Bull. Nov. Comp. Center, Num. Anal., 9 (2000), 91–99

Fast coding-decoding algorithm in fractal image compression via spherical ranges classification*

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The paper deals with the new classification of the ranges in image based on simple geometrical consideration. This approach leads to the fast algorithms for the training of fractal bases and fast coding-decoding processes in the image compression. Some numerical examples are also presented.

1. Introduction

One of the simplest algorithms oriented to the image compression is the use of the fractal bases. Formally, the fractal base is the fixed array of the simplest graphical or digital elements (fractals) which are able to code (with the acceptable loss of accuracy) the ranges of the real image. In this approximation, it is possible to execute the following simple operations with the fractals: scaling, orthogonal transformations (rotations, reflections), summing with the constant (changing of the brightness), multiplying to the constant (contrasting). After these coding of the ranges of the initial image, we need to store (instead of range) only the number of fractal in the base and parameters of its transformation. The volume of this information is usually less than initial one. In the decoding process, we have to replace the range by the transformed fractal from the base. This is the general computational scheme.

Let us note the main problems arising in the construction of the fractal image compression algorithm. 1) How to assemble the fractal base? Would we desire to organize the universal fractal base for the effective coding of all images from the different applications? How to restrict the volume of the universal fractal base? Maybe, the creation (teaching) of the base, oriented to the concrete application is a more reasonable approach. 2) The main and the most important part for assembling the fractal base is the problem of range classification. Together with the fast and reasonable forming of the classes, we need to organize the fast search of the "middle" point (cluster

*Supported by Interschool Scientific-Technical Program *Fundamental higher-school research in the field of natural sciences and humanities. Universities of Russia* under Grant 99-11-16.

center or gravity center) in the class, and we would like to refuse from the exhausting search. From this point of view we need to provide the convexity (at any sense) of every class. We will construct the classification algorithm for the fast assembling of the fractal base and for the fast code-decode processing. Let us note that the classifications suggested in [1–3] do not provide the convexity of the classes.

2. Spherical classification of the half-tone ranges

Let us consider the pixel range of the size $N \times N$ pixels, $N = 2^n$, $n \geq 2$ with the values of intensity at every point, divide it into four equal ranges of the size $2^{n-1} \times 2^{n-1}$ and calculate the mean values f_1, f_2, f_3, f_4 of intensity at every small square (Figure 1).

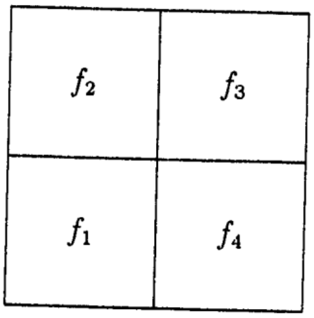

Figure 1

Let us execute the following transformations with the 4D-vector $\vec{f}$: add to $\vec{f}$ the constant vector $\vec{\lambda} = \lambda\vec{e}$, $\vec{e} = (1,1,1,1)$, $\lambda \in R^1$ to provide the condition $\sum_{i=1}^{4}(f_i + \lambda) = 0$; after that we execute the Euclidian normalization of the vector $\tilde{x} = (\tilde{x_1}, \tilde{x_2}, \tilde{x_3}, \tilde{x_4})$, $\tilde{x_i} = f_i + \lambda$, $i = \overline{1,4}$, and obtain the vector $\overline{x} = (x_1, x_2, x_3, x_4)$ with the components $x_i = \tilde{x}_i\Big(\sum\limits_{k=1}^{4} x_k^2\Big)^{-1}$.

Finally,

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ {x_1}^2 + {x_2}^2 + {x_3}^2 + {x_4}^2 = 1 \end{cases} \tag{1}$$

The set determined by (1) in R^4 is the intersection of unit 4D-sphere S_4 with the hyperplane containing the origin. Hence, this set is also unit 3D-sphere S_3 lying in the hyperplane (Figure 2). Let us introduce the following transformation of variables:

$$\begin{aligned} \xi_1 &= \frac{1}{2}(x_1 - x_2 + x_3 - x_4), \\ \xi_2 &= \frac{1}{2}(x_1 - x_2 - x_3 + x_4), \\ \xi_3 &= \frac{1}{2}(x_1 + x_2 - x_3 - x_4), \\ \xi_4 &= \frac{1}{2}(x_1 + x_2 + x_3 + x_4) = 0. \end{aligned} \tag{2}$$

It is clear that the variable $\xi_4 \equiv 0$ vanishes at 3D-sphere S_3 and the variables ξ_1, ξ_2, ξ_3 become its directing cosines, ${\xi_1}^2 + {\xi_2}^2 + {\xi_3}^2 = 1$.

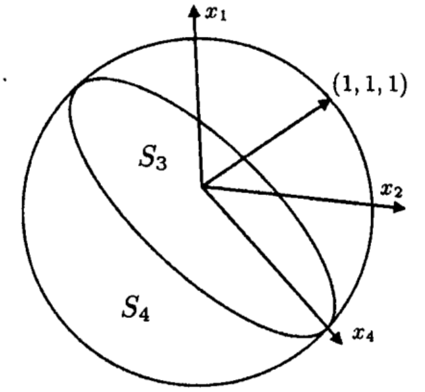

Figure 2

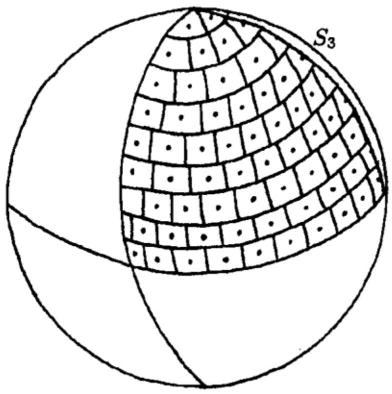

Figure 3

Thus every $N \times N$-range corresponds to the point at the unit 3D-sphere. Moreover, this point is invariant with respect to adding of the constant (brightness), to the multiplying to the constant (contrasting), and, also to the size of the range (scaling). The signs of the directing cosines show in what of eight octants of the 3D-sphere our point is lying. To reduce the fractal base which we want to construct and to provide partially the rotation-reflection operations with the fractal we restrict the consideration to the positive octant $\xi_1, \xi_2, \xi_3 \geq 0$ by changing of all signs to plus (but the initial information about the signs of cosines is necessary for the decoding process as we will see below). We restrict ourself by eight rotations-reflections which map the square to itself. Now it is very easy to organize the fast and effective classification of the ranges using their spherical characteristics. Let us divide the interval [0, 1], where the modulus of the first directing cosine is varying with the uniform mesh $0 \leq \xi_1^{(0)} \leq \xi_1^{(1)} \leq \ldots \leq \xi_1^{(S)}$. We say the range belongs to the class $\mathcal{K}_i$ if

$$\xi_1 \in [{\xi_1}^{(i-1)}, {\xi_1}^{(i)}) \tag{3}$$

Every class $\mathcal{K}_i$ corresponds with the "latitude stripe" on the 3D-sphere. Every stripe $\mathcal{K}_i$ can be divided with the subclasses $\mathcal{K}_{ij}$, $j = 1, \ldots, m_j$, submitted to the inequalities

$$\xi_2^{(j-1)} \leq \xi_2 < \xi_2^{(j)}. \tag{4}$$

In accordance with the "longitude" we ought to regulate the number of the mesh points in every stripe to provide the approximately equal squares of the arising spherical rectangulars.

The mesh steps with respect to ξ_1 and ξ_2 depend on the volume N (number of fractal) of the fractal base we want to organize.

Let us consider the class $\mathcal{K}_{ij}$. It is generated by the convex restrictions (4), (5). If we define the convex combination with two ranges of the equal size from $\mathcal{K}_{ij}$, it is easy to understand that this combination is also lying in the class $\mathcal{K}_{ij}$. Thus, every class generated by spherical classification is convex and form the convex cone in the space of vectors of the dimension $N \times N$.

We discuss now the "teaching" (assembling) of the fractal base, coding and decoding processes.

3. Teaching of the fractal base

Let us fix the size of teaching ranges. Structurally the process for the forming of the fractal base (FB) can be described with four consequent executions (Figure 4):

1) organization of the source of typical ranges,

2) classification of the ranges,

3) determination of the class centers (fractals),

4) fulfillment of FB with the fast search tools.

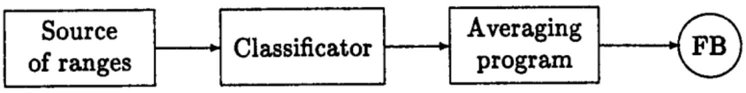

Figure 4. Teaching mode

As example, the source of ranges can be organized with the typical pictures (one or few) in the following way: take all ranges of the given size from the picture. After that we send this range flow to classification. In every arising class, its gravity center $\beta^{(k)}$ for the ranges R (as for $N \times N$ vectors) can be currently calculated by the recurrent formula

$$\beta_{n+1}^{(k)} = \frac{n}{n+1}\beta_n^{(k)} + \frac{1}{n+1}R. \tag{5}$$

After that we need to normalize $\beta_{n+1}^{(k)}$ to the unit sphere. This vector (as $N \times N$-range) become k-th fractal. Because of convexity, the fractal also belongs to the same class.

4. Coding process

If we want to code picture, let us divide it into elementary ranges $\{R_i\}_{i=1}^{M}$ with the same size as the size of fractals $\{C_i\}_{i=1}^{T}$ in FB (with the overlapping or not). Send every range $R_j = \{r_i^{(j)}\}_{i=1}^{N^2}$ to classification and find the numbers i, j of the class. Keep the signs of directing cosines ξ_1, ξ_2, ξ_3 to clarify what kind *rot* of eight rotations (reflections) we need to do with the fractal. After that let us approximate the range by optimal choice of additional brightness σ and contrast h by the minimizing of the mesh L_2-norm I, $I^2 = \sum_{i=1}^{N^2}(\sigma c_i^{(j)} + h - r_i^{(j)})^2$. It leads to 2×2-linear algebraic system with the solution

$$\begin{aligned} \sigma &= \frac{N^2 \sum_{i=1}^{N^2} c_i^{(j)} r_i^{(j)} - \sum_{i=1}^{N^2} c_i^{(j)} \sum_{i=1}^{N^2} r_i^{(j)}}{N^2 \sum_{i=1}^{N^2}\left(c_i^{(j)}\right)^2 - \left(\sum_{i=1}^{N^2} c_i\right)^2}, \\ h &= \frac{1}{N^2}\left(\sum_{i=1}^{N^2} r_i^{(j)} - \sigma \sum_{i=1}^{N^2} c_i^{(j)}\right), \end{aligned} \tag{6}$$

here $C_j = \{c_i^{(j)}\}_{i=1}^{N^2}$. The case $N^2 \sum_{i=1}^{N^2}\left(c_i^{(j)}\right)^2 = \left(\sum_{i=1}^{N^2} c_i\right)^2$ is not realized on practice, since according to norm condition C_i we have

$$\sigma = \sum_{i=1}^{N^2} c_i^{(j)} r_i^{(j)}, \qquad h = \frac{1}{N^2}\sum_{i=1}^{N^2} r_i^{(j)} \tag{7}$$

To reduce the volume of the final code array (Figure 5) we usually do the quantization (by any way) of the real number σ and h to store few bits only.

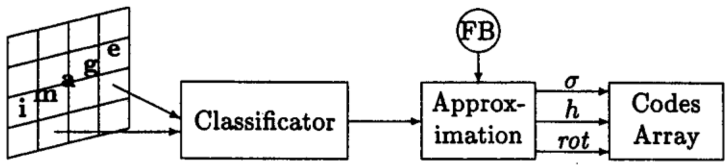

Figure 5. Coding mode

5. Decoding process

The decoding is the simplest block among the described ones (Figure 6). During the scanning of the code array we have the complete information on the position of the range in the initial image, on the number of fractal which we need to transform with the known parameters rot, σ, h (see Figure 6).

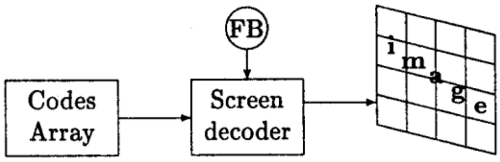

Figure 6. Decoding mode

If we have the overlapping of the ranges in coding process we need to average the corresponding transformed fractals.

6. Numerical experiments

Example 1. The coding the X-ray image by fractal base constructed on Akkem mountain lake (Altay).

We have selected two principally different images: the digitized photography of the well-known Russian Akkem mountain lake (Figure 7) and medical X-ray, we call "Xray" (Figure 8). The base consisting of 256 fractals was built on image Akkem. In fractal base construction, we took 16 nodes uniform mesh with respect to ξ_1 and, applying our classification, every class was divided by nonuniform mesh with respect to ξ_2. We use the 8×8 dimension for every fractal. This base was applied for coding the "Xray". Here we used 7 and 5 bits for keeping parameters σ and h, respectively.

For experimental clarity, the teaching mode of base for this image was turn off. As the result of the coding the compression ratio was 51 times while JPEG technology provided only 29 with the quality coefficient $q = 75\%$. Note that coding time was about 1 sec while standard algorithm of fractal image compression provides about 90 sec.

Example 2. Artificial fractal base.

The coding parameters were selected as in Example 1. For the compression of "Xray" the artificial base was applied, containing also 256 fractals of the size 8×8. The results of "Xray" reconstruction with this base were approximately the same (Figures 9, 10) as in Example 1. Coding time and compression ratio were also the same for both experiments.

We believe, that the artificial base building as described above will decode images of all sorts in acceptable quality.

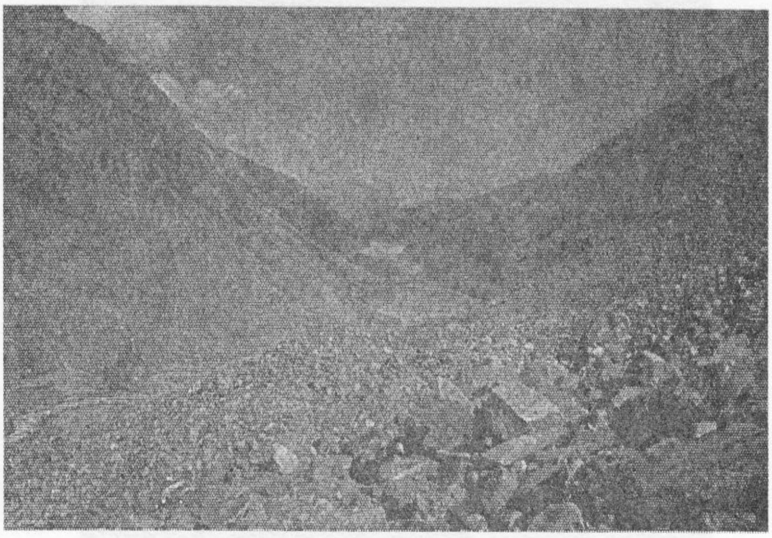

Figure 7. Original "Akkem" image used for fractal base construction

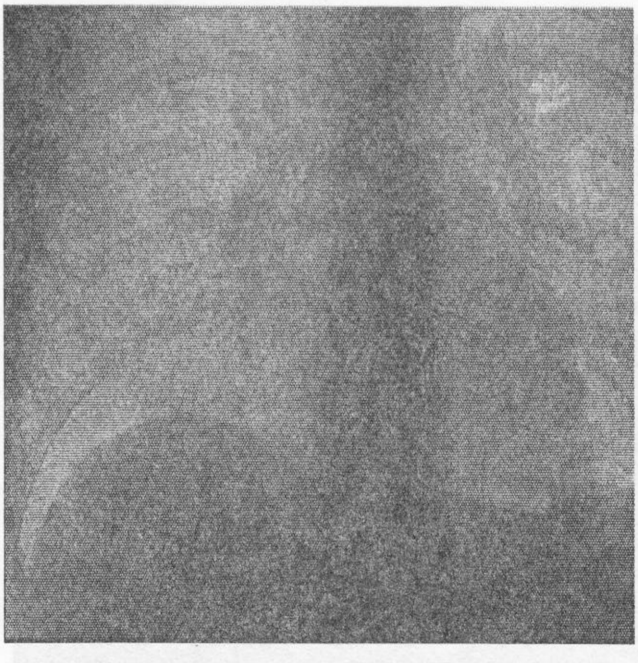

Figure 8. Original medical image "Xray"

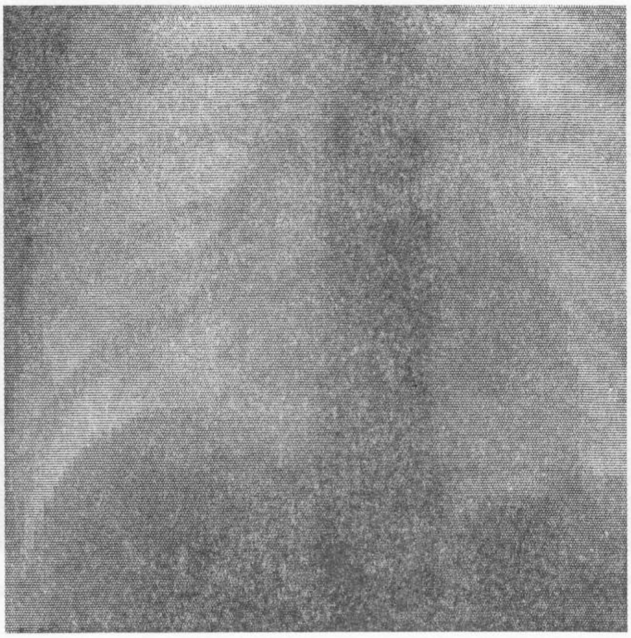

Figure 9. Reconstructed image with artificial base

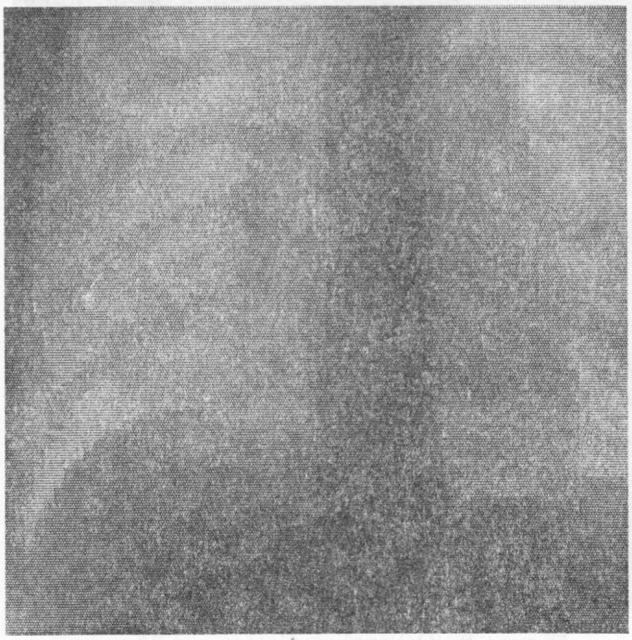

Figure 10. Reconstructed image with Akkem's base

7. Conclusion and future work

Spherical classification combined with fractal image compression algorithm provides a significant speedup of encoding time. Please, pay your attention that the coding process can be combined with the teaching mode to improve current fractal base. We have described here the experiments in which we used the ranges of fixed dimension. In future, we are going to combine this classification with the well-known quadtree algorithm to improve the quality of decoded image using different sizes of ranges. The problem of artifacts which always arises in fractal tools is also the subject of future studies.

References

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