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Orographic retardation of a cold atmospheric front*

M.S. Yudin

Abstract. The propagation of a cold front over a mountain is simulated by a local non-hydrostatic finite difference model. New conservative difference operators are proposed for the advection terms inside the computational domain. Temperature transport is calculated with semi-Lagrangian scheme and data produced by the finite difference model. Comparisons are made between model calculations and theoretical and experimental results obtained by other authors. The model realistically describes the effect of orography on the front propagation.

1. Introduction

This paper is intended to be a preliminary study to simulate the propagation of a cold atmospheric front near a lake. The mesoscale meteorological phenomena that take place at the coastal sites have received much attention of researchers in recent years. This is due to the fact that these phenomena may have a considerable impact on the local climate.

The problem of simulating the local climatic characteristics of aerosol transfer in the atmosphere in the vicinity of a water basin is also important from a computational view point. It is necessary to use efficient numerical algorithms in the domains of an abrupt variation of calculated fields without using mesh refinement and suppression of spurious oscillations near the front of aerosol cloud propagation. In this paper, a simple finite-difference scheme is used, which is based on a semi-Lagrangian approach. This scheme is accurate and robust.

Hydrostatic models have played a great role in the simulation of atmospheric flows that occur at land-water boundaries [1]. Later, the advent of nonhydrostatic models enabled a wider class of flow phenomena to be simulated, specifically small-scale phenomena as, for instance, sea breeze front propagation.

This paper originated from an attempt to realistically simulate the meteorological flows that occur at water-land boundaries. Here, even with low topography, the changes in roughness from water to land and local terrain

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variations produce dynamic effects that are difficult for simulating by hydrostatic models.

Special methods have been developed to correct the hydrostatic approach by parameterizing the nonhydrostatic terrain effects in hydrostatic models [2, 3].

In this paper, a preliminary investigation is carried out to simulate the cold front propagation over a steep mountain in three dimensions to test the proposed discretizations for the advection terms.

The paper is organized as follows: the model equations and the discretizations are described in Section 2. Section 3 is devoted to a numerical scheme for the advection transport of temperature in the atmosphere. The calculation algorithm here is based on a semi-Lagrangian approach. In Section 4, results of model simulation of the orographic retardation of a cold front by a steep mountain are discussed. Conclusions to the paper are given in Section 5.

2. Model equations

The necessary input data for the pollutant transport calculation are obtained from a small–scale meteorological model. The basic equations for motion, heat, moisture and continuity of a non-hydrostatic version of the model in a terrain–following coordinate system are as follows:

$$
\begin{aligned}
&\frac{dU}{dt} + \frac{\partial P}{\partial x} + \frac{\partial (G^{13}P)}{\partial \eta} = f_1(V - V_g) - f_2 W + R_u,\\
&\frac{dV}{dt} + \frac{\partial P}{\partial y} + \frac{\partial (G^{23}P)}{\partial \eta} = -f_1(U - U_g) + R_v,\\
&\frac{dW}{dt} + \frac{1}{G^{1/2}}\frac{\partial P}{\partial \eta} + \frac{gP}{C_s^2} = f_2 U + g\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}} + R_w\\
&\frac{d\theta}{dt} = R_\theta,\\
&\frac{ds}{dt} = R_s,\\
&\frac{1}{C_s^2}\frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial}{\partial \eta}\Big(G^{13}U + G^{23}V + \frac{1}{G^{1/2}}W\Big) = \frac{\partial}{\partial t}\Big(\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}}\Big),
\end{aligned}
$$

$U = \bar{\rho}G^{1/2}u$, $V = \bar{\rho}G^{1/2}v$, $W = \bar{\rho}G^{1/2}w$, $P = G^{1/2}p'$, where p', θ' are deviations from the basic state pressure $\bar{p}$ and the potential temperature $\bar{\theta}$, s is specific humidity, C_s is the sound wave speed, u_g, v_g are components of the geostrophic wind representing the synoptic part of the pressure, η is a terrain-following coordinate transformation:

$$\eta = \frac{H(z - z_s)}{(H - z_s)},$$

z_s is the surface height, H is the height of the top of the model domain. Here $H = \text{const}$,

$$G^{1/2} = 1 - z_s/H, \quad G^{13} = \frac{1}{G^{1/2}}\Big(\frac{\eta}{H} - 1\Big)\frac{\partial z_s}{\partial x}, \quad G^{23} = \frac{1}{G^{1/2}}\Big(\frac{\eta}{H} - 1\Big)\frac{\partial z_s}{\partial y}.$$

For an arbitrary function φ

$$\frac{d\varphi}{dt} = \frac{\partial}{\partial t} + \frac{\partial u\varphi}{\partial x} + \frac{\partial v\varphi}{\partial y} + \frac{\partial \omega\varphi}{\partial \eta},$$

where

$$\omega = \frac{1}{G^{1/2}} W + G^{13} u + G^{23} v.$$

The terms R_u, R_v, R_ω, R_θ, R_s refer to subgrid-scale processes. As turbulence parameterization, we use a simple scheme:

$$K_m = \begin{cases} l^2 \sqrt{\dfrac{1}{2} D^2 (1 - \mathrm{Ri})}, & \mathrm{Ri} < 1, \\ 0, & \mathrm{Ri} \geq 1, \end{cases}$$

$$\mathrm{Ri} = \frac{g(d\ln\theta/dz)}{D^2/2}, \qquad D = \nabla \underline{u} + \underline{u}\nabla.$$

In the hydrostatic version of the model, the vertical velocity is not determined from the third equation of motion, as is the case in the non-hydrostatic version, but calculated diagnostically with the help of the continuity equation. At the top of the model domain in the hydrostatic version, we have a free surface that satisfies an additional evolutionary equation.

We approximate the advective terms in the above described model by the following difference operators:

$$\begin{aligned}
\delta_d \varphi &= \frac{\varphi(d + \Delta d/2) - \varphi(d - \Delta d/2)}{\Delta d}, \\
\varphi^d &= \frac{\varphi(d + \Delta d/2) + \varphi(d - \Delta d/2)}{2}, \\
\mathrm{ADVX} &= \delta_x(u^x(\rho^x u)^x) + \delta_y(v^x(\rho^x u)^y) + \delta_z(\omega^x(\rho^x u)^z), \\
\mathrm{ADVY} &= \delta_x(u^y(\rho^y v)^x) + \delta_y(v^y(\rho^y v)^y) + \delta_z(\omega^y(\rho^y v)^z), \\
\mathrm{ADVZ} &= \delta_x(u^z(\rho^z w)^x) + \delta_y(v^x(v(\rho^z w)^y) + \delta_z(\omega^z(\rho^z w)^z), \\
\mathrm{ADVT} &= \delta_x(u(\rho\theta)^x) + \delta_y(v(\rho\theta)^y) + \delta_z(\omega(\rho\theta)^z).
\end{aligned}$$

A more detailed description of the model and the numerical algorithms used in calculations can be found in [4].

3. Advection of temperature

The problem examined in this study has very small physical diffusion, and the governing equations can be considered as almost inviscid and adiabatic. The schemes used for the advection of momentum were discussed in the previous section. Here we consider a suitable scheme for the advection of a scalar, e.g. temperature.

For fully hyperbolic problems explicit schemes have been widely used in the past and will probably continue to be used in the future. Traditionally, problems for which the governing equations are hyperbolic everywhere have been solved by the method of characteristics. Sometimes traditional methods of characteristics are slower than competitive finite difference methods. However, some finite difference schemes use a form of the governing equations such that knowledge of the characteristic locations can be exploited very efficiently [5].

A flux-corrected transport scheme developed by Smolarkiewicz was successfully applied to simulation of the propagation of an idealized atmospheric front by Schumann [6]. This is an example of a general technique of the predictor-corrector type in which large diffusion is introduced at the predictor stage and an almost equal amount of antidiffusion is introduced at the corrector stage.

In this paper, the advection of temperature is treated with an efficient semi-Lagrangian finite difference scheme [7, 8]. Here the advection of a scalar is calculated in two steps:

1. Determination of so-called departure points. This is the point from which the point under consideration is reached at the next time step.
2. Interpolation of values of the advected scalar from grid points on the departure point:

   $$x_D = x - \int u\,dt, \qquad f(x, t + \Delta t) = f(x_D, t).$$

   Here u is the velocity vector and Δt is the time step size.

At the first step, interpolation of the velocity vector is performed. The second step is devoted to interpolation of the advected scalar. The procedure of interpolation determines the accuracy of the method. For velocity, linear interpolation is used. At the second step, a third-order accuracy scheme will be used in this paper for reasons to be discussed below.

The total advection of temperature is split into horizontal and vertical components. For simplicity, we present here only a one-dimensional scheme. The scheme is designed as follows: an arbitrary function f is expanded into a Taylor series up to terms of the fourth order. The free coefficients of this expansion are determined through values of the function at grid points.

Denote $\lambda = (x_D - x_i)/\Delta x$. Here Δx is the grid size. By solving the resultant system of linear equations for the free coefficients, we finally obtain:

$$f(t+\Delta t) = f_i(1-\lambda/2-\lambda^2+\lambda^3/2) + f_{i+1}(\lambda+\lambda^2/2-\lambda^3/2) + \\ f_{i+2}(-\lambda/6-\lambda^2+\lambda^3/6) + f_{i-1}(-\lambda/3+\lambda^2/2-\lambda^3/6).$$

Experiments have been performed to compare various semi-Lagrangian schemes. Some conclusions follow:

1. The first-order schemes have large numerical diffusion.
2. The second-order schemes are nonmonotonic and have a small-scale wavelike structure.
3. In the third-order schemes, the above two effects are essentially reduced.
4. The schemes of order higher than 3 have a significant increase in cost but only a small increase in the solution quality.

Therefore, we will use the above third-order semi-Lagrangian scheme as a reasonable compromise between cost and accuracy.

4. Simulation of cold front propagation

In this section, we apply the above model to simulating the propagation of a cold atmospheric front over a steep obstacle in three dimensions. The input parameters are taken from [6]: the obstacle is a circular mountain with an axially symmetric sinusoidal height profile of 2 km and a surface diameter of 200 km. The computational domain is $600 \times 400 \times 6$ km.

Figure 1 shows results of simulations on the propagation of a cold atmospheric front with an asymptotic height of 4 km and a temperature jump of

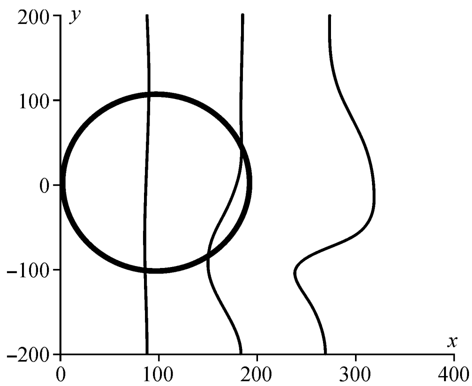

Figure 1

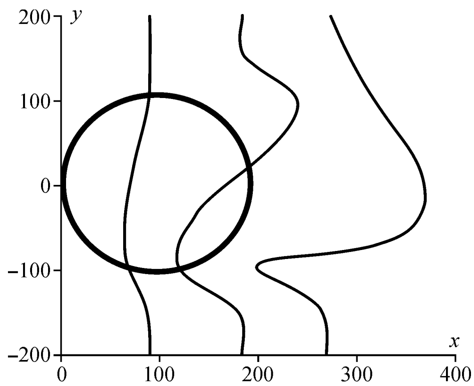

Figure 2

3 K for neutral stratification. Surface isochrones at a sequence of nondimensional time varying from 0 to 1 are shown. In Figure 2 the same is shown for a stably stratified atmosphere. The results show acceleration of the front on the northern side and retardation near the mountain center. Examination of the flow also shows strong anticyclonic motion over the mountain, which causes significant front deformation. It follows from Figure 2 that the stratification greatly increases the effects of acceleration and retardation. A qualitative agreement is observed between the above results and similar temperature patterns obtained in [6] with different schemes.

5. Conclusions

We have described an application of the nonhydrostatic version of a small-scale meteorological model to simulating flows that occur due to the retardation of a cold front by a steep mountain in three dimensions. The preliminary results of simulations described above were carried out to test the discretizations for the advection operators proposed in this paper. The simulation results are in qualitative agreement with the existing theoretical insight, observations, and calculations performed by other authors.

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