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Logical characterization of probabilistic τ-bisimulation equivalences

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Abstract. Stochastic Petri nets (SPNs) is a well-known model for quantitative analysis. We consider the class called DTWSPNs that is a modification of discrete time SPNs (DTSPNs) by transition labeling and weights. Transitions of DTWSPNs are labeled by actions that represent elementary activities and can be visible or invisible for an external observer. For DTWSPNs, interleaving and step probabilistic τ-bisimulation equivalences that abstract from invisible actions are introduced. A logical characterization of the equivalences is presented via formulas of the new probabilistic modal logics. This means that DTWSPNs are (interleaving or step) probabilistic τ-bisimulation equivalent if they satisfy the same formulas of the corresponding probabilistic modal logic. Thus, instead of comparing DTWSPNs operationally, one has to check the corresponding satisfaction relation only. The new interleaving and step logics are modifications of the probabilistic modal logic PML proposed by K.G. Larsen and A. Skou on probabilistic transition systems with visible actions.

Keywords: stochastic Petri nets, invisible transitions, interleaving and step semantics, equivalences, bisimulation, modal logics.

1. Introduction

Stochastic Petri nets (SPNs) are an extension of a well-known model of *Petri nets (PNs)*. SPNs are a model for quantitative analysis of discrete event dynamic systems having uncertain behaviour. SPNs are based on the concept of stochastic time delays and combine advantages of PNs with the possibility to evaluate quantitative characteristics of real-time processes. SPNs are essentially a high-level language for specification and performance analysis of computing systems. To avoid high complexity of stochastic systems analysis, only special distributions of time delays are used in SPNs. But this does not restrict their modeling power much, since the distributions are in most cases adequate to the real nature of the modeled systems execution, otherwise the good approximation is always possible. The dynamic behaviour of SPNs is described by the corresponding Markov chains (MCs).

Parameters of firing delay distributions or conditional firing probabilities are associated with transitions of SPNs. In these cases, marking change probabilities are distributed along continuous or, respectively, discrete time scale. Initially, only *continuous time SPNs (CTSPNs)* have been considered [28, 18]. A CTSPN is analyzed with the use of the corresponding continuous time MC (CTMC). Later on, *discrete time SPNs (DTSPNs)* have been proposed as well [29]. Analysis of a DTSPN is accomplished via the corresponding discrete time MC (DTMC). Usually, continuous models take nonnegative real numbers, and discrete ones take natural numbers as a time scale. Continuous formalisms usually use exponential distribution (sometimes called the power one), and discrete formalisms use geometric distribution (a special case of the Bernoulli one). This is due to the good "memoryless" property of those distributions. The property guarantees that the probability of the current state change depends on the probability of the previous one only. Thus, there is no need to remember execution history preceding the previous state change, and there is no difference between overall and residual delay distributions. In [27, 15], a class of *generalized SPNs (GSPNs)* has been defined as an extension of CTSPNs. GSPNs have exponential (with exponential time delay) and immediate (with zero time delay) transitions. Immediate transitions are used to resolve choices, whereas exponential ones model time delays of elementary executions. Thus, GSPNs allow for a more flexible system specification than CTSPNs. Analysis of a GSPN is based on the treatment of the corresponding semi-Markov MC (SMC), from which the embedded DTMC (EDTMC) is extracted.

For SPNs and GSPNs, labeling of transitions by actions representing elementary activities has been introduced [10, 11]. The actions can be visible or invisible. The symbol τ is used to denote the action that is invisible for an "external observer". The actions that are not labeled by τ are considered as visible ones. When labeling is defined, it becomes possible to introduce equivalence notions for the mentioned SPN classes. The equivalences are used to compare the stochastic behaviour of SPNs. The relations are called τ-equivalences, if they abstract from invisible actions.

For continuous-time models, the following probabilistic relations have been proposed in the literature. The interleaving probabilistic τ-bisimulation equivalence has been defined in [7, 8] on Markov process algebras, in [9, 12] — on stochastic automata, in [3] — on probabilistic transition systems, in [10] — on CTSPNs, and in [11] — on GSPNs. In [22, 20], the interleaving bisimulation equivalence for CTMCs has been proposed. The paper [7] presents different equivalences on Markov process algebras, and [9] — on stochastic automata. In [10], the interleaving performance bisimulation equivalence for CTSPNs has been defined. In [11], interleaving, performance and ϵ-approximated performance bisimulation relations for GSPNs have been presented. A wide range of probabilistic equivalence notions for unlabeled CTMCs has been considered in [19].

For discrete-time models, the labeling and equivalence notions have been much less explored up to the recent time. This especially concerns the relations that respect step semantics when time is discrete. To fill the gap, in [5, 6], a new class of DTSPNs with labeled transitions called DTWSPNs has been introduced. The dynamic behavior of this class of nets is characterized by steps instead of single transitions. The underlying stochastic process is still a DTMC, however, transitions of the DTMC describe the sets of transitions that fire concurrently. Thus, commonly used notions of bisimulation or trace equivalence for probabilistic processes [13, 25, 26] are not adequate for this type of models. A wide range of trace and bisimulation probabilistic τ-equivalences for DTWSPNs have been proposed in interleaving and step semantics for DTWSPNs in [5, 6]. As for equivalences in the discrete-time setting, one can additionally mention a very recent paper [4] only, where interleaving bisimulation and τ-bisimulation relations have been proposed on DTMCs. Nevertheless, DTWSPNs are more convenient for modeling. Note also that no back or back-forth equivalences for DTMCs as well as no relations in step semantics have been defined in that paper.

A characterization of equivalences via modal logics is used to change the operational reasoning on systems behaviour by the logical one. Moreover, such an interpretation elucidates the nature of the equivalences defined in an operational manner. On the other hand, we have an operational characterization of logical equivalences as a result. Importance of modal characterization for behavioural equivalences was explained in [1].

In the literature, a number of characterizations for probabilistic equivalences have been proposed. In [25, 26], the interleaving probabilistic bisimulation equivalence for probabilistic transition systems has been characterized via the probabilistic modal logic PML that is a probabilistic extension of a well-known HML [21]. In [16], it was shown that the interleaving probabilistic bisimulation equivalence of labelled Markov processes can be characterized by a simple negation-free modal logic. Note that this result was obtained for a continuous probabilistic model and requires no finite branching assumption, unlike [25, 26]. In [23], PML was adapted to a special type of probabilistic transition systems, and it was shown that the resulting logic characterizes the interleaving probabilistic bisimulation equivalence on the model considered. In the paper [2], for the interleaving probabilistic bisimulation equivalence on finite Markov processes, a characterization in terms of the temporal logic $pCTL^*$ has been proposed. In [24], the interleaving τ-testing equivalence for a generalization of the transition systems from [25, 26] has been provided with a characterization by a new quantitative extension of HML. The interleaving probabilistic τ-bisimulation equivalence for labelled concurrent MCs has been described in [17] by formulas of the temporal logic $pCTL^*$. In [4], τ-bisimulation equivalences for DTMCs and CTMCs have been interpreted via temporal logics $PCTL$ and CSL, respectively.

In this paper, we present a characterization of interleaving and step probabilistic τ-bisimulation equivalences from [5, 6] via two new probabilistic modal logics based on PML. This means that DTWSPNs are bisimulation equivalent if they satisfy the same formulas of the corresponding probabilistic logic, i.e., if they are logically equivalent. This provides one with the possibility for logical reasoning on probabilistic equivalences for DTWSPNs. It is worth mentioning that step probabilistic equivalences have not been logically interpreted before at all, not within the DTWSPNs framework only.

The paper is organized as follows. In Section 2, DTWSPNs are introduced. Probabilistic τ-bisimulation equivalences for DTWSPNs are defined in Section 3. A logical characterization of the equivalences is presented in Section 4. The concluding Section 5 contains a review of the results obtained, as well as the perspective directions for further research.

2. Labeled DTSPNs with weights

In this section, a class of labeled DTSPNs with transition weights is proposed.

Definition 1. A finite *multiset* M over a set X is a mapping $M : X \to \mathbb{N}$ such that $|\{x \in X \mid M(x) > 0\}| < \infty$.

The *set of all finite multisets* over X is denoted by $\mathbb{N}_f^X$. When $\forall x \in X\ M(x) \leq 1$, M is a proper set. The *cardinality* of a multiset M is defined in such a way: $|M| = \sum_{x \in X} M(x)$. We write $x \in M$ if $M(x) > 0$, and $M \subseteq M'$ if $\forall x \in X\ M(x) \leq M'(x)$. We define $(M + M')(x) = M(x) + M'(x)$ and $(M - M')(x) = \max\{0, M(x) - M'(x)\}$.

Definition 2. *A (labeled) DTSPN with weights (DTWSPN)* is a tuple $N = (P_N, T_N, W_N, \Lambda_N, \Omega_N, L_N, M_N)$, where

- $P_N = \{p, q, \ldots\}$ is the set of *places*;
- $T_N = \{t, u, \ldots\}$ is the set of *transitions*;
- $W_N : (P_N \times T_N) \cup (T_N \times P_N) \to \mathbb{N}$ is the *arc weights* function;
- $\Lambda_N : T_N \to \mathbb{R}_+$ is the *transition weights* function;
- $\Omega_N : T_N \to (0; 1]$ is the transition *conditional probabilities* function;
- $L_N : T_N \to Act_\tau$ is the transition *labeling* function that assigns action names to transitions, where Act is the set of visible actions, $\tau \notin Act$ is the invisible action, and $Act_\tau = Act \cup \{\tau\}$;
- $M_N \in \mathbb{N}_f^{P_N}$ is the *initial marking.*

In the graphical representation of DWSPNs, places are depicted by circles, transitions — by squares with the labelling action symbols inside. The names of places and transitions are depicted near them when needed. If the names are omitted but used, it is supposed that the places and transitions are numbered from left to right and from top to down. Arc weights are depicted by directed arcs of the corresponding multiplicity. Markings are depicted by black dots called "tokens" in the corresponding places. The token quantity is the multiplicity of a place in the marking. Transition weights and conditional probabilities are depicted near the corresponding transitions when needed. If they are not depicted, it is supposed that all transitions have equal weights and conditional probabilities.

In [5, 6], the transition relation $M \stackrel{A}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$ is defined, where $\mathcal{P}$ is the *probability of reaching* $\widetilde{M}$ *from* M *by internal steps followed by a visible step* labeled by $A \in I\!N_f^{Act} \setminus \emptyset$.

We will write $M \stackrel{A}{\twoheadrightarrow} \widetilde{M}$ if $\exists \mathcal{P} > 0 \ M \stackrel{A}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$. For a one-element multiset of actions $A = \{a\}$ we write $M \stackrel{a}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$ and $M \stackrel{a}{\twoheadrightarrow} \widetilde{M}$.

Definition 3. For a DTWSPN N we introduce the following notions.

- The *visible reachability set* $RS^*(N)$ is the minimal set of markings such that
  - $M_N \in RS^*(N)$;
  - if $M \in RS^*(N)$ and $M \stackrel{A}{\twoheadrightarrow} \widetilde{M}$, then $\widetilde{M} \in RS^*(N)$.
- The *visible reachability graph* $RG^*(N)$ is a directed labeled graph with the set of nodes $RS^*(N)$ and an arc labeled with A, $\mathcal{P}$ between nodes M and $\widetilde{M}$ if $M \stackrel{A}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$ and $\mathcal{P} > 0$.
- The *visible underlying DTMC* $DTMC^*(N)$ is a DTMC with the state space $RS^*(N)$ and a transition $M \twoheadrightarrow_{\mathcal{Q}} \widetilde{M}$ whenever at least one arc between M and $\widetilde{M}$ exists in $RG^*(N)$. The probability $\mathcal{Q}$ is computed as

$$\mathcal{Q} = \sum_{\{A \mid M \stackrel{A}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}\}} \mathcal{P}.$$

In particular cases, interleaving behaviour of DTWSPNs should be compared. In interleaving semantics, we abstract from steps labeled by non-singleton multisets of visible actions. After the abstraction, we have to normalize probabilities of the remaining one-action steps. For this, the *interleaving transition relation* is introduced. Let N be a DTWSPN and $M, \widetilde{M} \in RS^*(N),\ a \in Act$. We write $M \stackrel{a}{\twoheadrightarrow\!\!\!\rightarrow}_{\mathcal{Q}} \widetilde{M}$, if $M \stackrel{a}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$ and

$$\mathcal{Q} = \frac{\mathcal{P}}{\sum_{\{b \in Act,\ \widetilde{M} \in RS^*(N) \mid M \stackrel{b}{\twoheadrightarrow}_{\mathcal{P}'} \widetilde{M}\}} \mathcal{P}'}.$$

3. Probabilistic τ-bisimulation equivalences

Bisimulation equivalences completely respect the points of choice of an external observer in the behavior of a modeled system. In this section, a parameterized definition of probabilistic τ-bisimulation equivalences is presented.

To define probabilistic bisimulation equivalences, we have to consider a bisimulation as an *equivalence* relation which partitions the states of the *union* of the visible reachability graphs $RG^*(N)$ and $RG^*(N')$ of two nets N and N' which are compared. For nets N and N' to be bisimulation equivalent, their initial markings M_N and $M_{N'}$ have to be related by a bisimulation having the following transfer property: two markings are related if in each of them the same (multisets of) actions can occur, and the resulting markings *belong to the same equivalence class.* In addition, sums of probabilities for all such occurrences should be the same for both markings. Thus, in our definitions, we follow the approach of [25, 26]. Hence, the difference between bisimulation and trace equivalences is that we do not consider *all possible* occurrences of (multisets of) actions from the initial markings, but only such that lead (stepwise) to markings *belonging to the same equivalence class.*

First, we introduce several helpful notations. Let for a DTWSPN N $\mathcal{L} \subseteq RS^*(N)$. For some $M \in RS^*(N)$ and $A \in \mathbb{N}_f^{Act}$ we write $M \stackrel{A}{\twoheadrightarrow}_{\mathcal{Q}} \mathcal{L}$ if

$$\mathcal{Q} = \sum_{\{\widetilde{M} \in \mathcal{L} \mid M \stackrel{A}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}\}} \mathcal{P}.$$

We will write $M \stackrel{A}{\twoheadrightarrow} \mathcal{L}$ if $\exists \mathcal{Q} > 0 \; M \stackrel{A}{\twoheadrightarrow}_{\mathcal{Q}} \mathcal{L}$. For a one-element multiset of actions $A = \{a\}$ we write $M \stackrel{a}{\twoheadrightarrow}_{\mathcal{Q}} \mathcal{L}$ and $M \stackrel{a}{\twoheadrightarrow} \mathcal{L}$.

In similar way, we define the notion $M \stackrel{a}{\rightharpoonup}_{\mathcal{Q}} \mathcal{L}$ based on the interleaving transition relation.

Let X be some set. We denote the cartesian product $X \times X$ by X^2. Let $\mathcal{E} \subseteq X^2$ be an equivalence relation on X. Then an *equivalence class* (w.r.t. $\mathcal{E}$) of an element $x \in X$ is defined by $[x]_{\mathcal{E}} = \{y \in X \mid (x, y) \in \mathcal{E}\}$. The equivalence $\mathcal{E}$ partitions X in the *set of equivalence classes* $X/_{\mathcal{E}} = \{[x]_{\mathcal{E}} \mid x \in X\}$.

Definition 4. Let N be a DTWSPN. An *equivalence* relation $\mathcal{R} \subseteq RS^*(N)^2$ is a $\star$-*probabilistic* τ-*bisimulation* between markings M_1 and M_2 of DTWSPN N, $\star \in$ {*interleaving, step*}, denoted by $\mathcal{R} : M_1 \underline{\leftrightarrow}_{\star p}^{\tau} M_2$, $\star \in \{i, s\}$, if $\forall \mathcal{L} \in RS^*(N)/_{\mathcal{R}}$

- $\forall x \in Act$ and $\hookrightarrow = \rightharpoonup$, if $\star = i$;
- $\forall x \in \mathbb{N}_f^{Act}$ and $\hookrightarrow = \twoheadrightarrow$, if $\star = s$;

$$M_1 \stackrel{x}{\hookrightarrow}_{\mathcal{Q}} \mathcal{L} \;\Leftrightarrow\; M_2 \stackrel{x}{\hookrightarrow}_{\mathcal{Q}} \mathcal{L}.$$

Markings M_1 and M_2 are $\star$-*probabilistic* τ-*bisimulation equivalent*, $\star \in${*interleaving, step*}, denoted by $M_1 \underline{\leftrightarrow}^{\tau}_{\star p} M_2$, if $\exists \mathcal{R} : M_1 \underline{\leftrightarrow}^{\tau}_{\star p} M_2,\ \star \in \{i, s\}$.

To introduce bisimulation between DTWSPNs N and N', we should consider a "composite" set of reachable states $RS^*(N) \cup RS^*(N')$.

Definition 5. Let N and N' be DTWSPNs. A relation $\mathcal{R} \subseteq (RS^*(N) \cup RS^*(N'))^2$ is a $\star$-*probabilistic* τ-*bisimulation* between N and N', $\star \in${*interleaving, step*}, denoted by $\mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{\star p} N'$, if $\mathcal{R} : M_N \underline{\leftrightarrow}^{\tau}_{\star p} M_{N'},\ \star \in \{i, s\}$.

DTWSPNs N and N' are $\star$-*probabilistic* τ-*bisimulation equivalent*, $\star \in${*interleaving, step*}, denoted by $N \underline{\leftrightarrow}^{\tau}_{\star p} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{\star p} N',\ \star \in \{i, s\}$.

It is easy to show that the union of two (interleaving or step) probabilistic τ-bisimulations is also a (interleaving or step) probabilistic τ-bisimulation. Hence, the largest bisimulation relation exists, and it is unique up to the ordering of equivalence classes. Consequently, for a given DTWSPN, equivalent DTWSPNs with a minimal state space exist. Thus, the bisimulations can be used for reduction if to compute the minimal equivalent representation of a DTWSPN.

4. Logical characterization

In this section, a logical characterization of probabilistic τ-bisimulation equivalences is accomplished via formulas of probabilistic modal logics. The results obtained could be interpreted as an operational characterization of the corresponding logical equivalences. DTWSPNs are considered as logically equivalent if they satisfy the same formulas.

4.1. Logic *IPML*

The probabilistic modal logic PML has been introduced in [25] on probabilistic transition systems without invisible actions for logical interpretation of the interleaving probabilistic bisimulation equivalence. On the basis of PML, we propose a new interleaving modal logic $IPML$ used for characterization of the interleaving τ-bisimulation equivalence.

Definition 6. Let $\top$ denote the truth and $a \in Act,\ \mathcal{P} \in (0; 1]$. A *formula* of $IPML$ is defined as follows:

$$\Phi ::= \;\top \mid \neg\Phi \mid \Phi \wedge \Phi \mid \langle a \rangle_{\mathcal{P}} \Phi$$

We define $\langle a\rangle\Phi = \exists\mathcal{P} > 0\ \langle a\rangle_{\mathcal{P}}\Phi$.

IPML denotes the set of *all formulas of the logic $IPML$*.

Definition 7. Let N be a DTWSPN and $M \in RS^*(N)$. The *satisfaction relation* $\models_N \subseteq RS^*(N) \times$ **IPML** is defined as follows:

1. $M \models_N \top$ — always;
2. $M \models_N \neg\Phi$, if $M \not\models_N \Phi$;
3. $M \models_N \Phi \wedge \Psi$, if $M \models_N \Phi$ and $M \models_N \Psi$;
4. $M \models_N \langle a\rangle_{\mathcal{P}}\Phi$, if $\exists\mathcal{L} \subseteq RS^*(N)\ M \stackrel{a}{\twoheadrightarrow}_{\mathcal{Q}} \mathcal{L},\ \mathcal{Q} \geq \mathcal{P}$ and $\forall\widetilde{M} \in \mathcal{L}$ $\widetilde{M} \models_N \Phi$.

Note that $\langle a\rangle_{\mathcal{Q}}\Phi$ implies $\langle a\rangle_{\mathcal{P}}\Phi$, if $\mathcal{Q} \geq \mathcal{P}$.

Definition 8. We write $N \models_N \Phi$, if $M_N \models_N \Phi$. DTWSPNs N and N' are *logically equivalent* in $IPML$, denoted by $N =_{IPML} N'$, if $\forall\Phi \in$ **IPML** $N \models_N \Phi\ \Leftrightarrow\ N' \models_{N'} \Phi$.

Let N be a DTWSPN and $M \in RS^*(N)$, $a \in Act$. The set of *next* to M markings *after occurrence of a visible action a* (the *visible image set*) is defined as $VisImage(M, a) = \{\widetilde{M} \mid M \stackrel{a}{\twoheadrightarrow} \widetilde{M}\}$. A DTWSPN N is a *image-finite* one, if $\forall M \in RS^*(N)\ \forall a \in Act\ |VisImage(M, a)| < \infty$.

Theorem 1. *For image-finite DTWSPNs N and N'*

$$N \underline{\leftrightarrow}_{ip}^{\tau} N'\ \Leftrightarrow\ N =_{IPML} N'.$$

Proof. As the subsequent Theorem 2, but with the use of marking changes due to occurrence of single actions instead of multisets of actions and the interleaving transition relation. □

Hence, in the interleaving semantics, we obtained a logical characterization of the probabilistic τ-bisimulation relation or, symmetrically, an operational characterization of the probabilistic modal logic equivalence.

Example 1. In Figure 1 $N \neq_{IPML} N'$, because for $\Phi = \langle a\rangle_1\langle b\rangle_{\frac{1}{2}}\top$ we have $N \models_N \Phi$, but $N' \not\models_{N'} \Phi$, since only in the DTWSPN N' an action a can occur so that no action b can occur afterwards.

Figure 2 presents usual reachability graphs of DTWSPNs from Figure 1. Figure 3 presents their visible reachability graphs. Markings are depicted in a vector form according to the standard place numbering.

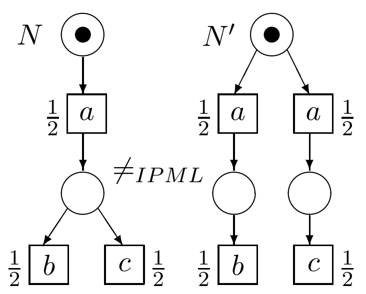

Figure 1. Differentiating power of $=_{IPML}$

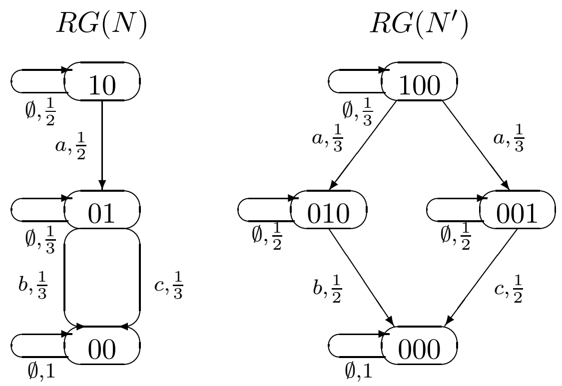

Figure 2. Reachability graphs of DTWSPNs from Figure 1

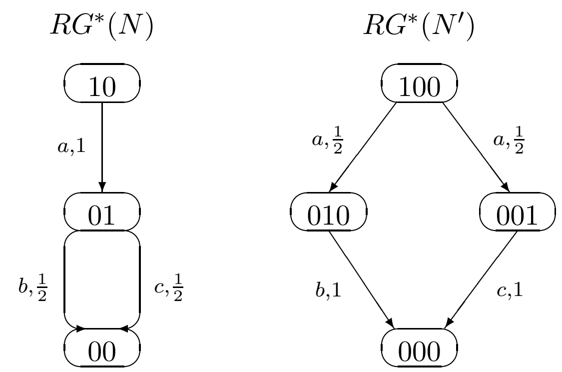

Figure 3. Visible reachability graphs of DTWSPNs from Figure 1

4.2. Logic $SPML$

On the basis of PML, we propose a new step modal logic $SPML$ used for characterization of the step τ-bisimulation equivalence.

Definition 9. Let $\top$ denote the truth and $A \in I\!N_f^{Act}$, $\mathcal{P} \in (0;1]$. A *formula* of $SPML$ is defined as follows:

$$\Phi ::= \ \top \mid \neg\Phi \mid \Phi \wedge \Phi \mid \langle A\rangle_{\mathcal{P}}\Phi.$$

We define $\langle A\rangle\Phi = \exists\mathcal{P} > 0\ \langle A\rangle_{\mathcal{P}}\Phi$.

SPML denotes the set of *all formulas of the logic $SPML$*.

Definition 10. Let N be a DTWSPN and $M \in RS^*(N)$. The *satisfaction relation* $\models_N \subseteq RS^*(N) \times$ **SPML** is defined as follows:

1. $M \models_N \top$ — always;
2. $M \models_N \neg\Phi$, if $M \not\models_N \Phi$;
3. $M \models_N \Phi \wedge \Psi$, if $M \models_N \Phi$ and $M \models_N \Psi$;
4. $M \models_N \langle A\rangle_{\mathcal{P}}\Phi$, if $\exists\mathcal{L} \subseteq RS^*(N)\ M \stackrel{A}{\twoheadrightarrow}_{\mathcal{Q}} \mathcal{L},\ \mathcal{Q} \geq \mathcal{P}$ and $\forall\widetilde{M} \in \mathcal{L}$ $\widetilde{M} \models_N \Phi$.

Note that $\langle A\rangle_{\mathcal{Q}}\Phi$ implies $\langle A\rangle_{\mathcal{P}}\Phi$, if $\mathcal{Q} \geq \mathcal{P}$.

Definition 11. We write $N \models_N \Phi$, if $M_N \models_N \Phi$. DTWSPNs N and N' are *logically equivalent* in $SPML$, denoted by $N =_{SPML} N'$, if $\forall\Phi \in$ **SPML** $N \models_N \Phi \ \Leftrightarrow\ N' \models_{N'} \Phi$.

Let N be a DTWSPN and $M \in RS^*(N),\ A \in I\!N_f^{Act}$. The set of *next* to M markings *after occurrence of a multiset of visible actions A* (the *visible image set*) is defined as $VisImage(M, A) = \{\widetilde{M} \mid M \stackrel{A}{\twoheadrightarrow} \widetilde{M}\}$. A DTWSPN N is a *image-finite* one, if $\forall M \in RS^*(N)\ \forall A \in I\!N_f^{Act}\ |VisImage(M, A)| < \infty$.

Theorem 2. *For image-finite DTWSPNs N and N'*

$$N \underline{\leftrightarrow}_{sp}^{\tau} N' \ \Leftrightarrow\ N =_{SPML} N'.$$

Proof. ($\Leftarrow$) To simplify the presentation, we propose the *indicator function* Γ that recovers a DTWSPN by a marking belonging to it. For a DTWSPN N and $M \in RS^*(N)$ we define $\Gamma(M) = N$.

Let us define the equivalence relation $\mathcal{R} = \{(M_1, M_2) \in (RS^*(N) \cup RS^*(N'))^2 \mid \forall \Phi \in \textbf{SPML}\ M_1 \models_{\Gamma(M_1)} \Phi \ \Leftrightarrow\ M_2 \models_{\Gamma(M_2)} \Phi\}$. We have $(M_N, M_{N'}) \in \mathcal{R}$. Let us prove that $\mathcal{R}$ is an interleaving probabilistic τ-bisimulation.

Assume that $M_N \overset{A}{\twoheadrightarrow}_{\mathcal{P}} \mathcal{L} \in RS^*(N) \cup RS^*(N')/_{\underline{\leftrightarrow}_{ip}^{\tau}}$. Let $M_{N'} \overset{A}{\twoheadrightarrow}_{\mathcal{P}'_1} M'_1, \dots, M_{N'} \overset{A}{\twoheadrightarrow}_{\mathcal{P}'_i} M'_i, M_{N'} \overset{A}{\twoheadrightarrow}_{\mathcal{P}'_{i+1}} M'_{i+1}, \dots, M_{N'} \overset{A}{\twoheadrightarrow}_{\mathcal{P}'_n} M'_n$ be changes of the marking $M_{N'}$ as a result of occurrence of the multiset of actions A. Since DTWSPN N' is an image-finite one, the number of such marking changes is finite. The marking changes are ordered so that $M'_1, \dots M'_i \in \mathcal{L}$ and $M'_{i+1}, \dots M'_n \notin \mathcal{L}$.

Then $\exists \Phi_{i+1}, \dots, \Phi_n \in \textbf{SPML}$ such that $\forall j\ (i+1 \le j \le n)\ \forall M \in \mathcal{L}\ M \models_{\Gamma(M)} \Phi_j$, but $M'_j \not\models_{N'} \Phi_j$. We have $M_N \models_N \langle A \rangle_{\mathcal{P}} (\wedge_{j=i+1}^{n} \Phi_j)$ and $M_{N'} \models_{N'} \langle A \rangle_{(1-\sum_{j=1}^{i} \mathcal{P}'_j)} \neg (\wedge_{j=i+1}^{n} \Phi_j)$.

Assume that $\mathcal{P} > \sum_{j=1}^{i} \mathcal{P}'_j$. Then $M_{N'} \models_{N'} \langle A \rangle_{(1-\mathcal{P})} \neg (\wedge_{j=i+1}^{n} \Phi_j)$ and $M_{N'} \not\models_{N'} \langle A \rangle_{\mathcal{P}} (\wedge_{j=i+1}^{n} \Phi_j)$ what contradicts $(M_N, M_{N'}) \in \mathcal{R}$. Hence, $\mathcal{P} \le \sum_{j=1}^{i} \mathcal{P}'_j$. Consequently, $M_{N'} \overset{A}{\twoheadrightarrow}_{\mathcal{P}'} \mathcal{L},\ \mathcal{P} \le \sum_{j=1}^{i} \mathcal{P}'_j \le \mathcal{P}'$. By symmetry of $\underline{\leftrightarrow}_{ip}^{\tau}$, we have $\mathcal{P} \ge \mathcal{P}'$. Thus, $\mathcal{P} = \mathcal{P}'$, and $\mathcal{R}$ is an interleaving probabilistic τ-bisimulation.

($\Rightarrow$) It is sufficient to consider only the case $\langle A \rangle_{\mathcal{P}} \Phi$, since all other cases are trivial. Let for DTWSPNs N and N' $N \underline{\leftrightarrow}_{ip}^{\tau} N'$. Then $M_N \underline{\leftrightarrow}_{ip}^{\tau} M_{N'}$. Assume that $M_N \models_N \langle A \rangle_{\mathcal{P}} \Phi$. Then $\exists \mathcal{L} \subseteq RS^*(N) \cup RS^*(N')$ such that $M_N \overset{A}{\twoheadrightarrow}_{\mathcal{Q}} \mathcal{L},\ \mathcal{Q} \ge \mathcal{P}$ and $\forall M \in \mathcal{L}\ M \models_{\Gamma(M)} \Phi$.

Let us define $\widetilde{\mathcal{L}} = \bigcup \{\mathcal{K} \in RS^*(N) \cup RS^*(N')/_{\underline{\leftrightarrow}_{ip}^{\tau}} \mid \mathcal{K} \cap \mathcal{L} \ne \emptyset\}$. Then $\forall \widetilde{M} \in \widetilde{\mathcal{L}}\ \exists M \in \mathcal{L}\ M \underline{\leftrightarrow}_{ip}^{\tau} \widetilde{M}$. Since $\forall M \in \mathcal{L}\ M \models_{\Gamma(M)} \Phi$, we have $\forall \widetilde{M} \in \widetilde{\mathcal{L}}\ \widetilde{M} \models_{\Gamma(\widetilde{M})} \Phi$ by the induction hypothesis.

Since $\mathcal{L} \subseteq \widetilde{\mathcal{L}}$, we have $M_N \overset{A}{\twoheadrightarrow}_{\widetilde{\mathcal{Q}}} \widetilde{\mathcal{L}},\ \widetilde{\mathcal{Q}} \ge \mathcal{Q}$. Since $\widetilde{\mathcal{L}}$ is the union of the equivalence classes w.r.t. $\underline{\leftrightarrow}_{ip}^{\tau}$, we have $M_N \underline{\leftrightarrow}_{ip}^{\tau} M_{N'}$ implies $M_{N'} \overset{A}{\twoheadrightarrow}_{\widetilde{\mathcal{Q}}} \widetilde{\mathcal{L}}$. Since $\widetilde{\mathcal{Q}} \ge \mathcal{Q} \ge \mathcal{P}$, we have $M_{N'} \models_{N'} \langle A \rangle_{\mathcal{P}} \Phi$. Therefore, DTWSPN N' satisfies all the formulas that N does. By symmetry of $\underline{\leftrightarrow}_{ip}^{\tau}$, DTWSPN N satisfies all the formulas that N' does. Thus, the sets of formulas satisfiable for N and N' coincide. □

Hence, in the step semantics, we obtained a logical characterization of the probabilistic τ-bisimulation relation or, symmetrically, an operational characterization of the probabilistic modal logic equivalence.

Example 2. In Figure 4 $N \underline{\leftrightarrow}_{ip}^{\tau} N'$ but $N \ne_{SPML} N'$, because for

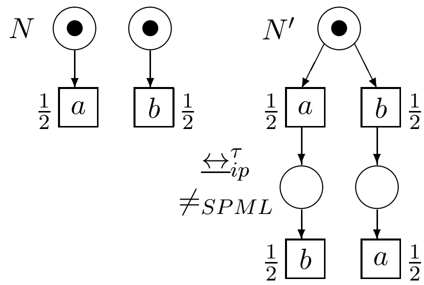

Figure 4. Differentiating power of $=_{SPML}$

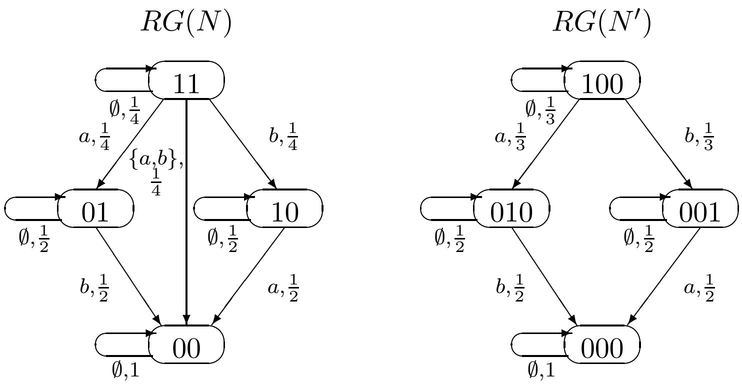

Figure 5. Reachability graphs of DTWSPNs from Figure 4

$\Phi = \langle\{a,b\}\rangle_{\frac{1}{3}}\top$ we have $N \models_N \Phi$, but $N' \not\models_{N'} \Phi$, since only in the DTWSPN N' actions a and b cannot occur concurrently.

Figure 5 presents usual reachability graphs of DTWSPNs from Figure 4. Figure 6 presents their visible reachability graphs. Markings are depicted in a vector form according to the standard place numbering.

5. Conclusion

In this paper, for a new class of DTSPNs with labeled and weighted transitions called DTWSPNs, we proposed a logical characterization of probabilistic τ-bisimulation equivalences in interleaving and step semantics. For this, we defined a new probabilistic modal logics $IPML$ and $SPML$ based on PML proposed by K.G. Larsen and A. Skou. The result provides one with a better understanding of basic features of the equivalences. Additionally, this gives the possibility for logical reasoning on stochastic behaviour

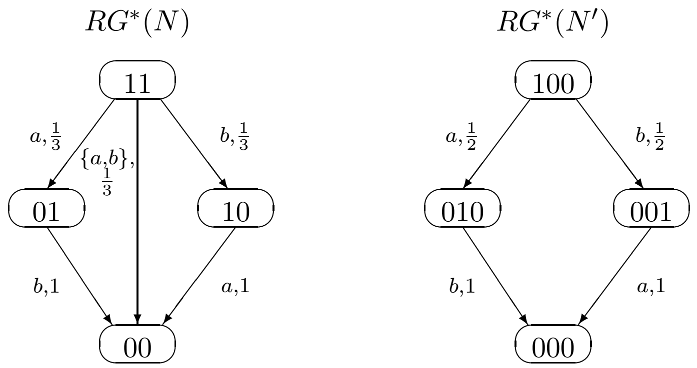

Figure 6. Visible reachability graphs of DTWSPNs from Figure 4

resemblance, while before it was possible in operational manner only.

Further research direction could be in logical interpretation of the remaining probabilistic τ-bisimulation equivalences for DTWSPNs proposed in [5, 6], i.e., back and back-forth ones. For this, one has to design the interleaving and step probabilistic analogues of the back-forth logic BFL proposed in [14] on event structures. BFL has the special back modalities corresponding to the back moving along the history of computations by one action. For characterization of the step relations, one should be able to go back by a multiset of actions. To interpret logically the probabilistic equivalences respecting back moves, one has to express somehow all the complex requirements from the definitions of these relations by means of a modal logic.

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