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Investigation of weak equivalence notions for event structures*

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1. Introduction

The notion of bisimulation equivalence has been introduced by D.M.R.Park [10]. Informally, two processes are bisimilar if their possible behaviors have the same branching structure, i.e. any behavior of one system can be reproduced by the other system. The great importance and usefulness of bisimulations in the theory of concurrent systems are evident. Mathematically, bisimulation is a very pleasant notion. It leads to a natural behavioral abstraction from transition systems. Algebraically, in the setting of CCS-like languages, bisimulations lead to elegant and simple laws [5]. Moreover, bisimulation equivalence has a beautiful characterization in terms of the Hennessy-Milner logic [5]. In [8] the concept of back-and-forth bisimulations has been introduced. In a back-and-forth bisimulation the agents can simulate each others' behavior not only in the direction of the arrows but also when going back in their history. This kind of bisimulations is interesting because of its connection with temporal and modal logics. These logics give rise to equivalences on transition systems and Kripke structures and it appears to be very useful to give operational characterizations of these equivalences. In the world of temporal and modal logics, there has been a lot of interest in past-tense operators. If one is looking for the operational characterizations of the equivalences induced by logics with a past-time operator, it seems natural to consider back-and-forth bisimulations.

Classical bisimulation theory deals with transition systems. Meanwhile it is possible to adapt the basic notion to systems with a richer structure, e.g. event structures [9] containing all features considered by many different logics. A simple way to do this is to view an event structure as a transition system by considering the graph of its global states. But this does not take a richer structure into account. Different bisimulation equivalences between event structures and their preservation under the action refinement are considered in [2–4]. An advantage of event structures is their ability to naturally represent and study the basic relations — causality, concurrency and conflict (nondeterministic choice) — between the events of structures. It

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is known that the variants of back-and-forth bisimulations capture intuition concerning causality and (implicitly) concurrency but not a conflict between event occurences in the structures. Attempting to get around this lack, we introduce a number of variants of bisimulations which explicitly reflect all the relations between events.

Over the years several semantics of the silent step have been introduced. The silent step normally denoted by the symbol τ, is due to Milner [6]. It enables abstraction from the internal activity. There are several options for defining an equivalence distinguishing just "visible" behaviors of the processes. In the literature, different kinds of such equivalences have been investigated in the interleaving semantics for transition systems and process algebras.

In this paper, for all considered strong bisimulations (not distinguishing a silent move τ from other actions), namely, interleaving, step, pomset and history-preserving, there have been introduced the variants explicitly reflecting conflict and concurrency relations between events in a structure; the weak variants of the mentioned bisimulations are defined and relationships between all considered equivalence notions are established.

Action refinement is a very important operation for the top-down design of concurrent systems, since it allows the level of abstraction to be changed by replacing an action with a system of subactions. Recently, this operation has attracted much attention ([3, 4]). Naturally, one would like a semantics to induce a congruence with respect to all operations of interest. For the refinement operator, it turns out that this is not the case for interleaving, step, and pomset-based semantics. In [3] it has been shown that the history preserving bisimulation is a congruence with respect to the action refinement for event structures without silent moves.

In this paper, for all recently introduced equivalence notions there has been established whether they are preserved under refinement operation or not.

The paper is organized as follows. Section 2 introduces the basic framework, labelled prime event structures, and related notions.

In Section 3, strong bisimulations and their variants (back, conflict-preserving and concurrency-preserving) are considered. A variant of an τ-bisimulation which takes into account the structure of a maximal configuration is introduced. The complete hierarchy of the considered strong bisimulations is built.

Weak bisimulations are introduced in Section 4, where interrelations between different kinds of equivalence notions are established.

Section 5 defines the operation of action refinement for event structures with silent actions. The behaviors of all considered bisimulation notions under the action refinement is investigated.

Conclusion summarizes the basic results.

2. Event structures

Here we consider the labelled prime event structures with silent actions (let us call them simply event structures) as a basic model of concurrent processes. An event structure consists of a set of event occurrences partially ordered by a causality relation. In addition, the structure contains a conflict relation between the events. Two events that are neither causally related nor in conflict are said to be concurrent. The subsets of events corresponding to executions in an event structure are called configurations which must be conflict-free and left-closed with respect to causality.

Definition 2.1. An *event structure* over an alphabet *Act* ($\tau \notin Act$) the symbol τ represents an internal action) is a 4-tuple $\mathcal{E} = (E, <, \#, l)$, where

- E is a countable set of events;
- $< \subseteq E \times E$ is an irreflexive partial order (the *causality relation*) satisfying the *principle of finite causes*:
  $\forall e \in E \circ \{d \in E \mid d < e\}$ is finite;
- $\# \subseteq E \times E$ is an irreflexive, symmetric relation (the *conflict relation*) satisfying the *principle of conflict heredity*:
  $\forall e_1, e_2, e_3 \in E \circ e_1 < e_2 \;\&\; e_1 \# e_3 \Rightarrow e_2 \# e_3$;
- $l : E \to Act_\tau$ is a labelling function, where $Act_\tau = Act \cup \{\tau\}$. □

Through the paper, we assume a set *Act* of action names (labels) to be fixed. The components of an event structure $\mathcal{E}$ are denoted by $E_\mathcal{E}$, $<_\mathcal{E}$, $\#_\mathcal{E}$, and $l_\mathcal{E}$. If it is clear from the context, the index $\mathcal{E}$ is omitted. For an event structure $\mathcal{E}$ we denote:
$id = \{(e, e) \mid e \in E\}$;
$\leq \; = \; < \cup \; id$;
$\smile \; = (E \times E) \setminus (\leq \cup \leq^{-1} \cup \; \#)$ (concurrency);
$co = \; \smile \cup \; id$.

An event structure $\mathcal{E}$ is called *empty* iff $E_\mathcal{E} = \emptyset$; *finite* iff $E_\mathcal{E}$ is finite; *conflict-free* iff $\#_\mathcal{E} = \emptyset$. Two event structures $\mathcal{E}$ and $\mathcal{F}$ are *isomorphic* ($\mathcal{E} \cong \mathcal{F}$) iff there exists a bijection between their sets of events preserving the relations $<$, $\#$, and labelling.

In graphic representations only the immediate conflicts — not the inherited ones — are pictured. The $<$-relation is represented by arcs omitting those derivable by transitivity. Following these conventions, a trivial example of an event structure is shown in Fig. 1, where $E = \{e_1, e_2, e_3, e_4\}$, $< \; = \{(e_1, e_3), (e_1, e_4), (e_2, e_3), (e_2, e_4)\}$, $\# = \{(e_3, e_4), (e_4, e_3)\}$, $l(e_1) = a$, $l(e_2) = b$, $l(e_3) = \tau$, $l(e_4) = b$.

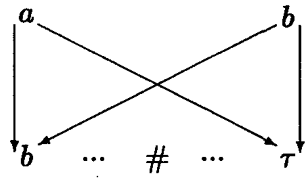


Figure 1

We will frequently give algebraic expressions (see [1]) for our examples, to make them easier to understand. The algebraic syntax includes the primitive constructs: sequential composition (;), parallel composition (||), and sum (+). The operation ; (||, +, respectively) may be easily "interpreted" by indicating that all events in one component are in the $<$-relation ($\smile$-relation, $\#$-relation, respectively) with all events in the other one.

The states of an event structure are called configurations. An event can occur in a configuration only if all the events in its past have occurred. Two events that are in conflict can never both occur in the same stretch of the behavior.

Before giving the formal definition of a configuration, we need some preparations. Let $\mathcal{E}$ be an event structure and $C \subseteq E_{\mathcal{E}}$. Then $\uparrow C = \{e \in E_{\mathcal{E}} \mid \exists e' \in C \diamond e' \leq_{\mathcal{E}} e\}$ and $\downarrow C = \{e \in E_{\mathcal{E}} \mid \exists e' \in C \diamond e \leq_{\mathcal{E}} e'\}$. For a singleton $e \in E_{\mathcal{E}}$, we will write $\uparrow e$ instead of $\uparrow \{e\}$ and $\downarrow e$ instead of $\downarrow \{e\}$.

Definition 2.2. A *configuration* of the event structure $\mathcal{E}$ is the subset $C \subseteq E_{\mathcal{E}}$ such that

- $\forall e, e' \in C \diamond \neg(e \#_{\mathcal{E}} e')$ (conflict-free);
- $\forall e, e' \in E_{\mathcal{E}} \diamond e \in C \ \& \ e' \leq_{\mathcal{E}} e \Rightarrow e' \in C$ (left-closed). □

Let $\mathcal{C}(\mathcal{E})$ denote the set of all configurations of $\mathcal{E}$. A configuration $C \in \mathcal{C}(\mathcal{E})$ is called *maximal* iff the following holds: $C' \in \mathcal{C}(\mathcal{E}) \ \& \ C \subseteq C' \Rightarrow C = C'$, i. e. C is a maximal set. Let $\mathcal{RC}(\mathcal{E})$ denote the set of maximal configurations of $\mathcal{E}$.

It is clear that $\downarrow e \in \mathcal{C}(\mathcal{E})$ for each $e \in E_{\mathcal{E}}$. Let $\mathcal{LC}(\mathcal{E}) = \{\downarrow e \mid e \in E_{\mathcal{E}}\}$ denote the set of *local configurations* of $\mathcal{E}$ and $\mathcal{LC}_{vis}(\mathcal{E}) = \{\downarrow e \in \mathcal{LC}(\mathcal{E}) \mid l_{\mathcal{E}}(e) \in Act\} \cup \{\emptyset\}$ denote the set of *visible local configurations* of $\mathcal{E}$. Additionally, we assume $\mathcal{LC}_0(\mathcal{E}) = (\mathcal{LC}(\mathcal{E}) \cup \{\emptyset\})$.

Let $C' \subseteq C \in \mathcal{C}(\mathcal{E})$. Then C' is called a *step* if $\forall e_1, e_2 \in C' \diamond \neg(e_1 <_{\mathcal{E}} e_2)$; the *restriction* of $\mathcal{E}$ to C' is defined as $\mathcal{E} \lceil C' = (C', <_{\mathcal{E}} \cap (C' \times C'), \#_{\mathcal{E}} \cap (C' \times C'), l_{\mathcal{E}} \mid_{C'})$. We use $pom_{\mathcal{E}}(C) = \{(\mathcal{E} \lceil (C'' \setminus C))/\cong \mid C \subseteq C'' \in \mathcal{C}(\mathcal{E})\}$ to denote the set of *pomsets* of C. We denote by C' not only the set itself, but also the labelled partial order it induces by restricting $<_{\mathcal{E}}$ and $l_{\mathcal{E}}$ to C'. It will (hopefully) be clear from the context what is meant. We will use the following notation for $C \subseteq E_{\mathcal{E}}$ and $p \in pom_{\mathcal{E}}(C)$:

$vis(C) = \{e \in C \mid l_{\mathcal{E}}(e) \in Act\}$; $vis(p) = p \lceil vis(E_p)$;
$hart(C) = \downarrow vis(C) \cap \uparrow vis(C)$; $hart(p) = p \lceil hart(E_p)$.

Definition 2.3. Let $\mathcal{E}$ be an event structure and C, $C' \in \mathcal{C}(\mathcal{E})$. Then

(i) $C \rightarrow_{\mathcal{E}} C' \overset{def}{\iff} C \subseteq C'$,
$\mapsto_{\mathcal{E}}$ denotes $\rightarrow_{\mathcal{E}}|_{\mathcal{LC}_0^2(\mathcal{E})}$;

(ii) $C \xrightarrow{p}_{\mathcal{E}} C' \overset{def}{\iff} C \rightarrow_{\mathcal{E}} C'$ and $C' \setminus C = p$, where $p \in pom_{\mathcal{E}}(C)$,
$\overset{p}{\mapsto}_{\mathcal{E}}$ denotes $\xrightarrow{p}_{\mathcal{E}}|_{\mathcal{LC}_0^2(\mathcal{E})}$;

(iii) $C \overset{\epsilon}{\Rightarrow}_{\mathcal{E}} C' \overset{def}{\iff} C \xrightarrow{p}_{\mathcal{E}} C'$ and $vis(p) = \emptyset$,
$\overset{\epsilon}{\models\!\!\Rightarrow}_{\mathcal{E}}$ denotes $\overset{\epsilon}{\Rightarrow}_{\mathcal{E}}|_{\mathcal{LC}_0^2(\mathcal{E})}$;

(iv) $C \overset{p}{\Rightarrow}_{\mathcal{E}} C' \overset{def}{\iff} \exists C_1, C_2 \diamond C \overset{\epsilon}{\Rightarrow}_{\mathcal{E}} C_1 \xrightarrow{p}_{\mathcal{E}} C_2 \overset{\epsilon}{\Rightarrow}_{\mathcal{E}} C'$ and $p = hart(p)$,
$C \overset{p}{\models\!\!\Rightarrow}_{\mathcal{E}} C' \overset{def}{\iff} \exists C_1, C_2 \diamond C \overset{\epsilon}{\models\!\!\Rightarrow}_{\mathcal{E}} C_1 \overset{p}{\mapsto}_{\mathcal{E}} C_2 \overset{\epsilon}{\models\!\!\Rightarrow}_{\mathcal{E}} C'$ and $p = hart(p)$;

(v) $C \uparrow_{\mathcal{E}} C' \overset{def}{\iff} \exists C'' \in \mathcal{C}(\mathcal{E}) \diamond (C \rightarrow_{\mathcal{E}} C''\ \&\ C' \rightarrow_{\mathcal{E}} C'')$,
$\underline{\uparrow}_{\mathcal{E}}$ denotes $\uparrow_{\mathcal{E}}|_{\mathcal{LC}_0^2(\mathcal{E})}$;

(vi) $C \not\uparrow_{\mathcal{E}} C' \overset{def}{\iff} \neg(C \uparrow_{\mathcal{E}} C')$,
$\underline{\not\uparrow}_{\mathcal{E}}$ denotes $\not\uparrow_{\mathcal{E}}|_{\mathcal{LC}_0^2(\mathcal{E})}$;

(vii) $C \uparrow'_{\mathcal{E}} C' \overset{def}{\iff} \neg(C \not\uparrow_{\mathcal{E}} C' \vee C \rightarrow_{\mathcal{E}} C' \vee C' \rightarrow_{\mathcal{E}} C)$,
$\underline{\uparrow}'_{\mathcal{E}}$ denotes $\uparrow'_{\mathcal{E}}|_{\mathcal{LC}_0^2(\mathcal{E})}$. □

Lemma 2.1. Let $\mathcal{E}$ be an event structure, C and $C' \in \mathcal{C}(\mathcal{E})$, and $\downarrow d$ and $\downarrow d' \in \mathcal{LC}(\mathcal{E})$. Then

(i) $C \uparrow_{\mathcal{E}} C' \iff C \cup C' \in \mathcal{C}(\mathcal{E})$;

(ii) $C \not\uparrow_{\mathcal{E}} C' \iff \exists e \in C\ \exists e' \in C' \diamond e \#_{\mathcal{E}} e'$;

(iii) $C \uparrow'_{\mathcal{E}} C' \iff C \setminus C' \neq \emptyset \neq C' \setminus C \& \forall e \in C \setminus C' \forall e' \in C' \setminus C \diamond e \smile_{\mathcal{E}} e'$;

(iv) $\downarrow d \mapsto_{\mathcal{E}} \downarrow d' \iff d \leq_{\mathcal{E}} d'$;

(v) $\downarrow d\ \underline{\not\uparrow}_{\mathcal{E}} \downarrow d' \iff d \#_{\mathcal{E}} d'$;

(vi) $\downarrow d\ \underline{\uparrow}'_{\mathcal{E}} \downarrow d' \iff d \smile_{\mathcal{E}} d'$.

Proof follows easily from Definition 2.3. □

An event structure $\mathcal{E}$ is called an *event structure without autoconcurrency* if $\forall e, e' \in E_{\mathcal{E}} \diamond (e\ co_{\mathcal{E}}\ e'\ \&\ l_{\mathcal{E}}(e) = l_{\mathcal{E}}(e')) \Rightarrow e = e'$.

Lemma 2.2. Let $\mathcal{E}$ be an event structure without autoconcurrency and C, C', $C'' \in \mathcal{C}(\mathcal{E})$ with $C' \neq C''$. Then $C' \xrightarrow{p}_{\mathcal{E}} C$ and $C'' \xrightarrow{q}_{\mathcal{E}} C \Rightarrow p \not\simeq q$.

Proof. Let $C' \xrightarrow{p}_{\mathcal{E}} C$ and $C'' \xrightarrow{q}_{\mathcal{E}} C$. Assume the contrary, i.e. there exists an isomorphism $f : (C \setminus C') \to (C \setminus C'')$. Since $C' \neq C''$, there exist $e \in C'' \setminus C' \subseteq (C \setminus C')$ and $f(e) \in C' \setminus C'' \subseteq (C \setminus C'')$ such that $e \neq_{\mathcal{E}} f(e)$. Let us consider all possible relations between e and $f(e)$. Here $f(e) <_{\mathcal{E}} e$ ($e <_{\mathcal{E}} f(e)$) contradicts $C'' \in \mathcal{C}(\mathcal{E})$ ($C' \in \mathcal{C}(\mathcal{E})$); $e \#_{\mathcal{E}} f(e)$ contradicts $e, f(e) \in C \in \mathcal{C}(\mathcal{E})$; $e \smile_{\mathcal{E}} f(e)$ contradicts the fact that $\mathcal{E}$ is without autoconcurrency. $\square$

In the following we will consider only the event structures without autoconcurrency and call them simply event structures.

3. Strong Bisimulations

In this section we consider bisimulation notions investigated in [2, 4]. Besides, we introduce new variants of bisimulations aimed to reflect all the relations between events in a structure.

Definition 3.1. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures, $\mathcal{B} \subseteq \mathcal{C}(\mathcal{E}) \times \mathcal{C}(\mathcal{F})$, $\alpha \in \{i, s, p, h\}$ and $\beta \in \{a, b, c, r\}^*$. Then
(i) $\mathcal{B}$ is an *α-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $(\emptyset, \emptyset) \in \mathcal{B}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
- $\mathcal{E} \lceil C \cong \mathcal{F} \lceil D$ if $\alpha = h$,
- if $C \xrightarrow{p}_{\mathcal{E}} C'$ such that
  - p has at most one element if $\alpha = i$,
  - p is a step if $\alpha = s$,

  then there are D' and q such that $D \xrightarrow{q}_{\mathcal{F}} D'$, $p \cong q$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(ii) $\mathcal{B}$ is an *αb-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is an α-bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
- if $C' \xrightarrow{p}_{\mathcal{E}} C$ such that
  - p has at most one element if $\alpha = i$,
  - p is a step if $\alpha = s$,

  then there are D' and q such that $D' \xrightarrow{q}_{\mathcal{F}} D$, $p \cong q$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(iii) $\mathcal{B}$ is an *αa-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is an α-bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
- if $C \not\uparrow_{\mathcal{E}} C'$, there is D' such that $D \not\uparrow_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(iv) $\mathcal{B}$ is an *αc-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is an α-bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
- if $C \uparrow'_{\mathcal{E}} C'$, there is D' such that $D \uparrow'_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(v) $\mathcal{B}$ is an *αr-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is an α-bisimulation between $\mathcal{E}$ and $\mathcal{F}$, and the following holds:

- for all $R \in \mathcal{RC}(\mathcal{E})$ and $C \in \mathcal{C}(\mathcal{E})$, if $C \subseteq R$, there are $R' \in \mathcal{RC}(\mathcal{F})$ and $D \in \mathcal{C}(\mathcal{F})$ such that $D \subseteq R'$, $\mathcal{E} \lceil R \cong \mathcal{F} \lceil R'$ and $(C, D) \in \mathcal{B}$,
- and vice versa.

$\mathcal{E}$ and $\mathcal{F}$ are *$\alpha\beta$-bisimilar*, denoted $\mathcal{E} \approx_{\alpha\beta} \mathcal{F}$, if there exists an $\alpha\beta$-bisimulation $\mathcal{B}$ that is an $\alpha\gamma$-bisimulation for all $\gamma \in \beta$. □

We now turn our attention to showing how various bisimulation equivalences just defined are related to those earlier introduced.

Proposition 3.1. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures, $\alpha, \alpha' \in \{i, s, p, h\}$ and $\beta \in \{a, c, r\}^*$. Then

$$\mathcal{E} \approx_{\alpha\beta b} \mathcal{F} \iff \mathcal{E} \approx_{\alpha'\beta b} \mathcal{F}.$$

Proof. It is sufficient to prove that $\mathcal{E} \approx_{i\beta b} \mathcal{F} \iff \mathcal{E} \approx_{h\beta b} \mathcal{F}$.

'$\Rightarrow$' Assume $\mathcal{B}$ to be an $i\beta b$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$. We first show hb-bisimilarity of $\mathcal{B}$. Clearly, $(\emptyset, \emptyset) \in \mathcal{B}$. Let $(C, D) \in \mathcal{B}$. The proof consists of the three parts.

1. First of all, it is necessary to show that $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$. (The case with $C = \emptyset = D$ is trivial.) W.l.o.g. suppose $\emptyset \overset{a_1}{\to}_{\mathcal{E}} C_1 \ \ldots \ C_{n-1} \overset{a_n}{\to}_{\mathcal{E}} C_n = C$. Since $\mathcal{B}$ is an $i\beta b$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$, there are $D_n, D_{n-1}, \ldots, D_1$ and $a_n, \ldots, a_1$ such that $\emptyset \overset{a_1}{\to}_{\mathcal{F}} D_1 \ \ldots \ D_{n-1} \overset{a_n}{\to}_{\mathcal{F}} D_n = D$ and $(C_i, D_i) \in \mathcal{B}$ for all $1 \leq i \leq n$. We proceed by induction on n.

$n = 1$. Obvious.

$n > 1$. By the induction hypothesis, there exists an isomorphism $f_{n-1} : C_{n-1} \to D_{n-1}$. Since $\mathcal{B}$ is an $i\beta b$-bisimulation, we can extend f_{n-1} to a label-preserving bijection $f : C \to D$. Let us prove f to be an isomorphism. Assume $C \setminus C_{n-1} = \{e\}$. Since $C \in \mathcal{C}(\mathcal{E})$ and $D \in \mathcal{C}(\mathcal{F})$, it is sufficient to show that $e' <_{\mathcal{E}} e \iff f(e') <_{\mathcal{F}} f(e)$ for all $e' \in C_{n-1}$. Assume the contrary, i.e. $e' <_{\mathcal{E}} e$ and $\neg(f(e') <_{\mathcal{F}} f(e))$ for some $e' \in C_{n-1}$ (the converse case is symmetric). W.l.o.g. suppose $e' \lessdot_{\mathcal{E}} e$ and $l_{\mathcal{E}}(e') = a_{n-1}$. The case $f(e') =_{\mathcal{F}} f(e)$ contradicts the fact that f is a label-preserving bijection. The case $f(e) <_{\mathcal{F}} f(e')$ contradicts $D_{n-1} \in \mathcal{C}(\mathcal{F})$. The case $f(e) \smile_{\mathcal{F}} f(e')$ remains to be considered. Then we have $D'_{n-1} = D \setminus \{f(e')\} \in \mathcal{C}(\mathcal{F})$, $D'_{n-1} \subset D$, and $D \setminus D'_{n-1} = a_{n-1}$. Hence $D'_{n-1} \overset{a_{n-1}}{\to}_{\mathcal{F}} D$ by Definition 2.2. Since $\mathcal{B}$ is an $i\beta b$-bisimulation, there exists C'_{n-1} such that $C'_{n-1} \overset{a_{n-1}}{\to}_{\mathcal{E}} C$ and $(C'_{n-1}, D'_{n-1}) \in \mathcal{B}$. Let $C \setminus C'_{n-1} = \{e''\}$. We consider all possible relations between events e' and e''.

- $e'' \leq_{\mathcal{E}} e'$. Then $e'' <_{\mathcal{E}} e$, contradicting $C'_{n-1} \in \mathcal{C}(\mathcal{E})$, because $e \in C'_{n-1}$ but $e'' \notin C'_{n-1}$.
- $e' <_{\mathcal{E}} e''$. Then we get the contradiction $e'' \notin C$, since $C \in \mathcal{C}(\mathcal{E})$.
- $e' \#_{\mathcal{E}} e''$. This contradicts $e', e'' \in C$.

- $e' \smile_{\mathcal{E}} e''$. Since $l(e') = l(e'')$, we get a contradiction to the fact that $\mathcal{E}$ is without autoconcurrency.

Thus $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$.

2. Suppose $C \xrightarrow{p'}_{\mathcal{E}} C'$. W.l.o.g. assume $C \xrightarrow{a_1}_{\mathcal{E}} C_1 \ \dots \ C_{n-1} \xrightarrow{a_n}_{\mathcal{E}} C_n = C'$. Since $\mathcal{B}$ is an $i\beta b$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$, there are $D_1, \dots, D_{n-1}, D_n$, such that $D \xrightarrow{a_1}_{\mathcal{F}} D_1 \ \dots \ D_{n-1} \xrightarrow{a_n}_{\mathcal{F}} D_n = D'$ and $(C_i, D_i) \in \mathcal{B}$ for all $1 \leq i \leq n$. Let $D' \setminus D = q'$. Then we have $D \xrightarrow{q'}_{\mathcal{F}} D'$, by Definition 2.3(ii). Since $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$ and $\mathcal{E}\lceil C' \cong \mathcal{F}\lceil D'$ (by the above proof), it is clear that $p' \cong q'$.

3. Suppose $C'' \xrightarrow{p''}_{\mathcal{E}} C$. The proof is similar to that of case 2.
Thus $\mathcal{B}$ is an $h\beta b$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$, by Definition 3.1.

'$\Leftarrow$' This easily follows from Definition 3.1. □

Proposition 3.2. Let $\mathcal{E}$, $\mathcal{F}$ be event structures, $\alpha \in \{i, s, p, h\}$ and $\beta \in \{a, r\}^*$. Then

(i) $\mathcal{E} \approx_{\alpha\beta b} \mathcal{F} \Longleftrightarrow \mathcal{E} \approx_{h\beta c} \mathcal{F}$;

(ii) $\mathcal{E} \approx_{\alpha\beta b} \mathcal{F} \Longleftrightarrow \mathcal{E} \approx_{\alpha\beta bc} \mathcal{F}$.

Proof. (i) '$\Rightarrow$' According to Proposition 3.1, we can set $\alpha = h$. Assume $\mathcal{B}$ to be an $h\beta b$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$. It is necessary to show $h\beta c$-bisimilarity of $\mathcal{B}$. Suppose $(C, D) \in \mathcal{B}$ and $C \uparrow'_{\mathcal{E}} C'$. By Definition 2.2 and Lemma 2.1(i), we have $C \neq C'$ and $C \cup C' = C'' \in \mathcal{C}(\mathcal{E})$. This implies $C \subset C''$ with $p = C'' \setminus C$ and $C' \subset C''$ with $p' = C'' \setminus C'$. Hence $C \xrightarrow{p}_{\mathcal{E}} C''$ and $C' \xrightarrow{p'}_{\mathcal{E}} C''$, due to Definition 2.2. Since $\mathcal{B}$ is an $h\beta b$-bisimulation, we have $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$ and can find D'' and q such that $D \xrightarrow{q}_{\mathcal{F}} D''$ and $(C'', D'') \in \mathcal{B}$. Once again by $h\beta b$-bisimilarity of $\mathcal{B}$, there exist D' and q' such that $D' \xrightarrow{q'}_{\mathcal{F}} D''$ and $(C', D') \in \mathcal{B}$. Note that $\mathcal{E}\lceil C' \cong \mathcal{F}\lceil D'$ and $\mathcal{E}\lceil C'' \cong \mathcal{F}\lceil D''$, due to Definition 3.1(i). It is necessary to show $D \uparrow'_{\mathcal{F}} D'$. Let us proceed by contradiction. According to Definition 2.2, only two cases are admissible, because $D \uparrow_{\mathcal{F}} D'$.

1. $D \rightarrow_{\mathcal{E}} D'$. This implies $D \subseteq D'$, due to Definition 2.3. If $D = D'$, then $\mathcal{E}\lceil(C'' \setminus C) \cong \mathcal{F}\lceil(D'' \setminus D) = \mathcal{F}\lceil(D'' \setminus D') \cong \mathcal{C}\lceil(C'' \setminus C')$ contradicting Lemma 2.2, because $C \neq C'$. Let us consider the case $D \subset D'$. Assume $f : D' \rightarrow C'$ to be an isomorphism and $C_0 = f(D)$. Obviously, $C_0 \subset C'$. We now show $C_0 \in \mathcal{C}(\mathcal{E})$. Clearly, C_0 is conflict-free. Assume $e \in C_0$ and $e' \in E_{\mathcal{E}}$ such that $e' <_{\mathcal{E}} e$. Since $e \in C'$ and $C' \in \mathcal{C}(\mathcal{E})$, $e' \in C'$. So, $f^{-1}(e) \in D$, $f^{-1}(e') \in D'$ and $f^{-1}(e') <_{\mathcal{F}} f^{-1}(e)$. Since $D \in \mathcal{C}(\mathcal{F})$, we have $f^{-1}(e') \in D$. This implies $e' \in C_0$, i.e. $C_0 \in \mathcal{C}(\mathcal{E})$. Hence $C_0 \rightarrow_{\mathcal{E}} C'$ and $C_0 \rightarrow_{\mathcal{E}} C''$, by Definition 2.3. Clearly, $\mathcal{E}\lceil(C'' \setminus C_0) \cong \mathcal{E}\lceil(C'' \setminus C)$. Then $C_0 \neq C$ contradicts Lemma 2.2 and $C_0 = C$ contradicts Definition 2.3, because $C_0 \rightarrow_{\mathcal{E}} C'$ and $C_0 \uparrow'_{\mathcal{E}} C'$.

2. $D' \rightarrow_{\mathcal{E}} D$. Symmetric to case 1.

'$\Leftarrow$' Assume $\mathcal{B}$ to be a minimal $h\beta c$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$. Let us first show $i\beta b$-bisimilarity of $\mathcal{B}$. Suppose $(C, D) \in \mathcal{B}$ and $C' \xrightarrow{a}_{\mathcal{E}} C$. This implies $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$, by Definition 3.1(i), and $C \setminus C' = a$, by Definition 2.2. Let $f : C \to D$ be an isomorphism and $C \setminus C' = \{e\}$. Take $D' = D \setminus \{f(e)\}$. Clearly, $D \setminus D' = a$ and $\mathcal{E}\lceil C' \cong \mathcal{F}\lceil D'$. Then $D' \in \mathcal{C}(\mathcal{F})$. Hence $D' \xrightarrow{a}_{\mathcal{F}} D$, due to Definition 2.2. Suppose the contrary, i.e. $(C', D') \notin \mathcal{B}$. By $h\beta c$-bisimilarity of $\mathcal{B}$, there are C'', D'' and p, q such that $(C'', D'') \in \mathcal{B}$, $\mathcal{E}\lceil C'' \cong \mathcal{F}\lceil D''$ and $C'' \xrightarrow{p}_{\mathcal{E}} C$, $D'' \xrightarrow{q}_{\mathcal{F}} D$. Note, $p \cong q$. Thus $C' \uparrow_{\mathcal{E}} C''$ (and $D' \uparrow_{\mathcal{F}} D''$), due to Definition 2.2. In the case $C' = C''$ (or $D' = D''$), we have $a = p \cong q$ (or $a = q \cong p$). Then $D' \neq D''$ (or $C' \neq C''$) contradicts Lemma 2.2 and $D' = D''$ (or $C' = C''$) contradicts $(C', D') \notin \mathcal{B}$. Hence $C' \neq C''$ (and $D' \neq D''$). Obviously, $C \not\subset C'$ (and $D' \not\subset D''$). By Definition 2.2., we have $\neg(C' \rightarrow_{\mathcal{E}} C'')$ (and $\neg(D' \rightarrow_{\mathcal{F}} D'')$) and two cases remain to be considered.

1. $C'' \rightarrow_{\mathcal{E}} C'$ (and $D'' \rightarrow_{\mathcal{F}} D'$). Let $C' \setminus C'' = p'$ (and $D' \setminus D'' = q'$). Then $C'' \xrightarrow{p'}_{\mathcal{E}} C'$ (and $D'' \xrightarrow{q'}_{\mathcal{F}} D'$), according to Definition 2.2. By $h\beta c$-bisimilarity of $\mathcal{B}$, there are D''' (and C''') and q'' (and p'') such that $D'' \xrightarrow{q''}_{\mathcal{F}} D'''$ (and $C'' \xrightarrow{p''}_{\mathcal{E}} C'''$) and $(C', D''') \in \mathcal{B}$ (and $(C''', D') \in \mathcal{B}$). Since $(C', D') \notin \mathcal{B}$, it holds that $D' \neq D'''$ (and $C' \neq C'''$). Once again by $h\beta c$-bisimilarity of $\mathcal{B}$, we have $\mathcal{E}\lceil C' \cong \mathcal{F}\lceil D'''$ (and $\mathcal{E}\lceil C''' \cong \mathcal{F}\lceil D'$) and can find $\tilde{D}$ (and $\tilde{C}$) and a such that $D''' \xrightarrow{a}_{\mathcal{F}} \tilde{D}$ (and $C''' \xrightarrow{a}_{\mathcal{E}} \tilde{C}$) and $(C, \tilde{D}) \in \mathcal{B}$ (and $(\tilde{C}, D) \in \mathcal{B}$). From Lemma 2.2 it follows that $D \neq \tilde{D}$ (and $C \neq \tilde{C}$), because $D' \neq D'''$ (and $C' \neq C'''$). It is easy to check that $\mathcal{B}' = \mathcal{B} \setminus \{(C, D)\}$ is an $h\beta c$-bisimulation contradicting the minimality of $\mathcal{B}$.

2. $C' \uparrow'_{\mathcal{E}} C''$ (and $D' \uparrow'_{\mathcal{F}} D''$). Then $C'' \setminus C' = a$ (and $D'' \setminus D' = a$) and $C' \setminus C'' = p$ (and $D' \setminus D'' = q$). Due to $h\beta c$-bisimilarity of $\mathcal{B}$, we can find D''' (and C''') such that $D'' \uparrow'_{\mathcal{F}} D'''$ (and $C'' \uparrow'_{\mathcal{E}} C'''$) and $(C', D''') \in \mathcal{B}$ (and $(C''', D') \in \mathcal{B}$). Since $(C', D') \notin \mathcal{B}$, it holds that $D' \neq D'''$ (and $C' \neq C'''$). We now show $D'' \setminus D''' = a$ (and $C'' \setminus C''' = a$) and $D''' \setminus D'' = q$ (and $C''' \setminus C'' = p$). By $h\beta c$-bisimilarity of $\mathcal{B}$, we have $\mathcal{E}\lceil C' \cong \mathcal{F}\lceil D'''$ (and $\mathcal{E}\lceil C''' \cong \mathcal{E}\lceil D'$). Since $\mathcal{E}\lceil C' \cong \mathcal{F}\lceil D'$, there exists an isomorphism $f : D' \to D'''$ (and an isomorphism $g : C' \to C'''$) such that $f(D' \cap D''') = D' \cap D'''$ (and $g(C' \cap C''') = C' \cap C'''$). Let $D'' \setminus D' = \{d\}$ (and $C'' \setminus C' = \{e\}$). Let us show $D'' \setminus D''' = \{d\}$ (and $C'' \setminus C''' = \{e\}$). Suppose the contrary, i.e. there is $d' \in D'' \setminus D'''$ (and $e' \in C'' \setminus C'''$) such that $d' \neq_{\mathcal{F}} d$ (and $e' \neq_{\mathcal{E}} e$). Then $d' \in D' \setminus D'''$ (and $e' \in C' \setminus C'''$) and $f(d') \in D''' \setminus D'$ (and $g(e') \in C''' \setminus C'$). Clearly, $d' \neq_{\mathcal{F}} f(d')$ (and $e' \neq_{\mathcal{E}} g(e')$). Four cases remain to be considered.

- $d' \smile_{\mathcal{F}} f(d')$ (and $e' \smile_{\mathcal{E}} g(e')$). This contradicts the fact that $\mathcal{F}$ (and $\mathcal{E}$) is without autoconcurrency.

- $d' \#_{\mathcal{F}} f(d')$ (and $e' \#_{\mathcal{E}} g(e')$). By Lemma 2.1(ii), we have $D'' \not\uparrow_{\mathcal{F}} D'''$ (and $C'' \not\uparrow_{\mathcal{E}} C'''$) contradicting Definition 2.2, since $D'' \uparrow'_{\mathcal{F}} D'''$ (and $C'' \uparrow'_{\mathcal{E}} C'''$).
- $d' <_{\mathcal{F}} f(d')$ (and $e' <_{\mathcal{E}} g(e')$). This contradicts $D''' \in \mathcal{C}(\mathcal{F})$ (and $C''' \in \mathcal{C}(\mathcal{E})$).
- $f(d') <_{\mathcal{F}} d'$ (and $g(e') <_{\mathcal{E}} e'$). This contradicts $D' \in \mathcal{C}(\mathcal{F})$ (and $C' \in \mathcal{C}(\mathcal{E})$).

Thus $D'' \setminus D''' = a$ (and $C'' \setminus C''' = a$). Moreover, $D'' \cap D''' = D'' \cap D'$ (and $C'' \cap C''' = C'' \cap C'$). Hence $D''' \setminus D'' = D''' \setminus (D''' \cap D'') = f(D' \setminus (D' \cap D'')) = f(D' \setminus D'') = q$ (and $C''' \setminus C'' = C''' \setminus (C' \cap C'') = g(C' \setminus (C' \cap C'')) = g(C' \setminus C'') = p$). Take $\widetilde{D} = D'' \cup D'''$ (and $\widetilde{C} = C'' \cup C'''$). This implies $D''' \xrightarrow{a}_{\mathcal{F}} \widetilde{D}$ (and $C''' \xrightarrow{a}_{\mathcal{E}} \widetilde{C}$) and $D'' \xrightarrow{p}_{\mathcal{F}} \widetilde{D}$ (and $C'' \xrightarrow{p}_{\mathcal{E}} \widetilde{C}$), by Definition 2.2 and Lemma 2.1(i). W.l.o.g. assume $(C, \widetilde{D})$, $(\widetilde{C}, D) \in \mathcal{B}$. Since $D' \neq D'''$ (and $C' \neq C'''$), we have $D \neq \widetilde{D}$ (and $C \neq \widetilde{C}$), due to Lemma 2.2. It is easy to check that $\mathcal{B}' = \mathcal{B} \setminus \{(C, D)\}$ is an $h\beta c$-bisimulation contradicting the minimality of $\mathcal{B}$.

Thus $\mathcal{B}$ is an $\alpha\beta b$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$ according to Proposition 3.1.

(ii) '$\Rightarrow$' This follows from point (i) of the proposition and Definition 3.1.

'$\Leftarrow$' This follows from Definition 3.1. □

Proposition 3.3. Let $\mathcal{E}$, $\mathcal{F}$ be event structures, $\beta \in \{a, b, c\}^*$. Then

$$\mathcal{E} \approx_{h\beta} \mathcal{F} \Longleftrightarrow \mathcal{E} \approx_{h\beta r} \mathcal{F}.$$

Proof. '$\Rightarrow$'. Suppose $\mathcal{E} \approx_{h\beta} \mathcal{F}$ and $\mathcal{B}$ to be an $h\beta$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$, $R \in \mathcal{RC}(\mathcal{E})$ and $C \in \mathcal{C}(\mathcal{E})$ such that $C \subseteq R$. Then $C \xrightarrow{p}_{\mathcal{E}} R$ by Definition 2.3. Since $\mathcal{B}$ is an $h\beta$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$, there exists a configuration $D \in \mathcal{C}(\mathcal{F})$ such that $(C, D) \in \mathcal{B}$ and $\mathcal{E} \lceil C \cong \mathcal{F} \lceil D$. Due to Definition 3.1(i), there exists $R' \in \mathcal{C}(\mathcal{F})$ such that $D \xrightarrow{q}_{\mathcal{F}} R'$, $p \cong q$, $(R, R') \in \mathcal{B}$ and $\mathcal{E}\lceil R \cong \mathcal{F}\lceil R'$. According to Definition 2.3(i), $D \subseteq R'$. It is necessary to show $R' \in \mathcal{RC}(\mathcal{F})$. Assume the contrary, i.e. there exists $R'' \in \mathcal{RC}(\mathcal{F})$ such that $R' \subset R''$. Due to Definition 2.3, we have $R' \xrightarrow{q'}_{\mathcal{F}} R''$ and $q' \neq \emptyset$. Then there exists $\tilde{R}$ such that $R \xrightarrow{p'}_{\mathcal{E}} \tilde{R}$ and $p' \cong q'$ according to Definition 3.1. This implies $R \subset \tilde{R}$ by Definition 2.3, which contradicts $R \in \mathcal{RC}(\mathcal{E})$. The converse case is similar.

'$\Leftarrow$' This follows from Definition 3.1. □

We now introduce a number of bisimulations which are directly defined on the domain of local configurations of the event structures. As it has been shown in [11], these notions are useful to discover a match for the equivalence induced by the logic L_1 [7].

Definition 3.2. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures, $\mathcal{B} \subseteq \mathcal{LC}_0(\mathcal{E}) \times \mathcal{LC}_0(\mathcal{F})$ and $\beta \in \{a, b, c, r\}^*$. Then

(i) $\mathcal{B}$ is a *local bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $(\emptyset, \emptyset) \in \mathcal{B}$ and for all $(C, D) \in \mathcal{B}$ the following holds:

- if $C \stackrel{p}{\mapsto}_{\mathcal{E}} C'$, then there exist D' and q such that $D \stackrel{q}{\mapsto}_{\mathcal{F}} D'$, $p \cong q$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(ii) $\mathcal{B}$ is a *local b-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a local bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:

- if $C' \stackrel{p}{\mapsto}_{\mathcal{E}} C$, then there exist D' and q such that $D' \stackrel{q}{\mapsto}_{\mathcal{F}} D$, $p \cong q$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(iii) $\mathcal{B}$ is a *local a-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a local bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:

- if $C \, \underline{\not\uparrow}_{\mathcal{E}} C'$, then there exists D' such that $D \, \underline{\not\uparrow}_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(iv) $\mathcal{B}$ is a *local c-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a local bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:

- if $C \, \underline{\uparrow}'_{\mathcal{E}} C'$, then there exists D' such that $D \, \underline{\uparrow}'_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(v) $\mathcal{B}$ is a *local r-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ if $\mathcal{B}$ is a local bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and the following holds:

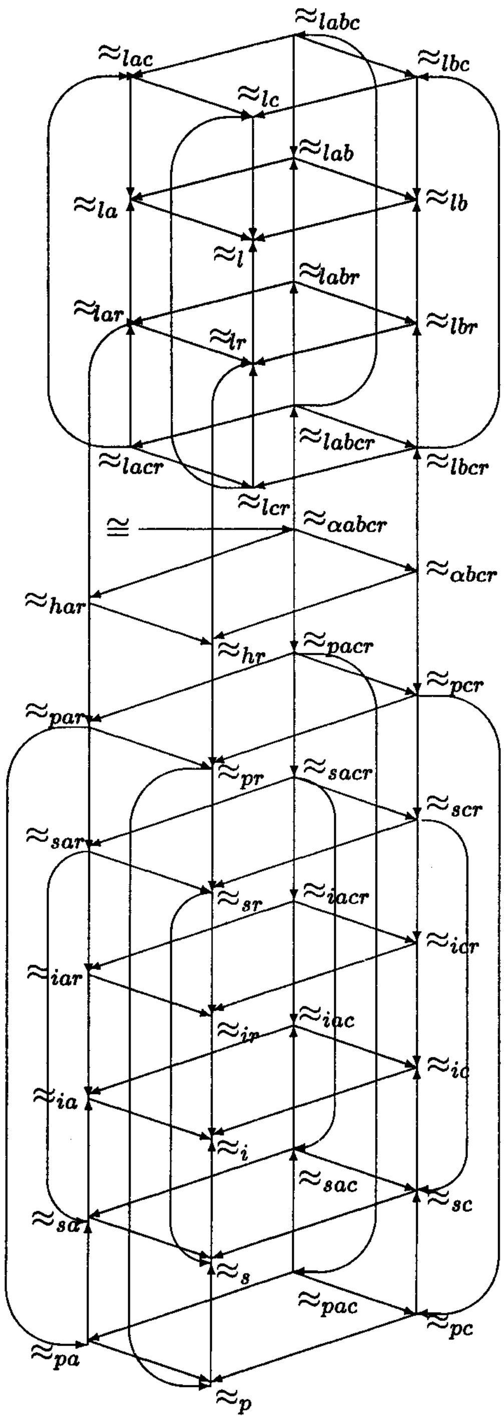
- for all $R \in \mathcal{RC}(\mathcal{E})$ if $\downarrow e \subseteq R$, then there exist $R' \in \mathcal{RC}(\mathcal{F})$ and $\downarrow d \subseteq R'$ such that $\mathcal{E} \lceil R \cong \mathcal{F} \lceil R'$ and $(\downarrow e, \downarrow d) \in \mathcal{B}$,
- and vice versa.

$\mathcal{E}$ and $\mathcal{F}$ are *locally β-bisimilar*, denoted by $\mathcal{E} \approx_{l\beta} \mathcal{F}$, if there exists a local β-bisimulation $\mathcal{B}$ which is a local γ-bisimulation for all $\gamma \in \beta$. □

Lemma 3.1. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures, $\mathcal{B}$ be a minimal local bisimulation between $\mathcal{E}$ and $\mathcal{F}$, and $(C, D) \in \mathcal{B}$. Then $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$.

Proof. Assume $\emptyset \stackrel{p}{\mapsto}_{\mathcal{E}} C$ and $\emptyset \stackrel{q}{\mapsto}_{\mathcal{F}} D$. We suppose the contrary, i.e. $p \not\cong q$. Since $\mathcal{B}$ is a local bisimulation, there exist $D' \in \mathcal{LC}_0(\mathcal{F})$ $(C' \in \mathcal{LC}_0(\mathcal{E}))$ and q' (and p') such that $\emptyset \stackrel{q'}{\mapsto}_{\mathcal{F}} D'$ (and $\emptyset \stackrel{p'}{\mapsto}_{\mathcal{F}} C'$), $p \cong q'$ (and $q \cong p'$) and $(C, D') \in \mathcal{B}$ (and $(C', D) \in \mathcal{B}'$). Obviously, $D \neq D'$ (and $C \neq C'$). Then $\mathcal{B}' = \mathcal{B} \setminus \{(C, D)\}$ is a local bisimulation between $\mathcal{E}$ and $\mathcal{F}$, which contradicts the minimality of $\mathcal{B}$. Hence, $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$. □

Proposition 3.4. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures and $\beta \in \{a, b, c, r\}^*$. Then
$$\mathcal{E} \approx_{h\beta} \mathcal{F} \Rightarrow \mathcal{E} \approx_{l\beta} \mathcal{F}.$$


$\approx_{labc}$
$\approx_{lac}$
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$\approx_{i}$
$\approx_{sac}$
$\approx_{sc}$
$\approx_{sa}$
$\approx_{s}$
$\approx_{pac}$
$\approx_{pc}$
$\approx_{pa}$
$\approx_{p}$

Figure 2

Proof. Assume $\mathcal{E} \approx_{h\beta} \mathcal{F}$ and $\mathcal{B}$ to be an $h\beta$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$. Let us define $\widetilde{\mathcal{B}} = \mathcal{B} \cap (\mathcal{LC}_0(\mathcal{E}) \times \mathcal{LC}_0(\mathcal{F}))$. It is necessary to show $l\beta$-bisimilarity of $\widetilde{\mathcal{B}}$. Let us prove the case with $\beta = \lambda$ (the remaining cases are similar). Clearly, $(\emptyset, \emptyset) \in \widetilde{\mathcal{B}}$. Suppose $(C, D) \in \widetilde{\mathcal{B}}$ and $C \overset{p}{\mapsto}_{\mathcal{E}} C'$. Then $(C, D) \in \mathcal{B}$, due to the construction of $\widetilde{\mathcal{B}}$, and $C \overset{p}{\to}_{\mathcal{E}} C'$, due to Definition 2.3. Since $\mathcal{B}$ is an $h\beta$-bisimulation, we have $\mathcal{E}\lceil C \cong \mathcal{F}\lceil D$ and can find D' and q such that $D \overset{q}{\to}_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$. Moreover, $\mathcal{E}\lceil C' \cong \mathcal{F}\lceil D'$, by Definition 3.1(i). This implies $p \cong q$ and $D' \in \mathcal{LC}_0(\mathcal{F})$. Hence $D \overset{q}{\mapsto}_{\mathcal{F}} D'$, due to Definition 2.3, and $(C', D') \in \widetilde{\mathcal{B}}$, due to the construction of $\widetilde{\mathcal{B}}$. Thus $\widetilde{\mathcal{B}}$ is an $l\beta$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$, by Definition 3.2. □

Theorem 3.1. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures and $\gamma, \delta \in \bigcup_{\beta \in \{a,b,c,r\}^*} (\{\alpha\beta \mid \alpha \in \{i, s, p, h\}\} \cup \{l\beta\})$. Then $\mathcal{E} \approx_\gamma \mathcal{F}$ implies $\mathcal{E} \approx_\delta \mathcal{F}$ iff there is a directed path from $\approx_\gamma$ to $\approx_\delta$ in Fig. 2.

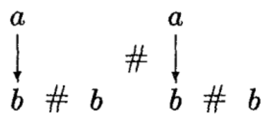
Proof. '$\Leftarrow$' All the implications in Fig. 2 follow from Definitions 3.1, 3.2 and Propositions 3.1–3.4.

'$\Rightarrow$' Now we show that it is impossible to draw any arrow from one equivalence to the other so that there is no directed path from the first equivalence to the second one in the graph in Fig. 2. For this purpose, we give the following examples.

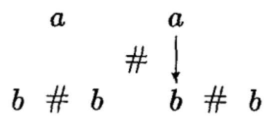
The event structures $\mathcal{E}_1 = (a; b) + (a; b)$ and $\mathcal{F}_1 = (a; b) + (a; (b + b))$ are $\alpha\beta$-bisimilar but not isomorphic.

Next we consider the event structures $\mathcal{E}_2$ and $\mathcal{F}_2$:

$\mathcal{E}_2$



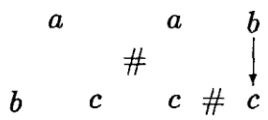
$\mathcal{F}_2$



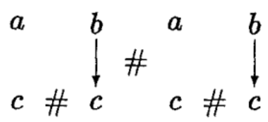
which are $\alpha'\beta'$-bisimilar, but neither $\alpha' b\beta'$- nor $h\beta$- nor $l\beta$-bisimilar for $\alpha' \in \{i, s, p\}$ and $\beta' \in \{a, c, r\}^*$.

The event structures $\mathcal{E}_3$ and $\mathcal{F}_3$:

$\mathcal{E}_3$

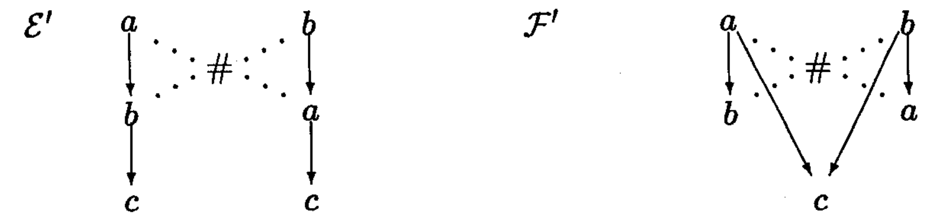


$\mathcal{F}_3$



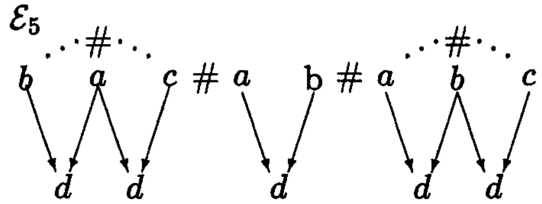
are $\alpha'\beta'$-bisimilar, but neither $\alpha' b\beta'$- nor $\alpha''\beta$-bisimilar for $\alpha' \in \{i, s\}$, $\alpha'' \in \{p, h\}$ and $\beta' \in \{a, c, r\}^*$.

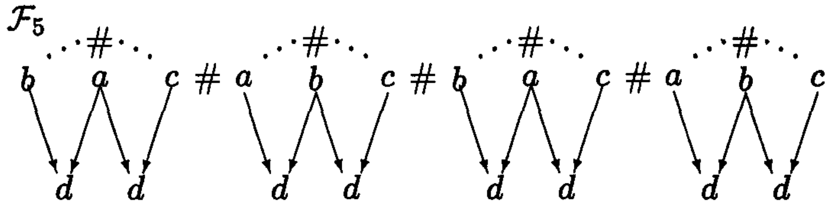
Let us consider the event structures $\mathcal{E}'$ and $\mathcal{F}'$:



The composed structures $\mathcal{E}_4 = (d \parallel \mathcal{E}') + (d \parallel \mathcal{F}')$ and $\mathcal{F}_4 = (d \parallel (\mathcal{E}' + \mathcal{F}')) + (d \parallel (\mathcal{E}' + \mathcal{F}'))$ are $i\beta'$-bisimilar, but neither $ib\beta'$- nor $\alpha'\beta$-bisimilar for $\alpha' \in \{s, p, h\}$ and $\beta' \in \{a, c, r\}^*$.

The event structures $\mathcal{E}_5$ and $\mathcal{F}_5$:

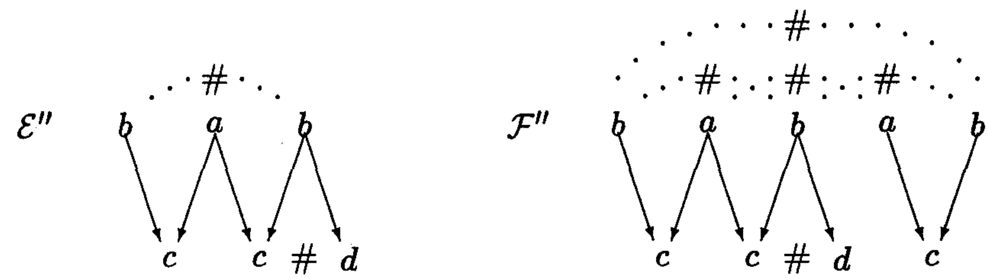




are $\alpha\beta'$ and $l\beta'$-bisimilar, but neither $\alpha b\beta'$- nor $\alpha c\beta'$- nor $lc\beta'$- nor $lb\beta'$-bisimilar for $\beta' \in \{a, r\}^*$.

The event structures $\mathcal{E}_6 = a; b$ and $\mathcal{F}_6 = a; (b + b)$ are $\alpha\beta'$- and $l\beta'$-bisimilar, but neither $\alpha a\beta'$- nor $la\beta'$-bisimilar for $\beta' \in \{b, c, r\}^*$.

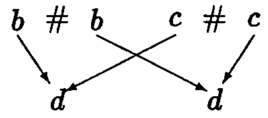
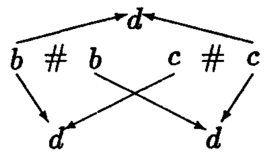
Let us consider the event structures $\mathcal{E}''$ and $\mathcal{F}''$:

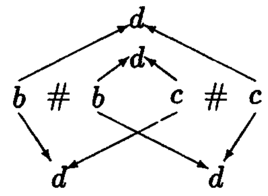


The composed structures $\mathcal{E}_7 = \mathcal{E}'' + \mathcal{E}''$ and $\mathcal{F}_7 = \mathcal{F}'' + \mathcal{F}''$ are $l\beta'$-bisimilar, but neither $lb\beta'$- nor $\alpha\beta$-bisimilar for $\beta' \in \{a, c, r\}^*$.

The event structures $\mathcal{E}_8 = (a \parallel (b + c)) + (a \parallel b) + (b \parallel (a + c))$ and $\mathcal{F}_8 = (a \parallel (b + c)) + (b \parallel (a + c)) + (a \parallel (b + c)) + (b \parallel (a + c))$ are $\alpha\beta'$- and $l\beta''$-bisimilar, but neither $\alpha c\beta''$- nor $\alpha b\beta'$- nor $lc\beta''$-bisimilar for $\beta' \in \{a, r\}^*$ and $\beta'' \in \{a, b, r\}^*$.

Let us finally consider the event structures $\mathcal{E}_9$, $\mathcal{F}_9$, and $\mathcal{E}_{10}$:

$\mathcal{E}_9$  $\mathcal{F}_9$ 

$\mathcal{E}_{10}$ 

The event structures $\mathcal{E}_9$ and $\mathcal{F}_9$ are $l\beta$-bisimilar, but not $\alpha\beta$-bisimilar. The event structures $\mathcal{E}_{10}$ and $\mathcal{F}_9$ are $l\beta'$-bisimilar, but not $lr\beta'$-bisimilar for $\beta' \in \{a, b, c\}^*$. □

4. Weak bisimulations

Taking into account the "invisible" nature of a step τ, we treat weak bisimulation equivalences as variants of strong bisimulations. In this section we investigate the difference between strong and weak variants of the considered bisimulations for event structures.

Definition 4.1. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures, $\mathcal{B} \subseteq \mathcal{C}(\mathcal{E}) \times \mathcal{C}(\mathcal{F})$, $\alpha \in \{i, s, p, h\}$ and $\beta \in \{a, b, c\}^*$. Then

(i) $\mathcal{B}$ is a *weak α-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $(\emptyset, \emptyset) \in \mathcal{B}$ and for all $(C, D) \in \mathcal{B}$ the following holds:

- $\mathcal{E} \lceil vis(C) \cong \mathcal{F} \lceil vis(D)$ if $\alpha = h$,
- if $C \stackrel{p}{\Rightarrow}_{\mathcal{E}} C'$ so that
  - p has at most one element if $\alpha = i$,
  - p is a step if $\alpha = s$,

  then there are D' and q such that $D \stackrel{q}{\Rightarrow}_{\mathcal{F}} D'$, $vis(p) \cong vis(q)$ and $(C', D') \in \mathcal{B}$,
- and vice versa;

(ii) $\mathcal{B}$ is a *weak αb-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a weak α-bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:

– if $C' \overset{p}{\Rightarrow}_{\mathcal{E}} C$ so that
  – p has at most one element if $\alpha = i$,
  – p is a step if $\alpha = s$,
  then there are D' and q such that $D' \overset{q}{\Rightarrow}_{\mathcal{F}} D$, $vis(p) \cong vis(q)$ and $(C', D') \in \mathcal{B}$,
– and vice versa;

(iii) $\mathcal{B}$ is a *weak αa-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a weak α-bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
  – if $C \not\uparrow_{\mathcal{E}} C'$, then there is D' such that $D \not\uparrow_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$,
  – and vice versa;

(iv) $\mathcal{B}$ is a *weak αc-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a weak α-bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
  – if $C \uparrow'_{\mathcal{E}} C'$, then there is D' such that $D \uparrow'_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$,
  – and vice versa.

$\mathcal{E}$ and $\mathcal{F}$ are *weakly $\alpha\beta$-bisimilar*, denoted by $\mathcal{E} \approx_{\tau\alpha\beta} \mathcal{F}$, if there exists a weak $\alpha\beta$-bisimulation $\mathcal{B}$ which is a weak $\alpha\gamma$-bisimulation for all $\gamma \in \beta$. □

Proposition 4.1. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures, $\alpha, \alpha' \in \{i, s, p, h\}$ and $\beta \in \{a, c\}^*$. Then

$$\mathcal{E} \approx_{\tau\alpha\beta b} \mathcal{F} \iff \mathcal{E} \approx_{\tau\alpha'\beta b} \mathcal{F}.$$

Proof is analogous to that of Proposition 3.1. □

Next we introduce the variants of bisimulations defined over the set of visible local configurations.

Definition 4.2. Let $\mathcal{LC}_{vis}(\mathcal{E}) \times \mathcal{LC}_{vis}(\mathcal{F})$ and $\beta \in \{a, b, c\}^*$. Then

(i) $\mathcal{B}$ is a *weak local bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $(\emptyset, \emptyset) \in \mathcal{B}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
  – if $C \overset{p}{\models\!\!\Rightarrow}_{\mathcal{E}} C'$ and $C' \in \mathcal{LC}_{vis}(\mathcal{E})$, then there are $D' \in \mathcal{LC}_{vis}(\mathcal{F})$ and q such that $D \overset{q}{\models\!\!\Rightarrow}_{\mathcal{F}} D'$, $vis(p) \cong vis(q)$ and $(C', D') \in \mathcal{B}$,
  – and vice versa;

(ii) $\mathcal{B}$ is a *weak local b-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a weak local bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
  – if $C' \overset{p}{\models\!\!\Rightarrow}_{\mathcal{E}} C$ and $C' \in \mathcal{LC}_{vis}(\mathcal{E})$, then there are $D' \in \mathcal{LC}_{vis}(\mathcal{F})$ and q such that $D' \overset{q}{\models\!\!\Rightarrow}_{\mathcal{F}} D$, $vis(p) \cong vis(q)$ and $(C', D') \in \mathcal{B}$,
  – and vice versa;

(iii) $\mathcal{B}$ is a *weak local a-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a weak local bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:
  – if $C \not\uparrow_{\mathcal{E}} C'$ and $C' \in \mathcal{LC}_{vis}(\mathcal{E})$, then there is $D' \in \mathcal{LC}_{vis}(\mathcal{F})$ such that $D \not\uparrow_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$,

– and vice versa;

(iv) $\mathcal{B}$ is a *weak local c-bisimulation* between $\mathcal{E}$ and $\mathcal{F}$ iff $\mathcal{B}$ is a weak local bisimulation between $\mathcal{E}$ and $\mathcal{F}$ and for all $(C, D) \in \mathcal{B}$ the following holds:

- if $C \ \uparrow'_{\mathcal{E}} \ C'$ and $C' \in \mathcal{LC}_{vis}(\mathcal{E})$, then there is $D' \in \mathcal{LC}_{vis}(\mathcal{F})$ such that $D \ \uparrow'_{\mathcal{F}} \ D'$ and $(C', D') \in \mathcal{B}$,
- and vice cersa.

$\mathcal{E}$ and $\mathcal{F}$ are *weak-locally β-bisimilar*, denoted by $\mathcal{E} \approx_{\tau l\beta} \mathcal{F}$, if there exists a weak local β-bisimulation $\mathcal{B}$ which is a weak local γ-bisimulation for all $\gamma \in \beta$. □

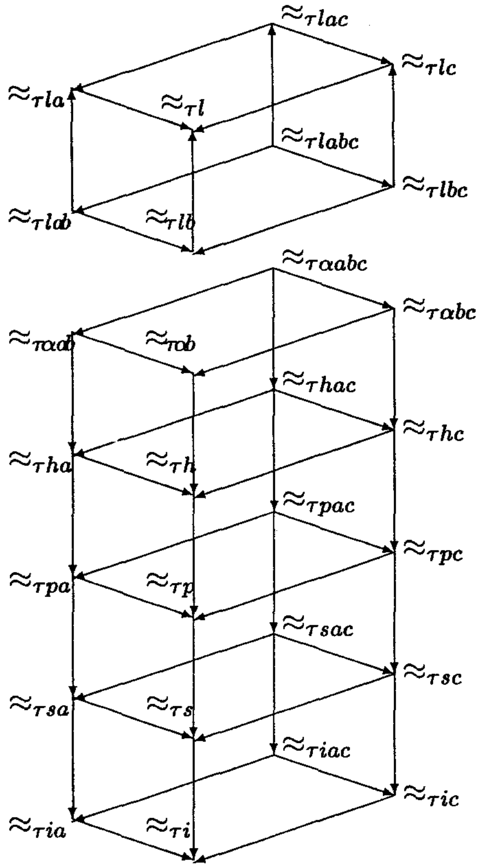


Figure 3

Theorem 4.1. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures and $\gamma, \delta \in \bigcup_{\beta \in \{a,b,c\}^*} (\{\alpha\beta \mid \alpha \in \{i, s, p, h\}\} \cup \{l\beta\})$. Then the following holds:

(i) $\mathcal{E} \approx_{\tau\gamma} \mathcal{F}$ implies $\mathcal{E} \approx_{\tau\delta} \mathcal{F}$ iff there is a directed path from $\approx_{\tau\gamma}$ to $\approx_{\tau\delta}$ in Fig. 3;

(ii) $\mathcal{E} \approx_{\gamma} \mathcal{F}$ implies $\mathcal{E} \approx_{\tau\delta} \mathcal{F}$ iff $\gamma = \delta$ or there is a directed path from $\approx_{\gamma}$ to $\approx_{\delta}$ in Fig. 2 and there is a directed path from $\approx_{\tau\gamma}$ to $\approx_{\tau\delta}$ in Fig. 3;

(iii) $\mathcal{E} \approx_{\tau\delta} \mathcal{F}$ does not imply $\mathcal{E} \approx_{\gamma} \mathcal{F}$ for all γ and δ.

Proof. (i) '$\Rightarrow$'. Now we show that it is impossible to draw any arrow from one equivalence to the other so that there is no directed path from the first equivalence to the second one in the graph in Fig. 3. For this purpose, we give the following examples.

The event structures $\mathcal{E}_{11}$ and $\mathcal{F}_{11}$:

$$\mathcal{E}_{11} \quad \begin{array}{c} a \\ \# \\ a \end{array} \qquad\qquad \mathcal{F}_{11} \quad \begin{array}{ccc} \tau & & \\ \downarrow & & \\ a & \# & a \end{array}$$

are $\tau\alpha\beta'$-bisimilar, but not $\tau\alpha\beta' c$-bisimilar for $\beta' \in \{a, b\}^*$.

Let us consider the event structures $\mathcal{E}'''$ and $\mathcal{F}'''$:

$$\mathcal{E}''' \quad \begin{array}{ccc} a & \# & a \\ \downarrow & & \# \\ \tau & \# & b \end{array} \qquad\qquad \mathcal{F}''' \quad \begin{array}{ccc} a & & \\ \downarrow & & \\ \tau & \# & b \end{array}$$

The composed structures $\mathcal{E}_{12} = \mathcal{E}''' + \mathcal{E}'''$ and $\mathcal{F}_{12} = \mathcal{F}''' + \mathcal{F}'''$ are $\tau\alpha\beta'$-bisimilar, but not $\tau\alpha\beta' b$-bisimilar for $\beta' \in \{a, c\}^*$.

Let us consider the event structures $\mathcal{E}''''$ and $\mathcal{F}''''$:

$$\mathcal{E}'''' \quad \begin{array}{c} a \\ \downarrow \\ b \end{array} \qquad\qquad \mathcal{F}'''' \quad \begin{tikzcd} a \arrow[d] & \# & a \arrow[dr] & & \tau \arrow[dl] \\ b & & & b & \end{tikzcd}$$

The composed structures $\mathcal{E}_{13} = (\mathcal{E}'''' + \mathcal{E}'''') \parallel \tau \parallel \tau$ and $\mathcal{F}_{13} = \mathcal{F}'''' \parallel \tau$ are $\tau\alpha\beta$-bisimilar, but not $\tau l\beta$-bisimilar.

The proof of the remaining cases is analogous to that of Theorem 3.1, using the examples $\mathcal{E}_1$–$\mathcal{E}_{10}$ and $\mathcal{F}_1$–$\mathcal{F}_9$, since the correspondent weak and strong bisimulations coincide for the class of event structures without silent action τ.

'$\Leftarrow$'. All the implications in Fig. 3 follow from Definitions 4.1, 4.2 and Proposition 4.1.

(ii) '$\Rightarrow$'. According to Definitions 3.1, 3.2, 4.1 and 4.2 it is easy to see that strong bisimulations imply the correspondent weak ones. Now we show that it is impossible to draw any arrow from $\approx_\gamma$ to $\approx_{\tau\delta}$ so that there is no directed path from $\approx_{\tau\gamma}$ to $\approx_{\tau\delta}$ in the graph in Fig. 3 or from $\approx_\gamma$ to $\approx_\delta$ in the graph in Fig. 2. For this purpose we consider the examples from Theorem 3.1.

The event structures

$\mathcal{E}_6$ and $\mathcal{F}_6$ are $\alpha\beta'$- and $l\beta'$-bisimilar, but neither $\tau\alpha a\beta'$- nor $\tau la\beta'$-bisimilar for $\beta' \in \{b,c\}^*$;

$\mathcal{E}_5$ and $\mathcal{F}_5$ are $\alpha\beta'$ and $l\beta'$-bisimilar, but neither $\tau\alpha b\beta'$- nor $\tau\alpha c\beta''$- nor $\tau lc\beta''$- nor $\tau lb\beta'$-bisimilar for $\beta' \in \{a\}^*$ and $\beta'' \in \{a,b\}^*$;

$\mathcal{E}_2$ and $\mathcal{F}_2$ are $\alpha'\beta'$-bisimilar, but neither $\tau\alpha' b\beta'$- nor $\tau h\beta$- nor $\tau l\beta$-bisimilar for $\alpha' \in \{i,s,p\}$ and $\beta' \in \{a,c\}^*$;

$\mathcal{E}_3$ and $\mathcal{F}_3$ are $\alpha'\beta'$-bisimilar, but not $\tau\alpha''\beta$-bisimilar for $\alpha' \in \{i,s\}$, $\alpha'' \in \{p,h\}$ and $\beta' \in \{a,c\}^*$;

$\mathcal{E}_4$ and $\mathcal{F}_4$ are $i\beta'$-bisimilar, but not $\tau\alpha'\beta$-bisimilar for $\alpha' \in \{s,p,h\}$ and $\beta' \in \{a,c\}^*$;

$\mathcal{E}_9$ and $\mathcal{F}_9$ are $l\beta$-bisimilar, but not $\tau\alpha\beta$-bisimilar;

$\mathcal{E}_7$ and $\mathcal{F}_7$ are $l\beta'$-bisimilar, but not $\tau lb\beta'$-bisimilar for $\beta' \in \{a,c\}^*$;

$\mathcal{E}_8$ and $\mathcal{F}_8$ are $l\beta''$-bisimilar, but not $\tau lc\beta''$-bisimilar for $\beta'' \in \{a,b\}^*$.

'$\Leftarrow$'. All the implications follow from Definitions 3.1, 3.2, 4.1, 4.2 and Theorems 3.1 and 4.1.

(iii) The example of the event structures $\mathcal{E}_{14} = (a;(b + (b;\tau))) + (a;b)$ and $\mathcal{F}_{14} = (a;b) + (a;b)$ shows that any weak bisimulation does not imply any strong one, since $\mathcal{E}_{14} \approx_{\tau\gamma} \mathcal{F}_{14}$, but $\mathcal{E}_{14} \not\approx_{\gamma} \mathcal{F}_{14}$. □

5. Bisimulations and action refinement

One of the most important features of the equivalence notion is its preservation by refinement of actions. Since we have introduced new bisimulations, it is interesting to see whether or not they are preserved by refinement. We use the definition of refinement from [3]. This operator allows one to design systems in a top-down style changing the level of abstraction by interpreting actions on a higher level by more complicated processes on a lower level. A refinement function is a function RF specifying, for each action a, a finite, conflict-free and nonempty event structure $RF(a)$ which is to be substituted for a. Interesting refinements will mostly refine only certain actions, hence replace most actions by themselves. However, for uniformity, we consider all actions to be refined assuming that the silent action τ is replaced by itself. Given an event structure $\mathcal{E}$ and a refinement function RF, we construct the refined event structure $RF(\mathcal{E})$ as follows. Each event e labelled by a is replaced by a disjoint copy, $\mathcal{E}_e$, of $RF(a)$. The causality and conflict structure is inherited from $\mathcal{E}$: every event which was causally before e will be causally before all events of $\mathcal{E}_e$, all events which causally followed e will causally follow all the events of $\mathcal{E}_e$, and all events in conflict with e will be in conflict with all the events of $\mathcal{E}_e$.

Definition 5.1. Let $\mathcal{E}$ be an event structure and RF be a *refinement function* (for $\mathcal{E}$) which associates a finite, conflict-free and nonempty event structure $RF(a)$ with each action $a \in Act$ and $RF(\tau) = (\{e\}, \emptyset, \emptyset, \tau)$. Then the *refinement* of $\mathcal{E}$ by RF is the event structure $RF(\mathcal{E}) = (E, <, \#, l)$ defined as follows:

- $E_{RF(\mathcal{E})} = \{(e, e') \mid e \in E_{\mathcal{E}},\ e' \in E_{RF(l_{\mathcal{E}}(e))}\}$;
- $(e, e') <_{RF(\mathcal{E})} (d, d') \iff (e <_{\mathcal{E}} d) \vee (e = d \ \&\ e' <_{RF(l_{\mathcal{E}}(e))} d')$;
- $(e, e') \#_{RF(\mathcal{E})} (d, d') \iff (e \#_{\mathcal{E}} d)$;
- $l_{RF(\mathcal{E})}(e, e') = l_{RF(l_{\mathcal{E}}(e))}(e')$. □

Note that the refinement $RF(\mathcal{E})$ of $\mathcal{E}$ is an event structure according to Definition 5.1 and the construction of the event structure $RF(a)$ for each a.

Proposition 5.1 ([4]). Let $\mathcal{E}$ be an event structure and RF be a refinement function (for $\mathcal{E}$). A set $\widetilde{C}$ is said to be a *configuration refinement* of $C \in \mathcal{C}(\mathcal{E})$ by RF if

- $\widetilde{C} = \cup_{e \in C}\{e\} \times C_e$, where $\forall e \in C : C_e \in \mathcal{C}(RF(e)) \setminus \{\emptyset\}$;
- $e \in busy(\widetilde{C}) \Rightarrow e$ is maximal in C w.r.t. $\leq_{\mathcal{E}}$,
  where $busy(\widetilde{C}) := \{e \in C \mid C_e$ is not the maximal configuration$\}$.

Then $\mathcal{C}(RF(\mathcal{E})) = \{\widetilde{C} \mid \widetilde{C}$ is a configuration refinement of $C \in \mathcal{C}(\mathcal{E})\}$. □

Proposition 5.2. Let $\alpha \in \{i, s, p\}$, $\beta \in \{a, b, c, r\}^*$ and $\beta' \in \{a, b, c\}^*$. Then the following equivalences are preserved under the operation of action refinement: $\approx_{h\beta}$, $\approx_{\alpha b\beta}$, $\approx_{l\beta}$, $\approx_{\tau h\beta'}$, $\approx_{\tau\alpha b\beta'}$ and $\approx_{\tau l\beta'}$.

Proof. We first show the preservation of $\approx_{h\beta}$ under the operation of action refinement. Let $\mathcal{E}$ and $\mathcal{F}$ be event structures, RF be a refinement function. Assume $\mathcal{B}$ to be an $h\beta$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$. Define $\widetilde{\mathcal{B}} = \{(\widetilde{C}, \widetilde{D}) \in \mathcal{C}(RF(\mathcal{E})) \times \mathcal{C}(RF(\mathcal{F}))| \ \exists (C, D) \in \mathcal{B}\ \exists f : C \to D \diamond pr_1(\widetilde{C}) = C, pr_1(\widetilde{D}) = D$, and f is an isomorphism such that $\forall e \in C \diamond C_e = D_{f(e)}\}$. It has been shown in [4] that $\widetilde{\mathcal{B}}$ is an h-bisimulation between $RF(\mathcal{E})$ and $RF(\mathcal{F})$. For $D \in \mathcal{C}(\mathcal{F})$ such that $(C, D) \in \mathcal{B}$, we define $ref_{\widetilde{C}}(D) = \bigcup_{d \in D}\{d\} \times D_d$ with $D_d = C_{f^{-1}(d)}$ for all $d \in D$ and for some isomorphism f from C onto D. Due to Definition 2.2 and Proposition 5.1, it is easy to see that $ref_{\widetilde{C}}(D) \in \mathcal{C}(RF(\mathcal{F}))$. By the construction of $\widetilde{\mathcal{B}}$, we get $(\widetilde{C}, ref_{\widetilde{C}}(D)) \in \widetilde{\mathcal{B}}$.

Now we show that $\widetilde{\mathcal{B}}$ is an $h\beta$-bisimulation between $\mathcal{E}$ and $\mathcal{F}$. Assume $(\widetilde{C}, \widetilde{D}) \in \widetilde{\mathcal{B}}$. Then $(C, D) \in \mathcal{B}$, by the construction of $\widetilde{\mathcal{B}}$. We consider the two cases.

$\beta = a$. We suppose $\widetilde{C} \not\uparrow_{RF(\mathcal{E})} \widetilde{C}'$. By Lemma 2.1(ii), there are $(e, g) \in \widetilde{C}$ and $(e', g') \in \widetilde{C}'$ such that $(e, g) \#_{RF(\mathcal{E})} (e', g')$, which implies $e \in C$, $e' \in C'$ and $e \#_{\mathcal{E}} e'$ due to Proposition 5.1 and Definition 5.1. Hence, $C \not\uparrow_{\mathcal{E}} C'$ again by Lemma 2.1(ii). Since $(C, D) \in \mathcal{B}$, there exists D' such that $D \not\uparrow_{\mathcal{F}} D'$

and $(C', D') \in \mathcal{B}$, due to Definition 3.1(iii). Assume $\widetilde{D}' = ref_{\widetilde{C}'}(D')$. Then $(\widetilde{C}', \widetilde{D}') \in \widetilde{\mathcal{B}}$. By Lemma 2.1(ii) there are $d \in D$ and $d' \in D'$ such that $d \ \#_{\mathcal{F}} \ d'$. Then, by Definition 5.1, we have $(d, g) \ \#_{RF(\mathcal{F})} \ (d', g')$ for all $g \in E_{RF(l_{\mathcal{F}}(d))}$ and $g' \in E_{RF(l_{\mathcal{F}}(d'))}$. According to Proposition 5.1, $(d, g) \in \widetilde{D}$ and $(d', g') \in \widetilde{D}'$ for some $g \in E_{RF(l_{\mathcal{F}}(d))}$ and $g' \in E_{RF(l_{\mathcal{F}}(d'))}$. Again by Lemma 2.1(ii), we get $\widetilde{D} \not\uparrow_{RF(\mathcal{F})} \widetilde{D}'$.

$\beta = c$. We suppose $\widetilde{C} \uparrow'_{RF(\mathcal{E})} \widetilde{C}'$. According to Proposition 5.1 and Definition 5.1, the following three cases are possible:

- $C \xrightarrow{p}_{\mathcal{E}} C'$. By Definition 3.1(i), there are D' and q such that $D \xrightarrow{q}_{\mathcal{F}} D'$, $p \cong q$ and $(C', D') \in \mathcal{B}$;
- $C' \xrightarrow{p}_{\mathcal{E}} C$. By Definition 3.1(ii), there are D' and q such that $D' \xrightarrow{q}_{\mathcal{F}} D$, $p \cong q$ and $(C', D') \in \mathcal{B}$;
- $C \uparrow'_{\mathcal{E}} C'$. By Definition 3.1(iv), there is D' such that $D \uparrow'_{\mathcal{F}} D'$ and $(C', D') \in \mathcal{B}$.

Assume $\widetilde{D}' = ref_{\widetilde{C}'}(D')$. Then $(\widetilde{C}', \widetilde{D}') \in \widetilde{\mathcal{B}}$. According to Lemma 2.1(iii), we have $(e, g) \smile_{RF(\mathcal{E})} (e', g')$ for all $(e, g) \in (\widetilde{C} \setminus \widetilde{C}')$ and $(e', g') \in (\widetilde{C}' \setminus \widetilde{C})$. We consider arbitrary $(e, g) \in (\widetilde{C} \setminus \widetilde{C}')$ and $(e', g') \in (\widetilde{C}' \setminus \widetilde{C})$. Assume $f : C \to D$ and $f' : C' \to D'$ to be isomorphisms. By Proposition 5.1 and the construction of $\widetilde{\mathcal{B}}$, we get $(f(e), g) \in (\widetilde{D} \setminus \widetilde{D}')$ and $(f(e'), g') \in (\widetilde{D}' \setminus \widetilde{D})$. It is sufficient to show that $(f(e), g) \smile_{RF(\mathcal{F})} (f(e'), g')$. We suppose the contrary, i. e. $\neg((f(e), g) \smile_{RF(\mathcal{F})} (f(e'), g'))$. Obviously, $(f(e), g) \neq_{RF(\mathcal{F})} (f(e'), g')$. Three cases remain to be considered.

- $(f(e), g) \ \#_{RF(\mathcal{F})} \ (f(e'), g')$, which implies $f(e) \in D$, $f(e') \in D'$ and $f(e) \ \#_{\mathcal{F}} \ f(e')$, according to Definition 5.1 and Proposition 5.1. By Lemma 2.1(ii), we have $D \not\uparrow_{\mathcal{F}} D'$, which contradicts Definition 2.3;
- $(f(e), g) <_{RF(\mathcal{F})} (f(e'), g')$. This contradicts $\widetilde{D}' \in \mathcal{C}(RF(\mathcal{F})$;
- $(f(e'), g') <_{RF(\mathcal{F})} (f(e), g)$. This contradicts $\widetilde{D} \in \mathcal{C}(RF(\mathcal{F})$.

Hence, $\widetilde{D} \uparrow'_{RF(\mathcal{F})} \widetilde{D}'$, due to Lemma 2.1(iii).

Therefore $\approx_{h\beta}$ is preserved under the action refinement, according to Definition 3.1 and Proposition 5.1.

The preservation of $\approx_{l\beta}$ under the action refinement can be proved by the similar way. The preservation of $\approx_{\alpha b\beta}$ follows from Proposition 3.1 and the above proof.

The preservation of $\approx_{\mathcal{T}h\beta'}$, $\approx_{\mathcal{T}\alpha b\beta'}$ and $\approx_{\mathcal{T}l\beta'}$ follows from Definitions 4.1, 4.2 and the preservation of the equivalences $\approx_{h\beta}$, $\approx_{\alpha b\beta'}$, $\approx_{l\beta}$ under the operation of action refinement. □

Proposition 5.3. Let $\alpha \in \{i, s, p\}$, $\beta \in \{a, c, r\}^*$ and $\beta' \in \{a, c\}^*$. Then $\approx_{\alpha\beta}$ and $\approx_{\tau\alpha\beta'}$ are not preserved under the operation of action refinement.

Proof. Let us consider the event structures $\mathcal{E}$ and $\mathcal{F}$:

The composed structures $\mathcal{E}_{15} = \mathcal{E} + \mathcal{F} + (a||b)$ and $\mathcal{F}_{15} = \mathcal{F} + \mathcal{F} + (a||b)$ are $\alpha'\beta$- and $\tau\alpha'\beta'$-bisimilar for $\alpha' \in \{i, s\}$. When refining an action b by the sequence $b_1 \rightarrow b_2$, we get $\mathcal{E}_{15} \not\approx_{\alpha'\beta} \mathcal{F}_{15}$ and $\mathcal{E}_{15} \not\approx_{\tau\alpha'\beta'} \mathcal{F}_{15}$.

The event structures $\mathcal{E}_4$ and $\mathcal{F}_4$ (see the proof of Theorem 3.1) are $p\beta$- and $\tau p\beta'$-bisimilar, but when refining an action a by the sequence $a_1 \rightarrow a_2$, we get $\mathcal{E}_4 \not\approx_{p\beta} \mathcal{F}_4$ and $\mathcal{E}_4 \not\approx_{\tau p\beta'} \mathcal{F}_4$. □

6. Conclusion

In this paper, in order to get equivalence notions nicely adapted to peculiarities of event structures, we have introduced a number of variants of bisimulation notions which consider all the relations between event occurrences in the structures. Quite close connections between the introduced and existing equivalences have been shown. A number of variants of bisimulations defined on the domain of local configurations of event structures have been put forward. It turned out that the variants of local bisimulations were coarser than the corresponding ones of history-preserving bisimulations and incomparable with interleaving, step and pomset bisimulations. For the class of labelled event structures with invisible actions, we have introduced weak variants of all considered bisimulation equivalences. Moreover, we have investigated the notion of r-bisimulation taking into account the structures of the maximal configurations. The lattice of interrelations between the equivalences under consideration has been established. For each bisimulation equivalence it has been shown, whether it is preserved under the operation of action refinement or not.

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