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A 3D numerical model for the Earth's mantle convection*

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Abstract. By now a substantial body of literature has evolved which deals with the numerical simulation of the 3D convective currents in the Earth's mantle (see, e.g., [1–4] and references therein). A crucial point is to estimate the reliability of numerical experimental results because the similarity principle fails when the dynamical processes that occur in the Earth's interior under the known physical parameters of the Earth's matter are simulated on the laboratory scale. Paper [4] is especially important for testing numerical models, because it contains results of the 3D simulation of convective currents in the mantle obtained by different authors dealing with model problems.

In solving the 3D problems of hydrodynamics, the variables vorticity and the vector potential appeared to be very useful [5]. Analysis of the known publications shows that these variables receive little attention in the numerical simulation of convective processes in the Earth's mantle. The present paper is an attempt to construct and test a numerical model of convective processes in the Earth's mantle using the above variables and the method of fractional steps [6].

1. Mathematical formulation and numerical solution of the problem

To describe currents in the upper mantle of the Earth, a well-known mathematical model is used including the non-dimensionalized equations [7]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial p}{\partial x} = 2\frac{\partial}{\partial x}\eta\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\eta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \tag{2}$$

$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial x}\eta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + 2\frac{\partial}{\partial y}\eta\frac{\partial v}{\partial y} + \frac{\partial}{\partial z}\eta\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \tag{3}$$

$$\frac{\partial p}{\partial z} = \frac{\partial}{\partial x}\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\eta\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + 2\frac{\partial}{\partial z}\eta\frac{\partial w}{\partial z} + \mathrm{Ra}\cdot T, \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \nabla^2 T + Q. \tag{5}$$

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Here u, v, w are the velocity vector components, p is pressure, T is temperature, Q is a source of heat release, Ra is the Rayleigh number, η is a dynamic viscosity.

The system of equations (1)–(5) is arranged so that at the initial time $t = t_0$ the initial conditions are given for the temperature alone: $T(x, y, z, t_0) = T_0(x, y, z)$. The boundary conditions at the sides are the conditions of symmetry, while at the bottom and at the top, the conditions of adhesion and fixed temperatures are set.

To construct a numerical model, the new dependent variables are used: the vorticity vector $\vec{\omega} = \nabla \times \vec{V}$ and the vector potential $\vec{\psi}$: $\vec{V} = \nabla \times \vec{\psi}$. The initial and the boundary conditions are formulated in terms of the new desired functions.

The finite difference algorithm of the problem solution is based on the method of fractional steps [6]. At each temporary layer (until reaching the steady-state mode), the calculation algorithm includes the following steps:

1. Given the known temperature distribution, vorticity, vector potential, the velocity components are calculated until the convergence of iteration processes is attained;
2. The temperature field is calculated.

The numerical model was tested solving the problem from [4]. The solution was sought for in a unit cube. The scale factor under the viscosity is $\eta_0 = 1.2 \cdot 10^{24}$, the Rayleigh number is $\mathrm{Ra} = 2 \cdot 10^4$.

The following parameters were calculated: the root-mean-square velocity V_{rms}, the Nusselt number Nu, the vertical component of the velocity w and the temperature T at angular points of the mean section of a convective layer; the heat flux $\vartheta = -\partial T/\partial z$ at angular points of the upper cube surface; the integral parameter calculated from the formula $\tau(x, z) = \int_0^Y \frac{\partial T}{\partial z}\, dy$ along the line parallel to the axis Y originated at the points $(0, 0.25)$, $(0.5, 0.25)$, $(1, 0.25)$ of the frontal XZ-plane; the mean temperature $T_m = \iint_{S_z} T\, dx\, dy$, calculated at the horizontal sections S_z at depths $z = 0.75$ and $z = 0.5$; the vertical component of the vorticity vector Ω at the point $(0.75, 0.25, 0.75)$. The dimension values (in the SI system), which were used in [4] and in this paper, were taken as follows:

$$d = 2700000\ \mathrm{m}, \quad \Delta T = 3700\,^\circ\mathrm{C}, \quad \chi = 10^{-6}\,\mathrm{m}^2/\mathrm{s}, \quad \alpha = 10^{-5}\,^\circ\mathrm{C}^{-1},$$
$$\rho = 3300\ \mathrm{kg/m}^3, \quad g_z = 10\ \mathrm{m/s}^2, \quad \eta_0 = 10^{24}\,\mathrm{Pa}\cdot\mathrm{s}.$$

The initial temperature distributions were taken as follows:

$$T(x, y, z, t_0) = T_0(x, y, z) = (1 - z) + 0.2\Big(\cos\frac{\pi x}{X} + \cos\frac{\pi y}{Y}\Big)\sin\pi z.$$

Parameter	Christensen's data on grid $32 \times 32 \times 64$	Authors' results on grids		
		$12 \times 12 \times 12$	$24 \times 24 \times 24$	$48 \times 48 \times 48$
Nu	3.03927	3.245	3.0397	3.040
$V_{\rm rms}$	35.132	37.97	35.360	35.07
$W(1,1,0.5)$	-58.23	-60.68	-58.262	-58.42
$T(1,1,0.5)$	0.23925	0.2326	0.2377	0.2393
$\vartheta(1,1)$	0.7684	0.80172	0.7804	0.7726
$\tau(1,0.25)$	-0.1388	-0.06761	-0.1341	-0.140
$T_m(0.50)$	0.58158	0.5844	0.5792	0.5815
$\omega^z(0.75,0.25,0.75)$	-11.125	-11.03	-11.203	-11.35
Computation time, s	—	65	119	4811

The results calculated for the variable from [4], the viscosity $\eta = \eta(T)$ were compared with Christensen's data as the most complete of those available in [4] and reported in the table.

A well-known efficient approach to solving problems of mathematical physics is the method of successive grids [8]. The results of its use in the present paper are illustrated in the table.

The direct computation on $48 \times 48 \times 48$ grid took 660 minutes using a PC with the Athlon 1000 processor. Thus, the time gain in computation on a grid succession is more than 8-fold.

A considerable gain in the computation time can also be reached using extrapolation according to Richardson [9].

In the process of calculation, additional control is given to the law of heat conservation as consequence to equations (1)–(5), initial and boundary conditions. The above results of numerical experiments indicate to a high efficiency and competitiveness of the devised numerical model. In more detail, some aspects of the development of the numerical model are given in [10, 11].

2. Modeling of the thermal convection under heterogeneous continental lithosphere

The problem of investigation of the thermal convective flows in the upper mantle below ancient continental platform will be examined as an example for the modeling of the real mantle processes. Thickened up to 200 km lithosphere is a particular quality of the platform, whereas the surrounding lithosphere is marked only with 120 km thickness according to geophysical data.

The Rayleigh number which determines the convective mode was chosen as $\mathrm{Ra} = 3.04 \cdot 10^5$ that corresponds to our knowledge about the conditions in the Earth's interior. The main model parameters were taken as follows:

$$d = 700000 \text{ m}, \quad \Delta T = 1800\,^\circ\text{C}, \quad \chi = 10^{-6}\,\text{m}^2/\text{s},$$
$$\alpha = 10^{-5}\,^\circ\text{C}^{-1}, \quad \rho = 3300 \text{ kg/m}^3, \quad g_z = 10 \text{ m/s}^2, \quad \eta_0 = 3 \cdot 10^{21}\,\text{Pa} \cdot \text{s}.$$

The small-scale thermal convection at the 200–350 km depth "astenospheric" level that was found below the thickened lithosphere is one of the principal results of the 3D modeling of the thermal convection in the upper mantle. The result confirms an assumption made in [12] in the 2D numerical experiments on the simulation of the thermal convection below the heterogeneous thickness lithosphere. The small-scale thermal convection is concentrated in the astenosphere as elongated sells with the horizontal size 500 km. This mode has developed at the periphery of the thickened lithosphere (Figure 1).

The small-scale convection has a geological sense associated with the superplume problem on the continents. The superplume conception is now

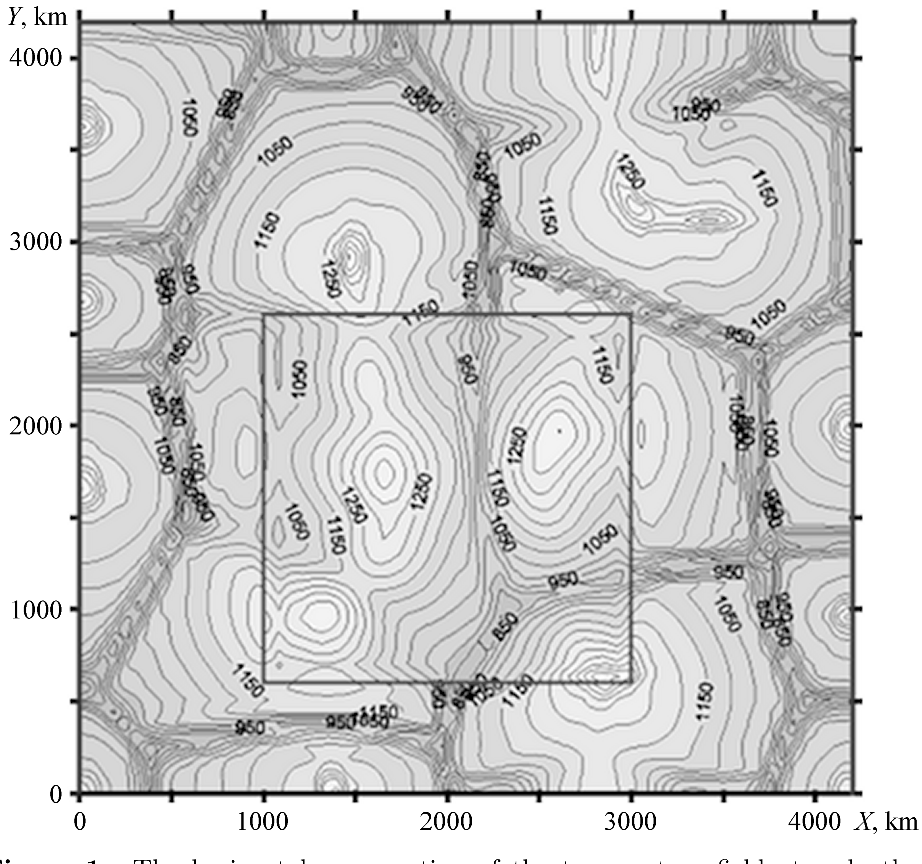

Figure 1. The horizontal cross-section of the temperature field at a depth of 220 km of the thermal convection model in the upper mantle beneath the continental lithosphere with a thickened platform marked with the thick line

widely used as a possible mantle mechanism for the most large-scale geological phenomena as a break-up of the supercontinents or the flood basalt flows [13]. In this paper, we consider the second phenomenon only. The permo-triasic magmatism occurred in the north part of Asia about 240–250 Myear ago. A huge flow of the flood basalt covered the west part of the Siberian platform, the West Siberian plate, the Karsk sea on the north and were found in Altai and in the south of Mongolia. Our modeling shows that the ascending convective flow under the thickened lithosphere is the most stable mode of the upper mantle thermal convection. It is possible to assume that the temperature of the ascending mantle flow was higher than usual in the large-scale flood basalt flows. Increasing of temperature to 200–400 °C, could be caused by appearance of the lower mantle superplume at the bottom of the upper mantle. The presence of the lower mantle material in Siberian lavas was confirmed by geochemical and isotopic data [14]. So, most likely that the main volume of the flood basalts was related to the central ascending mantle convective flow under the Siberian platform. At the same time, there were local magmatic events at the periphery of the platform. In the south there was marked granite and bimodal magmatism in Altai, the east Tuva, Baikal, Mongolia and China. The flood basalts of this age were found in the West Siberian platform in the west and on Taimyr, to the north of the Siberian platform (Figure 2). Such a periphery magmatism

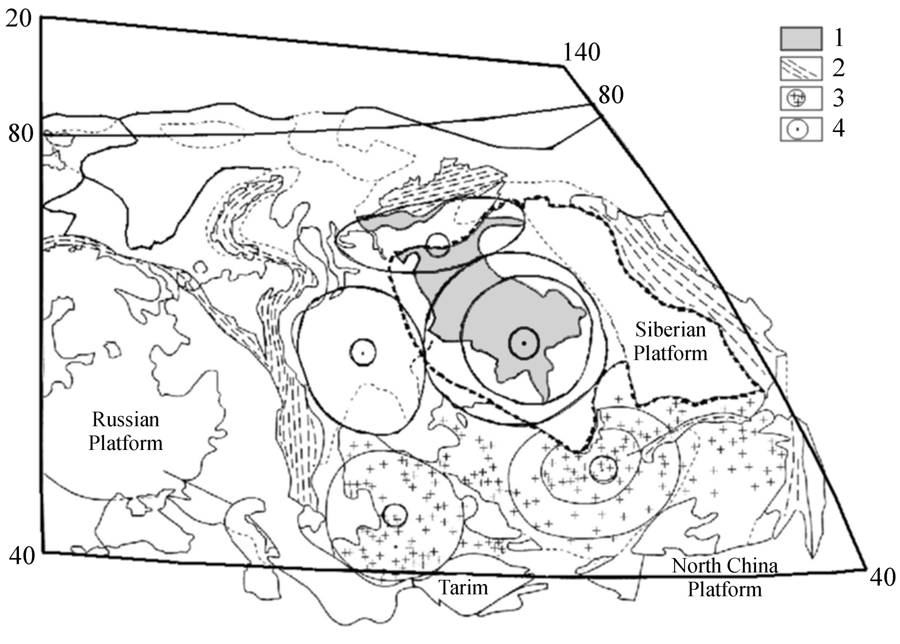

Figure 2. Areals of the permo-triassic magmatism in Asia [13]: 1) trapps of the Siberian platform, 2) fold belts, 3) granite and bimodal magmatism at the periphery of the platform, 4) the centers of the local upper mantle plumes

was born by a small-scale thermal convection when the upper mantle was thermally stimulated by the lower mantle superplume. Usually a small-scale flow brings a hot material with temperature 1300–1400 °C to the base of the lithosphere, but in the presence of the superplume this material would be higher at 100–200 °C. So, there were conditions for the partial decompressing melting at the periphery of the platform, where thickness of the lithosphere was about 120 km. Such a melting forms sources for periphery magmatism during the flood basalts events.

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