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A difference algorithm for a small-scale non-hydrostatic meteorological model

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A numerical scheme for the advection part of a small-scale mathematical-meteorological model developed earlier is proposed. The algorithm has a technical advantage when dealing with implicit difference schemes. The flow produced by the meteorological model serves as a basis for the calculation of pollutant transport over complex terrain. A simple Monte Carlo method is used for this purpose. Some results of a calculation for the transport of a passive substance over a steep hill are presented. A reasonably good agreement with the existing theory is achieved.

1. Introduction

A wide variety of theoretical and practical problems of interest are closely connected with meteorological phenomena on meso and micro spatial scales [1]. Important information on urban climate and pollutant transport is often based on a very scarce set of available data. Some mathematical-meteorological models developed in recent years can serve as useful interpolants for such data bases. These models are used as helpful tools, which produce such an amount of four-dimensional (space-time) information that one can never obtain from observations only.

Many problems of meso and micro scale dynamics require a non-hydrostatic treatment of the equations of motion. It is well-known [2] that the hydrostatic approximation is not valid when the resolvable horizontal and vertical scales are the same order.

One of the first successful models in the important field of non-hydrostatic modeling was created by Clark [3]. It was a three-dimensional limited-area anelastic finite difference model that used a terrain-following coordinate transformation. In the design of the model the emphasis was put on conservative aspects of the difference schemes employed. The discretized advective terms of these schemes conserved the first moments of the original system of equations (e.g., momentum, potential temperature). The discrete kinetic energy in Clark's algorithm was conserved provided the second time derivatives were negligible.

The purpose of this paper is to describe an algorithm for the advective terms in a small-scale model based on the artificial compressibility approach which was developed earlier by the author. The algorithm exactly conserves the first moments of the original equations and seems to have some technical advantages when dealing with implicit time schemes.

The equations of the model are given in Section 2. Section 3 describes the algorithm proposed for the advective terms in the model. A simple Monte Carlo method which simulates the transport of a pollutant using the information calculated by the meteorological model is briefly described in Section 4. Section 5 shows a calculation of polutant transport over a hill as an example, and Section 6 is devoted to conclusions.

2. The model

The basic equations of motion, heat, moisture and continuity in a terrain-following coordinate system are as follows:

$$\frac{dU}{dt} + \frac{\partial P}{\partial x} + \frac{\partial (G^{13}P)}{\partial \eta} = f_1(V - V_g) - f_2 W + R_u,$$

$$\frac{dV}{dt} + \frac{\partial P}{\partial y} + \frac{\partial (G^{23}P)}{\partial \eta} = -f_1(U - U_g) + R_v,$$

$$\frac{dW}{dt} + \frac{1}{G^{1/2}}\frac{\partial P}{\partial \eta} + \frac{gP}{C_s^2} = f_2 U + g\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}} + R_w$$

$$\frac{d\theta}{dt} = R_\theta,$$

$$\frac{ds}{dt} = R_s,$$

$$\frac{1}{C_s^2}\frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial}{\partial \eta}\Big(G^{13}U + G^{23}V + \frac{1}{G^{1/2}}W\Big) = \frac{\partial}{\partial t}\Big(\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}}\Big).$$

$U = \bar{\rho}G^{1/2}u$, $V = \bar{\rho}G^{1/2}v$, $W = \bar{\rho}G^{1/2}w$, $P = G^{1/2}p'$, where p', θ' are deviations from the basic state pressure $\bar{p}$ and potential temperature $\bar{\theta}$, s is specific humidity, C_s is the sound wave speed, u_g, v_g are the components of geostrophic wind representing the synoptic part of the pressure, η is a terrain-following coordinate transformation:

$$\eta = \frac{H(z - z_s)}{(H - z_s)},$$

z_s is the surface height, H is the height of the top of the model domain. Here $H = \text{const}$,

$$G^{1/2} = 1 - \frac{z_s}{H}, \quad G^{13} = \frac{1}{G^{1/2}}\Big(\frac{\eta}{H} - 1\Big)\frac{\partial z_s}{\partial x}, \quad G^{23} = \frac{1}{G^{1/2}}\Big(\frac{\eta}{H} - 1\Big)\frac{\partial z_s}{\partial y}.$$

In the above equations we use the following notation: for an arbitrary function φ

$$\frac{d\varphi}{dt} = \frac{\partial\varphi}{\partial t} + \frac{\partial u\varphi}{\partial x} + \frac{\partial v\varphi}{\partial y} + \frac{\partial\omega\varphi}{\partial\eta} = \frac{\partial\varphi}{\partial t} + \mathrm{ADV}\varphi,$$

where

$$\omega = \frac{1}{G^{1/2}} w + G^{13}u + G^{23}v.$$

The terms R_u, R_v, R_ω, R_θ, R_s refer to subgrid-scale processes. As the turbulence parameterization of the model, we use a simple scheme:

$$K_m = \begin{cases} l^2\sqrt{\frac{1}{2}D^2(1-\mathrm{Ri})}, & \mathrm{Ri} < 1, \\ 0, & \mathrm{Ri} \geq 1, \end{cases} \tag{1}$$

$$\mathrm{Ri} = \frac{g(d\ln\theta/dz)}{D^2/2}, \quad D = \nabla\underline{u} + \underline{u}\nabla,$$

where K_m is the vertical exchange coefficient, Ri is the local Richardson number, l is the Blackadar mixing length [2]. A more detailed description of the model and the numerical algorithms used can be found in [4, 5]. In this paper, the emphasis is put on the discretization of the advective terms only. This is described in the next section.

3. The algorithm

We approximate the advective terms in the model described above by the following difference operators:

$$\begin{aligned}
\mathrm{ADVU} &= \delta_x(u^x(\rho^x u^x)) + \delta_y(v^x(\rho^x u)^y) + \delta_\eta(\omega^x(\rho^x u)^\eta) \\
\mathrm{ADVV} &= \delta_x(u^y(\rho^y v)^x) + \delta_y(v^y(\rho^y v)^y) + \delta_\eta(\omega^y(\rho^y v)^\eta) \\
\mathrm{ADVW} &= \delta_x(u^\eta(\rho^\eta w)^x) + \delta_y(v^\eta(v(\rho^\eta w)^y) + \delta_\eta(\omega^\eta(\rho^\eta w)^\eta) \\
\mathrm{ADV}\theta &= \delta_x(u(\rho\theta)^x) + \delta_y(v(\rho\theta)^y) + \delta_\eta(\omega(\rho\theta)^\eta),
\end{aligned}$$

where

$$\begin{aligned}
\delta_d\varphi &= [\varphi(d+\Delta d/2) - \varphi(d-\Delta/2)]/\Delta d, \\
\varphi^d &= [\varphi(d+\Delta d/2) + \varphi(d-\Delta d/2)]/2,
\end{aligned}$$

and d could be any of the independent variables.

As in [3], it can be readily shown that these forms are exactly conservative with respect to the first moments..

4. The Monte Carlo model

A simple Monte Carlo model described in [6] was chosen to perform transport and diffusion simulations. This model is especially attractive because of its mathematical simplicity and flexibility. We describe it briefly as follows:

At time $t + \Delta t$ in the terrain-following coordinate system of Section 2 individual particles are located at

$$x_i(t + \Delta t) = x_i(t) + U_i(t)\Delta t.$$

The particle velocity U_i is divided into a mean velocity u_i, simulated by the model described in Section 2, and a turbulent component u'_i

$$U_i = u_i + u'_i.$$

The following algorithm is used:

$$\begin{aligned} u'_i(t + \Delta t) &= R_{L_i}(\Delta t)u'_i(t) + (1 - R_{L_i}(\Delta t)^2)^{1/2}\sigma'_{u_i}\Psi, \\ R_{L_i}(\Delta t) &= \exp(-\Delta t/T_{L_i}), \end{aligned}$$

where T_{L_i} and R_{L_i} are Lagrangian time scales and autocorrelations, Ψ is a random number generator. The velocity variances are

$$\sigma'_u = (2m_1E)^{1/2}, \qquad \sigma'_v = (2m_2E)^{1/2}, \qquad \sigma'_w = (2m_3E)^{1/2}.$$

For three types of stratification

$$\frac{\partial\theta}{\partial z} \le -0.5 \text{ K}/100 \text{ m}, \qquad \left|\frac{\partial\theta}{\partial z}\right| < 0.5 \text{ K}/100 \text{ m}, \qquad \frac{\partial\theta}{\partial z} \ge 0.5 \text{ K}/100 \text{ m},$$

we use the following coefficients $m_1 = 0.40, 0.54, 0.54$; $m_2 = 0.30, 0.30, 0.37$; $m_3 = 0.30, 0.16, 0.09$; $T_{L_i} = K(\sigma'_{u_i})^{-2}$.

The turbulent kinetic energy E is obtained from $K = l\sqrt{cE}$. Here c is an empirical constant commonly chosen as $c = 0.2$ [6].

5. Pollutant transport over a hill

In this section we give some results of a calculation for the transport of a passive substance over a steep hill.

A bell-shaped hill with a height of 500 m is located at the center of a 10 km × 10 km domain. The top of the domain is at 5 km. A geostrophic flow goes from the west, with $u_g = 5$ m/s, $v_g = 0$.

As the basic state, a standard atmospheric stratification $\frac{d\theta}{dz} = 3.5$ K/km is assumed. An absorbing layer is located above a height of approximately 1500 m. The computational grid consists of $31\times31\times16$ points, the horizontal grid size is $\Delta x = \Delta y = 333$ m, the vertical grid size Δz is variable, increasing with height. The hill is slowly inflated during the first 15 minutes of the computation. A tracer of 5000 particles was released east of the hill.

Figures 1 and 2 show a successive motion of the pollutant over the hill. They represent x–z cross-sections over the center of the hill at 12 and 24 min. Figures 3 and 4 show the distributions of the substance at two successive cross-sections further north. The pollutant flow is shifted to the north despite the symmetry of the initial conditions. This is in accordance with the existing theory, which describes the flow as essentially close to the Ekman spiral [7].

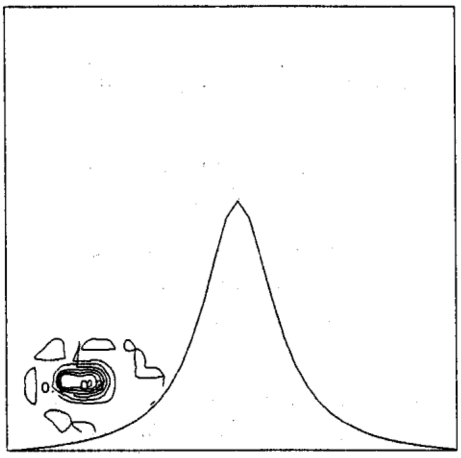

Figure 1

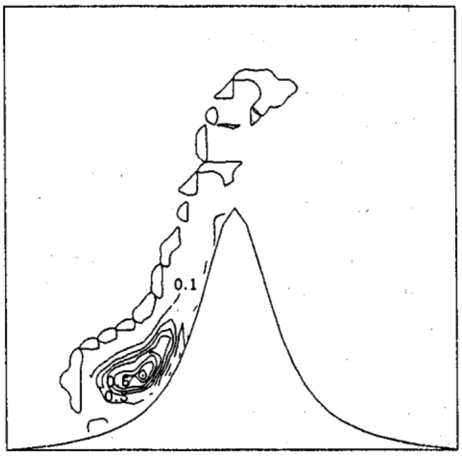

Figure 2

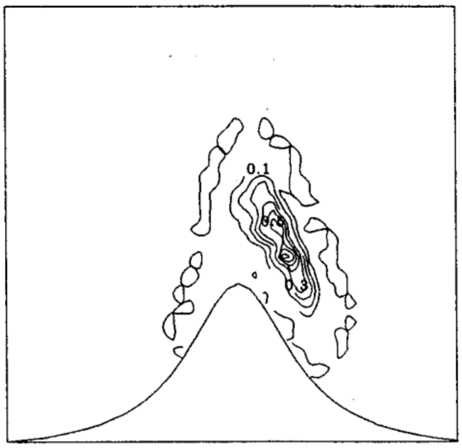

Figure 3

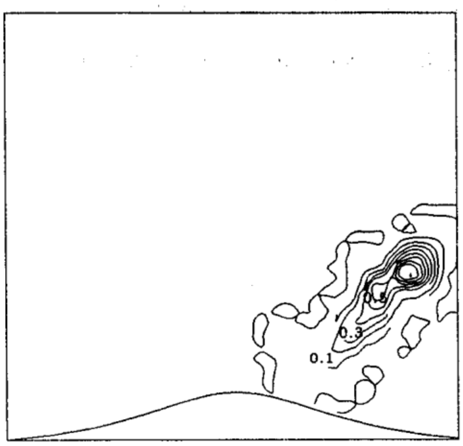

Figure 4

6. Conclusions

The results of the test simulation seem to testify that the algorithm proposed can be used as a helpful tool in numerical modeling of pollutant transport over a steep terrain.

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