence, in accordance with the above said its probability distribution density will be defined as

$$f_\xi(x) = \begin{cases} 1/\Delta x, & |x| \le \Delta x/2, \\ 0, & |x| > \Delta x/2. \end{cases} \tag{1}$$

Let us note the most important features of an error of the analog- digital transformation. An average value of a random value ξ, distributed by the uniform law (1) and its variance are easily determined and respectively equal to (cf. [2]):

$$\bar{\xi} = \int\limits_{-\infty}^{\infty} x\, f_\xi(x)\, dx = \frac{1}{\Delta x} \int\limits_{-\Delta x/2}^{\Delta x/2} x\, dx = 0, \tag{2}$$

$$\sigma_\xi^2 = \overline{\xi - \bar{\xi}} = \int\limits_{-\infty}^{\infty} x^2 f_\xi(x)\, dx = \frac{1}{\Delta x} \int\limits_{-\Delta x/2}^{\Delta x/2} x^2\, dx = \frac{\Delta x^2}{12}. \tag{3}$$

The following section will deal with the contribution of the ADC to an error of measurement of a vibroseismic signal.

3. Noise magnification factor

An error of measurement of a vibroseismic signal is determined by the three additive noise components present in the seismometric channel (at the input of the ADC). These are the microseismic noise n, the instrument noise ε and the ADC noise ξ. The instrument noise is so insignificant as compared to the microseismic noise that it can be neglected, or otherwise we can unite them together, still naming it microseismic. Thus, an error of measurement u is taken equal to the sum of two components: $u = n + \xi$.

Based on the fact that the microseismic noise is centered ($\bar{n} = 0$) and taking (2) into account, we arrive at the expression of the error variance of the measurement

$$\sigma_u^2 = \overline{(n+\xi)^2} = \overline{n^2} + 2\overline{n\xi} + \overline{\xi^2} = \sigma_n^2 + \sigma_\xi^2 + 2\overline{n\xi}. \tag{4}$$

The random values n and ξ are statistically independent, and as was already mentioned, their correlation in this case is equal to $\overline{n\xi} = \bar{n}\,\bar{\varepsilon} = 0$. In view of the given remark, and with allowance for expression (3), the variance σ_u^2, given by (4), will be written down in the following form:

$$\sigma_u^2 = \sigma_n^2 + \sigma_\xi^2 = \sigma_n^2 + \frac{\Delta x^2}{12}. \tag{5}$$

Let us define noise magnification factor of the analog-to-digital transformation as

$$\mathcal{K} = \frac{\sigma_u - \sigma_n}{\sigma_n} = \frac{\sigma_u}{\sigma_n} - 1. \tag{6}$$

Using formula (5) and definition (6), we come to

$$\mathcal{K} = \sqrt{1 + \frac{\Delta x^2}{12\sigma_n^2}} - 1 = \frac{\Delta x^2}{12\sigma_n^2}\left(1 + \sqrt{1 + \frac{\Delta x^2}{12\sigma_n^2}}\right)^{-1}. \tag{7}$$

For $\sigma_n/\Delta x \geq 1$ with deviation less than 2% from the exact expression (7) the factor under study can be written as

$$\mathcal{K} \approx \frac{1}{24}\left(\frac{\sigma_n}{\Delta x}\right)^{-2}. \tag{8}$$

It is necessary to note that the noise magnification factor $\mathcal{K}$ is strictly speaking, less than the value (8).

The ratio $\sigma_n/\Delta x$, included in (7), (8), represents nothing but a standard deviation of the microseismic noise expressed in the ADC quanta. Therefore for the practical application it is convenient to express this value by the binary ADC digits m as

$$\frac{\sigma_n}{\Delta x} = 2^m, \qquad m = 0, 1, 2, \dots\,. \tag{9}$$

By substituting it into (8), we obtain the expression

$$\mathcal{K} = \frac{1}{3 \cdot 2^{2m+3}}, \tag{10}$$

which establishes the dependence of the noise magnification factor of the ADC from a standard deviation of the microseismic noise expressed in digits. To illustrate, the table presents this dependence for the first 8 ADC digits.

ADC digits	0	1	2	3	4	5	6	7
$\mathcal{K}$, %	4.16	1.04	0.26	0.065	0.016	$4 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$2.5 \cdot 10^{-4}$

Based on the obtained results (the table and expressions (8), (10)), we come to a conclusion, that with the presence of the microseismic noise with $\sigma_n \geq \Delta x$ the basic contribution to an error of measurement of a vibroseismic signal brings in only the microseismic noise, instead of the ADC. Even if the level of the microseismic noise σ_n is equal to the quantum Δx ($k = 0$), the contribution of the ADC to magnification of an error of measurement does not exceed 4% (see the table), which is negligibly small.

4. Set-up of amplification factor of the seismometric channel

For generality we introduce the concept of the system amplification factor $\mathcal{E}$, describing the microseismic noise and the receiving-measuring equipment as a whole by defining it as the amplification factor of the seismometric channel for which the equality $\sigma_n = \Delta x$ is fulfilled. Thus a natural unit of measurement of an amplification factor is obtained, connecting a standard deviation of noise at the input of the equipment σ_m with the ADC quantum by the following dependence:

$$\mathcal{E} = \frac{\Delta x}{\sigma_m}. \tag{11}$$

Expressing an amplification factor of the seismometric channel H in terms of $\mathcal{E}$, we shall obtain a relative amplification factor h, which is defined as

$$h = \frac{H}{\mathcal{E}}. \tag{12}$$

In order to understand the sense of the value of the factor h let us transform it. Based on (11), (12) and the obvious equality $\sigma_m H = \sigma_n$, obtain

$$h = \frac{H}{\mathcal{E}} = \frac{\sigma_m H}{\sigma_m \mathcal{E}} = \frac{\sigma_n}{\Delta x}. \tag{13}$$

It becomes clear from the above result, that a standard deviation of the microseismic noise expressed in the ADC quanta is equal to a relative amplification factor h. Hence, regulating on of h directly results (according to (13)), in the change of the microseismic noise level in the seismometric channel.

Now we can again turn to the noise magnification factor of the ADC (7) and express it by the variable h. Rewriting (7) in terms of the result (13) we obtain

$$\mathcal{K} = \sqrt{1 + \frac{1}{12h^2}} - 1 = \frac{1}{12h^2}\left(1 + \sqrt{1 + \frac{1}{12h^2}}\right)^{-1}. \tag{14}$$

Studying this expression, we can distinguish two important qualitatively different variants:

1) $\mathcal{K} \geq 1$ with $h \leq 1/6$;

2) $\mathcal{K} \leq 0.04$ with $h \geq 1$.

The first variant corresponds to the case, in which the basic contribution to an error of measurement belongs to the ADC. It happens when measuring very large signals, directly taken from a vibrosource.

In the second variant the basic contribution to an error of measurement belongs to the microseismic noise (see the table). Measurement of vibroseismic signals at the Earth's vibrational research completely relates to this variant. In this case it is necessary to pay attention, to the fact that the magnification of a relative amplification factor h does not lead to the qualitative magnification of precision of measurement, and the results of experiments will be equivalent, both with small and with large values h, within the range $[1; \infty)$.

Such an indeterminacy of h allows us to provide such measurement conditions of vibroseismic signals, with which the ADC overload is completely absent.

For the cases with a large signal/noise ratio the regulation of an amplification factor is obvious and is not labor-consuming. Unexpected ejections of measured signals, leading to the ADC overload, are not observed here.

It is different, when a signal/noise ratio is less or equal to unit. Here the microseismic noise predominates over the vibroseismic signal. In this case the measurement signal is noise-like and with inaccurate set-up of the amplification factor can result in the ADC overload, which is inadmissible as was already said.

To realize the set-up correctly, it is necessary to take advantage of statistical properties of the microseismic noise. Experimentally it is determined, that if a path of microseismic noise is observed, its maximum ejection on the path would not exceed the level of the triple standard deviation. Having added the double overload margin, we shall arrive at a guaranteed dynamic range in which, practically, there is no ADC overload.

On this basis, we obtain a simple in realization technique of the correct set-up of an amplification factor. Observing a noise path of a measurement signal, we set up an amplification factor of the seismometric channel so that the maximum ejection of a signal does not exceed half a maximum of the ADC measured level. In some rare cases, depending on the character of the microseismic noise, it is necessary to some what increase to some extent the value of the overload, thus reducing an amplification factor. In this case, it is necessary to note that the precision of measurement remains the same.

5. Requirements to ADC

In the previous sections it was discussed that a level of microseismic noise, instead of the ADC quantum, determines the error of measurement of vibroseismic signals. Hence in this case the precision of measurement is not a criterion of the choice of the amount of digits of an analog-to-digitial converter – the basic engineering parameter.

The choice of the amount of digits is determined now only by the dynamic range of measured signals, i.e., the microseismic noise and a vibroseismic signal together. As the experience shows the dynamic range of the class all the measured signals in the vibroseismic sounding of the Earth with a safety margin is within the range with value 512.

Thus, for vibroseismic investigations in view of a sign of a signal a ten-digit ADC with the dynamic range 1024 suits completely. The further magnification of a digit grid of the ADC will not result in the increase of precision of measurements and hence has no sense.

6. Conclusions

- Theoretical study has shown that with the presence of the microseismic noise with $\sigma_n \geq \Delta x$, the basic contribution to an error of measurement of a vibroseismic signal brings in only the microseismic noise, instead of the ADC.
- It has been found that with a relative amplification factor $h \geq 1$ its magnification does not result in the increase of precision of measurement.
- The obtained result has a direct practical application of the technique of setting-up an amplification factor of the seismometric channel and of the choice of the ADC technical data.

References

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[2] D. Hudson, *Statistics for physicists*, Mir, Moscow, 1970, 36–37 (in Russian).